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INSTITUTE OF ACTUARIES.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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CONTENTS OF VOL. XLVIII.

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Approximate Valuation of Endowment Assurances. By W. PALIN ELDERTON, F.I.A., *Actuary and Manager of the Equitable Life Assurance Society.*

[Read before the Institute, 24 November 1913.]

I.—INTRODUCTORY.

IN the discussion that followed the reading of a paper before the Institute of Actuaries Students' Society, by E. A. Woodall on the valuation of special policies, F. Bedford explained that certain classes, such as joint life endowment assurances, could be valued conveniently by assuming that all the policies will mature at the same age, and calculating the net premium on the same assumption. This method is mentioned in the Students' *Journal*, vol. I, No. 2, pp. 24–26, where Mr. Woodall, in giving the method in the abstract of his paper and the discussion, explains how the method applies to certain classes, and writes: "Since the reserve for an endowment assurance maturing at "70 or 75 is practically the same as the reserve for an endowment "assurance of the same term maturing at 50, 55, 60, or 65, the "error of 10 years in the equivalent age is of little importance, "provided the net premium is calculated on the same assumptions." This quotation may be taken as the text for the present paper,

which discusses the application of the method to ordinary endowment assurances and the errors involved in its use.

The method was brought before the Institute in a slightly different form by Mr. H. Archer Thomson in his paper "On "the Treatment of Endowment Assurance Policies in Periodical "Valuations" (See *J.I.A.*, xxxiv, pp. 11–21, 48 and 49), where the grouping of Endowment Assurances about two or more ages at maturity was described and the use of one age at maturity was mentioned as "a check." The method does not appear to have attracted much attention in the discussion, and it would be interesting to know whether it has been used in practice, and if so, with what results.*

Before turning to the details of the paper I must add that certain preliminary work on endowment assurance valuations was done for me by F. Bedford, while at a later stage G. D. Millward and C. Carpmael have helped me in many ways; some of the test valuations being due almost entirely to the latter. Although I am actually responsible for the paper, and the opinions expressed in it, so much of the work has been done for me by others, that I almost feel that the paper is only mine by name.

II.—THEORY.

The underlying theory of the method is that ${}_{n}V_{x\overline{t}|}$ is approximately constant for any particular values of t and n regardless of the values of x, so that if an endowment assurance for 20 years effected at age 25 is valued exactly as if it had been an endowment assurance for 20 years (the same term) effected at 35 no great error will result. The first nine columns of Table I afford sufficient evidence of this, and a comparison of the values for policies maturing at earlier ages with those of policies maturing at 80 shows that even in this extreme case the errors tend to balance owing to the values at 80 being smaller for the policies of shorter terms and larger for policies of longer terms. A perusal of the table will also show that if all policies are valued for their correct terms as if they matured at, say, 55, the error in a total valuation could not be very large.

* Since the paper was read the author's attention has been drawn to the fact that in the form in which it is now submitted the method was first suggested by Mr. Thomson—for policies maturing at ages 50 to 65—in the discussion on Mr. Lidstone's "Some Remarks on the Valuation of Endowment Assurances in Groups." See *J.I.A.*, xxxiv, 91.

TABLE I.

Values of Endowment Assurances. O^{M} 3 per-cent.

No. of Years in force	At 40	At 45	At 50	At 55	At 60	At 65	At 70	At 75	At 80	Adjusted Value (See Sec. IV)	Value of $3\frac{1}{4}$ % Sinking Fund Policy
					10–years Endowment Assurance.						
1	8·52	8·47	8·41	8·34	8·25	8·12	7·93	7·64	7·26	8·34	8·62
5	45·59	45·42	45·21	44·94	44·54	43·93	43·02	41·63	39·58	44·97	46·00
					15–years Endowment Assurance.						
1	5·24	5·21	5·17	5·13	5·10	5·07	5·03	4·99	4·95	5·13	5·28
5	27·91	27·76	27·59	27·41	27·21	26·95	26·59	26·09	25·45	27·42	28·17
10	60·78	60·57	60·33	60·03	59·63	59·05	58·17	56·86	54·96	60·06	61·22
					20–years Endowment Assurance.						
1	3·63	3·62	3·60	3·58	3·57	3·59	3·63	3·71	3·84	3·58	3·63
5	19·29	19·23	19·11	19·00	18·93	18·93	19·00	19·15	19·47	19·02	19·36
10	41·82	41·65	41·43	41·20	40·99	40·78	40·53	40·24	39·97	41·22	42·07
15	68·34	68·15	67·91	67·63	67·28	66·80	66·11	65·12	63·73	67·65	68·74
					25–years Endowment Assurance.						
1	2·67	2·68	2·69	2·68	2·68	2·71	2·78	2·91	3·11	2·69	2·65
5	14·21	14·23	14·21	14·15	14·15	14·26	14·54	15·03	15·81	14·18	14·16
10	30·76	30·72	30·61	30·46	30·40	30·49	30·77	31·30	32·20	30·50	30·77
15	50·09	49·96	49·75	49·53	49·34	49·22	49·18	49·22	49·45	49·56	50·27
20	72·84	72·69	72·47	72·21	71·91	71·54	71·04	70·36	69·46	72·24	73·13
					30–years Endowment Assurance.						
1	2·04	2·06	2·08	2·10	2·11	2·14	2·21	2·34	2·57	2·10	2·01
5	10·85	10·94	11·01	11·04	11·08	11·21	11·55	12·17	13·13	11·05	10·77
10	23·51	23·62	23·66	23·63	23·66	23·87	24·41	25·37	26·86	23·66	23·41
15	38·27	38·30	38·25	38·14	38·11	38·28	38·77	39·66	41·10	38·18	38·25
20	55·50	55·43	55·29	55·10	54·95	54·92	55·05	55·40	56·09	55·13	55·64
25	75·79	75·67	75·50	75·28	75·02	74·73	74·38	73·97	73·47	75·31	76·07
					35–years Endowment Assurance.						
1	...	1·61	1·64	1·68	1·71	1·74	1·81	1·93	2·13	1·68	1·57
5	...	8·59	8·73	8·85	8·96	9·10	9·42	10·01	10·97	8·84	8·41
10	...	18·59	18·77	18·92	19·04	19·30	19·88	20·96	22·66	18·92	18·27
15	...	30·18	30·32	30·39	30·50	30·80	31·53	32·84	34·89	30·42	29·85
20	...	43·60	43·63	43·62	43·66	43·90	44·54	45·70	47·56	43·65	43·42
25	...	59·26	59·19	59·07	58·99	59·02	59·28	59·87	60·91	59·10	59·36
30	...	77·76	77·64	77·46	77·26	77·03	76·80	76·57	76·38	77·49	78·06
					40–years Endowment Assurance.						
1	...	...	1·32	1·36	1·40	1·45	1·51	1·61	1·79	1·36	1·25
5	...	...	7·00	7·18	7·37	7·57	7·86	8·37	9·23	7·19	6·68
10	...	...	15·11	15·39	15·66	15·99	16·54	17·54	19·19	15·39	14·52
15	...	...	24·46	24·74	25·00	25·41	26·17	27·57	29·80	24·74	23·73
20	...	...	35·19	35·39	35·61	36·04	36·19	38·46	40·90	35·42	34·52
25	...	...	47·58	47·66	47·81	48·15	48·89	50·24	52·40	47·70	47·20
30	...	...	62·04	62·01	62·01	62·12	62·48	63·22	64·52	62·04	62·06
35	...	...	79·20	79·08	78·93	78·77	78·62	78·53	78·56	79·11	79·51

III.—Preliminary Tests of Method of Sec. II.

In view of the possibility of good results being obtained, some practical trials were made and groups were formed roughly for this purpose from the Endowment Assurance Model Office (J. Buchanan, D.Sc., *J.I.A.*, vol. xli, pp. 18 *et seq.*). These groups were made up of policies effected at ages 15, 20, 25, 30, 35, 40, 45, 50, 55, and 60, maturing at ages 40, 45, 50, 55, 60, 65, and 70, and were arranged so that reserves were only required at the end of 5, 10, 15. 20, 25, 30, and 35 years. This saved work, and is a sufficiently accurate test. The groups were valued at O^M 3 per-cent on the assumption that all policies were non-participating and matured at 55. The total error was £138 (in defect) on a total value of just over £300,000, but when bonuses were valued the error in the value of the bonuses alone was about 7 per mille of the value of the bonuses, or rather over 1 per mille of the total reserve in defect. This and other investigations brought out the fact that, so far as the O^M Table is concerned, while 55 is the best age for non-profit business it is less satisfactory for with-profit business, because the largest bonuses are attached to policies maturing at the later ages. It would not be difficult to adjust the bonuses as the true average age would change very little from year to year, and if we assume the average maturity age to be 55 when it ought to be 60, we know that the understatement of the value of the bonuses is about 1 per-cent of the amount of the bonuses or ·7 per-cent of their value. The former is the easier adjustment. The adjustment would have to be borne in mind when estimating the cost of new bonus, &c.

An alternative would be to value at 60, but the policy-values are as a whole somewhat lower than those at 55, so that in some extreme cases where all policies were effected for short terms at comparatively early ages at maturity, the error might be 6 or 7 per mille in defect, and the bonuses 7 per mille in excess, or the whole valuation about 4 per mille in defect. Here again it would not be difficult to form a table to enable adjustments to be made, but, of course, an actuary would consider the circumstances of the particular office when deciding which average age at maturity should be chosen.

It would be unsatisfactory to assume a higher age than 60, not only because of the error in valuing the bonuses, but also because the net premiums would be overstated and the loadings diminished to too great an extent.

While my personal opinion is that in view of the saving of labour and simplification of the valuation it would be justifiable to value all policies as if they matured at 55 or 60 according to the circumstances of the particular office, I thought that it was worth while to make an attempt to overcome these little difficulties, and the next section shows a way in which this can be done.

IV.—Alternative Method.

To overcome the difficulties indicated in the previous section, it is necessary to have a method of valuation that gives policy-values approximately equal to those for policies maturing at 55; but also gives assurance-values a little more on the safe side, say, roughly, equal to those for policies maturing at 60. It is not difficult to arrange this, as policy-values are unaffected if the annuities-due are all increased or decreased in a constant ratio. If, therefore, the annuity-values for policies maturing at 55 ($\mathbf{a}_{55-n:\overline{n}}$) are decreased by 1·8 per-cent, the resulting annuity-values are slightly more than those for policies maturing at 60 ($\mathbf{a}_{60-n:\overline{n}}$), and the assurance-values slightly less. As Table I showed that the policy-values for terms of 20 years and under were a little low, the annuity-values were further decreased for 20 years and under by a ratio of one quarter per mille for the 20-year term to 1 per mille at the 5-year term. The annuity-values, assurance-values, and net premiums are given in Table A at the end of the paper, and the policy-values are shown in Table I, where they can be compared easily with the policy-values at 55, 60, and other ages at maturity. It seemed possible that there might be some slight distortion in the policy-values for the last few years of any term, and in order to test this Table II was prepared. The values in that table show that the policy-values are good average values.

Table II.

Values of Endowment Assurances. O^M 3 per-cent.

No. of years to run	10-years term				20-years term				30-years term			
	At 40	At 55	At 70	Adjusted Value (*See* Sec. IV)	At 40	At 55	At 70	Adjusted Value (*See* Sec. IV)	At 40	At 55	At 70	Adjusted Value (*See* Sec. IV)
1	88·29	87·96	86·81	87·73	93·19	92·92	92·16	92·79	94·79	94·59	94·07	94·50
2	77·02	76·48	74·68	76·29	86·63	86·17	84·94	86·06	89·77	89·44	88·62	89·36
3	66·16	65·51	63·43	65·43	80·31	79·72	78·26	79·68	84·94	84·51	83·56	84·49
4	55·69	55·01	52·92	55·03	74·22	73·55	72·00	73·57	80·28	79·80	78·83	79·82

The work of the valuation would be as follows for an O^{M} 3 per-cent valuation :

(1) Group policies according to unexpired term.

(2) Calculate net premiums from Table A.

(3) Value sums assured and premiums by the same table.

In other words the valuation of endowment assurances becomes as short and simple as if they were whole life assurances.

V.—VALUATION RESULTS.

The method just described was used for the valuation of the groups adopted for the preliminary trial, and Table III shows the result. The values, if all policies are valued as if they mature at 60 or 55, are also shown. The reversionary bonus was calculated at £1 per-cent for each year the policy was in force.

The errors are small throughout the table and are negligible so far as the method of Section IV is concerned. The valuation as at 55 understates the value of the bonuses by 7·34 per mille.

A second trial was made with groups in which the policies matured on the average at between 50 and 55, and were for short durations, about 50 per-cent being for 20 years. Judging from Table I it seemed probable that such a distribution would put a severe strain on the method ; Table IV gives the result. The objection to using age 60 in this case is that the net premiums would be overstated by about 5 per-cent and the loadings would be considerably reduced ; to a lesser extent this objection applies to the method of Section IV.

These results do not show so close an agreement. In the case of non-profit business the error would be 2 per mille in defect if the method of Section IV were used, and the errors for "age 60" would be over 6 per mille. The bonuses are naturally over-valued and in with-profit cases this reduces the total error. The error in the value of the bonuses is considerable, but is on the safe side from the office point of view ; the only caution necessary being that if reversionary bonuses are surrendered for cash, the cash values should be calculated accurately or by using a lower maturity age (say 50).

TABLE III.

Unexpired Term	Sum Assured	Reversionary Bonuses	Exact Valuation			Method of Sec. IV			As at Age 60			As at Age 55		
			Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together
5	161,500	29,250	111,335	25,388	136,723	111,672	25,428	137,100	111,114	25,398	136,512	111,611	25,354	136,965
10	192,000	26,175	86,217	19,942	106,159	86,366	19,936	106,302	85,980	19,958	105,938	86,320	19,843	106,163
15	195,000	20,825	57,780	14,021	71,801	57,731	14,012	71,743	57,595	14,032	71,627	57,670	13,884	71,554
20	145,500	13,025	30,472	7,785	38,257	30,359	7,763	38,122	30,372	7,784	38,156	30,318	7,667	37,985
25	91,000	6,800	13,515	3,624	17,139	13,392	3,599	16,991	13,465	3,613	17,078	13,386	3,541	16,927
30	36,000	2,300	3,930	1,102	5,032	3,837	1,083	4,920	3,896	1,088	4,984	3,840	1,061	4,901
35	10,000	500	752	217	969	719	210	929	737	211	948	718	204	922
Total	831,000	98,875	304,001	72,079	376,080	304,076	72,031	376,107	303,159	72,084	375,243	303,863	71,554	375,417
Error App. − True	...	...	...	...	...	+ 75	− 48	+ 27	− 842	+ 5	− 837	− 138	− 525	− 663
Per mille	...	...	...	...	...	+ 0·25	− 0·67	+ 0·07	− 2·78	+ 0·07	− 2·23	− 0·45	− 7·43	− 1·77

TABLE IV.

Un-expired Term	Sum Assured	Rever-sionary Bonuses	Exact Valuation			Method of Sec. IV			As at Age 60			As at Age 55		
			Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together
5	290,400	50,470	202,142	43,731	245,873	201,794	43,875	245,669	200,699	43,823	244,522	201,150	43,747	244,897
10	328,700	40,740	146,806	30,842	177,648	146,413	31,065	177,478	145,693	31,064	176,757	146,330	30,885	177,215
15	353,500	27,305	87,433	18,150	105,583	87,203	18,372	105,575	86,888	18,398	105,286	87,096	18,204	105,300
20	153,100	10,790	27,673	6,326	33,999	27,641	6,431	34,072	27,627	6,448	34,075	27,608	6,351	33,959
25	71,300	3,565	7,866	1,849	9,715	7,879	1,887	9,766	7,900	1,894	9,794	7,873	1,857	9,730
Total	1,197,000	132,870	471,920	100,898	572,818	470,930	101,630	572,560	468,807	101,627	570,434	470,057	101,044	571,101
Error { App. — True }	...	...	...	...	...	− 990	+ 732	− 258	−3,113	+ 729	−2,384	−1,863	+ 146	−1,717
Per mille	...	...	...	...	...	−2·10	+7·22	−0·45	− 6·64	+7·21	− 4·18	− 3·97	+1·45	− 3·00

Another example was taken with data more like those of Table III, and the following figures resulted:

TABLE V.

Un-expired Term	Sum Assured	Reversionary Bonuses	Exact Valuation			Method of Sec. IV			As at Age 60			As at Age 55		
			Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together
5	171,500	31,024	118,077	26,845	144,922	118,521	26,959	145,480	117,925	26,928	144,853	118,475	26,892	145,367
10	192,800	25,490	85,573	19,434	105,007	85,471	19,438	104,909	85,276	19,436	104,712	85,627	19,324	104,951
15	183,000	18,000	51,873	12,096	63,969	51,840	12,111	63,951	51,687	12,128	63,815	51,774	12,001	63,775
20	124,500	9,550	25,095	5,731	30,826	24,977	5,692	30,669	24,978	5,707	30,685	24,942	5,621	30,563
25	77,000	5,150	10,511	2,713	13,224	10,473	2,725	13,198	10,521	2,736	13,257	10,468	2,682	13,150
30	29,000	1,650	2,878	784	3,662	2,828	777	3,605	2,866	780	3,646	2,799	761	3,560
35	6,000	300	446	126	572	432	126	558	442	126	568	431	123	554
Total	783,800	91,164	294,453	67,729	362,182	294,542	67,828	362,370	293,695	67,841	361,536	294,516	67,404	361,920
Error (App.—True)	...	...	...	...	...	+ 89	+ 99	+ 188	− 758	+ 112	− 646	+ 63	− 325	− 262
Per mille	...	...	...	...	...	+ 0·30	+ 1·46	+ 0·52	− 2·58	+ 1·65	− 1·79	+ 0·21	− 4·85	− 0·73

TABLE VI.

Unexpired Term	Sum Assured	Reversionary Bonuses	Exact Valuation			Method of Sec. IV			As at Age 60			As at Age 55		
			Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together	Policies	Bonuses	Together
5	60,000	7,200	36,606	6,250	42,856	36,718	6,259	42,977	36,480	6,251	42,731	36,702	6,241	42,943
10	42,000	4,200	17,670	3,199	20,869	17,693	3,205	20,898	17,587	3,203	20,790	17,684	3,184	20,868
15	28,000	1,800	6,263	1,209	7,472	6,244	1,211	7,455	6,218	1,213	7,431	6,237	1,200	7,437
20	8,000	400	1,141	238	1,379	1,134	238	1,372	1,132	239	1,371	1,132	235	1,367
Total	138,00	13,600	61,680	10,896	72,576	61,789	10,913	72,702	61,417	10,906	72,323	61,755	10,860	72,615
Error { App. − True }	...	...	...	...	...	+ 109	+ 17	+ 126	− 263	+ 10	− 253	+ 75	− 36	+ 39
Per mille	...	...	...	...	...	+ 1·76	+ 1·56	+ 1·73	− 4·28	+ ·91	− 3·50	+ 1·21	− 3·31	+ 0·54

Table VI gives the result of a trial on policies of short durations maturing only at 40, 55, and 70, the number maturing at 55 being double the number at each of the other ages.

Further checks were made by adding the columns in Table I, &c., and as the results confirmed those given above, it is unnecessary to give further details, but it is perhaps advisable to refer to two other points connected with the approximate methods, namely, the maximum errors that might arise, and the effect of a large number of reductions of premium. The maximum error can be seen from Table I, and it will be noticed that the introduction of a £10,000 case maturing at 75 or 80, might mean an error of £500 in excess in the valuation if the policy were for a short term, or about £500 in defect if the policy were for a very long term—this means an error of 10 per-cent on the individual reserve, but the effect on the total valuation would be unimportant unless many such policies were introduced, and an inspection of Table I shows that it is unlikely that all these exceptional policies would show a large error in the same direction at any valuation. The error in valuing reductions of premium will be in the opposite direction to the error in valuing reversionary bonuses, and any office having a large number of reductions of premium would probably use values at 55 instead of 60 or assume higher annuity-values if the method of Section IV were adopted.

With the help of the examples given in this section a rough idea can be formed of the error in a valuation, but, if desired, a more accurate estimate of the error could be made in a few hours by valuing groups based on the new business and the business in force in the same way as that adopted in Tables III to VI.

VI.—Advantages of the Methods.

It will naturally be asked why it is suggested that another method of valuing endowment assurances should be discussed when the accurate year of birth method is available, and an approximation so close as that invented by Mr. Lidstone can be used if policies are grouped according to unexpired term. The answer is that Mr. Lidstone's method involves the calculation of the Z constant, and at the time when an actuarial staff is most pressed necessitates the calculation of average maturity ages and annuity and assurance-values. The year of birth method is simpler than the Z method when the actual valuation is made, as all the multiplication factors can be got

ready beforehand, but entails more work in the course of each valuation year.

The advantages of the suggested method are that:

(1) There are no constants.

(2) No average age has to be found when a valuation is being made.

(3) All the valuation factors are known beforehand.

In other words the method saves work throughout the year, and more important still, at the time when the valuation has to be made. The method is accurate and simple in application, and can be understood and worked as easily as the methods used for whole life assurances.

VII.—Objections to the Methods.

The objections to the method seem to be that:

(1) The expected mortality cannot be accurately estimated.

(2) The loadings may be distorted.

(3) Changes in the incidence of new business might mean that the valuation became less accurate.

(4) The method in Section IV is artificial, so that in the event of an alteration in valuation basis the valuation factors and net premiums could not be taken from the tables ordinarily published, but would have to be evolved specially.

(5) It may be necessary to value extreme cases individually.

Taking these objections in order (1) is certainly valid. The expected mortality can, however, be found approximately by using Buchanan's model office and checking the results occasionally or by forming a table giving the number of policies at each maturity age in various durations. Such a table* made once in every five years (say in the middle of the quinquennium) would enable us to make an estimate of mortality which should be fairly accurate. If for any office it is advisable to pay considerable attention to the mortality profit from endowment assurances as compared with other classes, such an office should, I think, use a year of birth grouping.

There is, however, another point in connection with the

* A table of the kind was given in "Frequency Curves and Correlation," p. 107.

profit from mortality that is perhaps worth mentioning. For analysing profits is it not right to work throughout as if the assumed valuation factors were the true factors? Applying this to the particular case we have assumed in making the valuation a certain annuity-value for each duration, and consequently we have assumed (or expected) the rates of mortality at each duration which give those annuity-values.* If there is any other mortality, and it is almost certain there will be, then a profit or loss from mortality must be shown unless an item of "error in valuation basis" is brought into analysis. If this view is correct the mortality difficulty disappears to some extent.

Objection (2) is of importance if there are so many extreme cases that the total of the suggested net premiums is very far from the total of those that would have been inserted if an exact valuation had been made. This will seldom happen, but it must be borne in mind that individual cases will have net premiums very wide of the mark and loadings in individual cases may be meaningless, although the totals will be reasonable. The analogous difficulty in the Z method is that individual policy-values are wide of the mark, although the value of the group is accurate. The loading difficulty would, I think, make the method inconvenient if bonuses were given by a contribution method depending on loadings (*e.g.*, Australian Mutual Provident Method and T. G. C. Browne's method).

Table I supplies the answer to objection (3): it is very hard to choose any possible series of cases showing considerable error in the total valuation.

Objection (4) is important. It does not appear, however, to be essential whether we reach our approximation to the truth by adjusting the annuity-values and all other functions *ab initio*, or whether we leave them unadjusted until we come to the final step in the valuation when we decide on the average annuity which is required to give an accurate valuation. In fact, from one point of view the suggested method is less artificial as it gives the value of each individual policy with fair accuracy, while as indicated above methods which use an average annuity-value with the true net premiums give policy-values which may be considerably in error in the individual cases of whch the group is composed.

* The rates of mortality giving the annuity-values of Table A are shown in the last column of that Table.

The objection should, however, be urged on a different ground, as the apparent artificiality of the method might mean that it could not be used with all tables of mortality. The answer is that with some tables we should not want it because if the values of policies maturing at 60 give satisfactory results for the valuation no modification of these values would be necessary, while, if modification is required, I do not think there would be any real difficulty in making it. If the method happened to be adopted in practice such points could be investigated when any new temporary annuity tables were calculated, and if adjustment proved necessary, suitable valuation annuity-values, &c., could be calculated and published. It is not difficult to see that an adjustment of values is generally possible because the policy-values remain unaltered if a is increased or decreased in a constant ratio; the use of a constant ratio will keep the reserves the same for the policies, but alter the reserves for the bonuses, while the use of a table of annuities altered with a varying ratio will modify the reserves entirely. A ratio decreasing with the term of the annuity will increase the reserves for policies, and a ratio increasing as the term decreases will decrease the reserves for policies.

The last objection, which holds with nearly every approximate method of valuation, is relatively unimportant. In view of the values shown in Table I, we should, I think, be justified in including every case and making no special allowance for the few extreme cases that arise in practice.

VIII.—APPLICATION OF METHOD TO AN OFFICE WHICH HAS ALREADY CALCULATED ITS NET PREMIUMS.

At the present time insurance offices have adopted the tables they wish to use, calculated the net premiums, and arranged their class books, &c., so that it is not worth while to calculate net premiums afresh even if the methods now suggested are considered of practical value. The labour-saving part of the method can, however, be introduced by re-making the last valuation at such a maturity age, M, that the new valuation gives the same result as the old. All new policies will be valued at M with net premiums calculated at that maturity age and not at the true maturity ages. It would be necessary to calculate extended tables for maturity age M similar to Tables B and C for maturity ages 55 and 60.

By this arrangement it would be possible to avoid the calculation of average ages, annuity-values, &c., in future, and, in fact, to simplify the arithmetical work of the valuation in the way already described. It would be advisable to alter the net premiums in extreme cases, *e.g.*, policies maturing at ages over 70 or under 45.

IX.—Valuation of Endowment Assurances without Grouping.

Having found that endowment assurances could be valued in last payment groups without a constant, it occurred to me that possibly they could be valued without any grouping by assuming that the O^M 3 per-cent value of an endowment assurance was approximately equal to the value at $3\frac{1}{4}$ per-cent of a $3\frac{1}{4}$ per-cent sinking fund policy (see last column of Table I). This was tried and, using the groups leading up to Table III, an error of approximately one per mille in excess resulted. Bonuses would have to be valued at $2\frac{3}{4}$ per-cent or $2\frac{5}{8}$ per-cent.

By means of constants the valuation of sums assured and premiums could be given separately, and by additional constants the expected mortality could be estimated without grouping.

It may be added that if worth while modifications of this method could be evolved, such, for instance, as a valuation based on the reserves of a sinking fund policy calculated throughout on the assumption that the rate of interest in the first year of assurance is i and in the nth year $i+(n-i)\delta$ where δ is a small quantity. The policies could still be valued without grouping.

X.—Conclusion.

Our investigation has so far been confined to the valuation of endowment assurances by the O^M Table with 3 per-cent interest, but, as has been indicated in Section VII, there seems no reason why the method should fail if a valuation by another table is required, and a few experiments have tended to confirm this view, and to show that more accurate results could be obtained by a table which more nearly follows Makeham's Law than the O^M.

Although, as pointed out in Section VII, it would in some circumstances be unsatisfactory to use the methods described in Sections I to VIII, they are, I think, of some practical use, and in case others may wish to consider them, Tables B and C giving functions that may be required are appended.

The methods of Section IX are mainly of interest on the theoretical side, but the mention of them may serve one good purpose if, owing to their being different from methods previously employed, it impresses on us the disadvantage of restricting methods of valuation in the way they seem to be restricted by the Assurance Companies Act, 1909, and may also serve to make us thankful that the administration of the Act is, at any rate in valuation matters, far more satisfactory than the Act itself.

TABLE A.

Table of Functions required in connection with method of Section IV.

O^M 3 per-cent.

Un-expired Term	a	A	π	q	Un-expired Term	a	A	π	q
1	1·000	·97087	...	...	31	18·541	·46288	·02481	·0056
2	1·933	·94369	...	·0369	32	18·903	·44942	·02377	·0054
3	2·818	·91793	·32579	·0312	33	19·256	·43915	·02280	·0053
4	3·666	·89323	·24365	·0256	34	19·599	·42916	·02189	·0051
5	4·486	·86928	·19378	·0205	35	19·934	·41939	·02104	·0049
6	5·276	·84633	·16041	·0183	36	20·260	·40990	·02023	·0048
7	6·035	·82422	·13658	·0170	37	20·576	·40070	·01948	·0047
8	6·766	·80294	·11867	·0158	38	20·884	·39172	·01876	·0046
9	7·472	·78237	·10470	·0149	39	21·184	·38299	·01808	·0045
10	8·152	·76256	·09354	·0140	40	21·476	·37449	·01744	·0045
11	8·809	·74343	·08439	·0133	41	21·758	·36627	·01683	·0044
12	9·445	·72490	·07676	·0126	42	22·033	·35826	·01626	·0043
13	10·061	·70696	·07027	·0118	43	22·299	·35052	·01572	·0043
14	10·656	·68964	·06472	·0114	44	22·558	·34297	·01520	·0042
15	11·232	·67285	·05990	·0110	45	22·810	·33563	·01471	·0042
16	11·793	·65651	·05567	·0103	46	23·054	·32852	·01425	·0041
17	12·335	·64072	·05194	·0099	47	23·294	·32153	·01380	·0040
18	12·862	·62538	·04862	·0095	48	23·528	·31472	·01337	·0039
19	13·373	·61049	·04565	·0091	49	23·756	·30808	·01297	·0038
20	13·870	·59602	·04297	·0087	50	23·978	·30161	·01258	·0037
21	14·354	·58192	·04054	·0083	51	24·198	·29521	·01220	·0035
22	14·825	·57694	·03832	·0080	52	24·416	·28886	·01183	·0033
23	15·282	·55489	·03631	·0077	53	24·629	·28265	·01147	·0032
24	15·728	·54191	·03446	·0074	54	24·839	·27654	·01115	·0030
25	16·163	·52923	·03274	·0071	55	25·048	·27045	·01079	·0028
26	16·585	·51694	·03117	·0067	56	25·254	·26444	·01047	·0026
27	16·997	·50495	·02971	·0065	57	25·460	·25845	·01015	·0024
28	17·398	·49327	·02836	·0063	58	25·664	·25250	·00983	·0022
29	17·789	·48188	·02709	·0060	59	25·870	·24650	·00953	·0019
30	18·170	·47078	·02591	·0058	60	26·075	·24053	·00924	·0016

TABLE B.

Table of functions for long terms if valuation is made at maturity age 55. O[M] 3 per-cent.

Unexpired Term n	$a_{55-n:n}$	$A_{55-n:\overline{n}}$	$\pi_{55-n:\overline{n}}$	q_{55-n}
46	23·471	·31638	·01348	·0033
47	23·714	·30930	·01304	·0032
48	23·953	·30234	·01262	·0031
49	24·185	·29558	·01222	·0030
50	24·412	·28897	·01184	·0029
51	24·636	·28245	·01146	·0027
52	24·856	·27604	·01111	·0026
53	25·074	·26969	·01075	·0025
54	25·288	·26346	·01041	·0023
55	25·501	·25725	·01009	·0021
56	25·711	·25114	·00977	·0019
57	25·919	·24508	·00946	·0017
58	26·128	·23899	·00915	·0014
59	26·337	·23291	·00884	·0012
60	26·546	·22682	·00854	·0010

NOTE.—The values of q were obtained by exterpolation below age 10, but were reduced arbitrarily more rapidly so as to make the policy values slightly larger. The resulting values of a are probably a little large, and A and π rather small. The bonuses will, however, always be small on the rare occasions when this table is required.

TABLE C.

Table of functions for long terms if valuation is made at maturity age 60.

Unexpired Term n	$a_{60-n:n}$	$A_{60-n:\overline{n}}$	$\pi_{60-n:\overline{n}}$	q_{60-n}
51	24·280	·29282	·01206	·0033
52	24·497	·28650	·01170	·0032
53	24·709	·28032	·01134	·0031
54	24·918	·27423	·01100	·0030
55	25·122	·26829	·01068	·0029
56	25·324	·26240	·01036	·0027
57	25·522	·25664	·01006	·0026
58	25·716	·25100	·00976	·0025
59	25·909	·24537	·00947	·0023
60	26·102	·23975	·00919	·0021

NOTE.—The values of q are the same as those in Table B. The other functions have been found from these values.

Abstract of the Discussion.

Mr. WILLIAM PENMAN, in opening the discussion, said that comment on the Paper naturally took the form of a comparison between Mr. Lidstone's method and the new method or methods now put forward. Mr. Lidstone's method was so familiar and gave such accurate results with so little labour, that the advocate of any rival method had no easy task. Briefly, Mr. Elderton suggested that the same arbitrary age at maturity should be assumed for all policies, and that the valuation net premiums and factors should be based on the assumed age at maturity. He pointed out that the same arbitrary age was not suitable both for the original policy and the bonus additions, and suggested the construction of a hypothetical table which would give the values desired both for the policies and the bonus additions. In so doing he had obtained a table based on the unexpired term alone. In other words he had eliminated age as a factor in the valuation, and the next step (which he merely indicated, without following the idea up) was to value on an interest basis only—with, if necessary, a varying rate of interest. Disregarding for the moment the further developments, he proposed first of all to confine himself to the proposition that endowment assurances might be valued on the assumption that they all matured at age 55—the net premiums as well as the valuation factors being taken accordingly.

He thought Table I was itself sufficient to show that the method was accurate, and Tables III, V, and VI confirmed the impression conveyed by Table I. Similarly Tables III, V, and VI showed that if age 60 be assumed as the maturity age, the values of the reversionary bonuses were obtained with great accuracy. He had omitted mention of Table IV, to which he did not attach much importance, except as a warning that discrimination was necessary in deciding whether or not the method was applicable to a given case. No doubt many private investigations would be made before the method was generally adopted, but he saw no reason for anticipating that the suggested method would not be found in practice to give results quite as accurate as the method now in general use. Mr. Lidstone's method worked very simply in practice, and no very considerable saving of labour was to be expected by the adoption of the suggested method. On the other hand, as Mr. Elderton had pointed out, there was a saving at the time when the final calculations were being made; such a saving, even if slight, was of considerable value, and the advantage certainly lay with the suggested method.

Table I was worthy of some further attention. An examination of the figures disclosed the fact that if age 55 be selected as the maturity age the values brought out for policies approaching maturity were too large, and those brought out for policies having a long unexpired term were too small. The accurate aggregate result arose, therefore, from a balance of errors, which was also the

case in Mr. Lidstone's method. It was, perhaps, worth calling attention to the fact that the errors in the two methods ran in opposite directions. Thus, Mr. Elderton's suggested method, with a normal distribution of business, gave values in excess of the true amount for policies having short unexpired durations, whilst Mr. Lidstone's method gave values in defect. For policies having long unexpired durations the relative positions were reversed. A comparison of the results given by the two methods group by group might be somewhat misleading, unless that fact was borne in mind, for close agreement was only to be expected in the aggregate result and in those groups having an unexpired term in the neighbourhood of twenty years.

With regard to the question whether the suggested method would come into general use or not, he was inclined to think that it would depend very much on the extent to which the total net premiums brought out differed from the total true net premiums. If the method involved an increase in the net premiums and a consequent reduction in the margin of loading it would be adopted with some reluctance. Mr. Elderton had not given much information on that point, and it would be interesting to know what result he anticipated. If the maturity ages by Mr. Lidstone's method averaged about 58 (as was frequently the case) then it seemed to him that the suggested method of treating all policies as maturing at age 55 would involve a reduction in the total net premiums. If that was correct, such a condition would certainly not be considered a hindrance to the adoption of the new method. The method of employing a hypothetical table did not appeal to him. It seemed clear that practically all the advantages of the system could be secured by valuing the policies as maturing at 55 and the bonus additions as maturing at age 60, which course seemed much to be preferred. The hypothetical table involved larger premiums and a smaller margin of loading, and it would not be readily understood.

The author had touched very briefly indeed on the question of valuations involving interest alone. Personally he would be grateful if Mr. Elderton would give some information as to how he proposed to value endowment assurances without any grouping. It seemed to him that some form of grouping was necessary, whether the policies were valued prospectively or by accumulation of premiums. It might be that he had misunderstood what the author meant in that respect. There were two other minor points to which he would like to refer, although one had no very direct bearing on the Paper itself. Table I, he thought, emphasized the fact that offices which treated rated-up lives at their rated-up ages for valuation purposes were not necessarily strengthening their office reserves so far as endowment assurances were concerned. The other point was that if on the face of it the method was suitable for endowment assurances, it was also very suitable for double-endowment assurances, and any office having a number of policies of that description on its books might find the method advocated in the Paper of use in

that direction, even if they did not see their way to adopt it for endowment assurances.*

Mr. DUNCAN C. FRASER said that he rose not with the object of making any extended remarks upon the very interesting Paper that had been read, but simply to express his agreement with its main proposition, and to put in some illustrative figures. In the valuations for which he had been responsible, he had always attached very great importance to the principle that the net premiums should be so calculated and the valuation ages so taken that the value of every individual policy fell in its correct curtate year of duration; and it had been his practice to use quinquennial ages of maturity for endowment assurances, adjusting the net premium age in each case to suit the maturing age taken. More recently he had found in an experimental valuation that if this method were extended and a single maturing age taken for all policies, a valuation could be obtained which for practical purposes might be treated as exact, the deviation from the result of an exact valuation being £6 per £10,000. He would not refer in detail to the actual figures showing the comparison between the exact and approximate valuations, but would hand them to the author.†

He agreed with the last speaker in regard to the hypothetical table. He objected to anything that involved manipulation of the tabular factors; he preferred to use them as they stood and to apply any necessary corrections to the final result.

Mr. R. E. UNDERWOOD, commenting on the author's suggestion that for the purpose of analyzing profits, it would be right "to work throughout as if the assumed valuation factors were the true factors", said that mortality profit usually fell under one of two headings—true mortality profit and valuation mortality profit. The first would be found by using a select mortality table in conjunction with reserves based on the same rate of interest as that assumed in the calculation of the premiums. As regarded the second—the only one usually calculated—valuation mortality profit was the excess of the expected strain on the valuation basis over the actual strain. One of the main objects in calculating that was to effect an agreement with the valuation surplus, and, unless they worked throughout as if the assumed valuation factors were the true factors, an important object of the investigation would be missed because they would have more than the usual difference between the actual surplus and the estimated surplus produced by analyzing their profits. To take an extreme case, if they experienced unusually heavy mortality on short-term endowment assurances on very young lives, and normal mortality in other respects, the loss from mortality on the young lives would be much heavier, if the actual strain

* [In view of the fact that the reserves—unlike the premiums—for Double Endowment Assurances are materially affected by the age at entry, we have invited Mr. Elderton to refer to this suggestion in his supplementary observations. His remarks on the subject will be found on p. 32.—EDS.]

† [The figures will be found in Mr. Elderton's communication on p. 31.—EDS.]

were reckoned on true ages, than if it were reckoned on the ages assumed in the actual valuation basis. That heavy loss, however, would not be reflected in the valuation itself, and would therefore cause a discrepancy in the surplus. It might be argued that as the new method was only an approximation to a correct valuation they ought to employ the correct factors in calculating their mortality profit; but he would suggest that, as the so-called "mortality profit" was purely relative, and meaningless apart from the valuation assumptions, they must follow those assumptions—to whatever they were an approximation—in order to obtain an intelligible result. If it were desired to compare mortality profits, the comparison could be instituted equally well between profits derived from any such assumptions, provided they were uniform.

The author remarked: "If for any office it is advisable to pay considerable attention to the mortality profit from endowment assurances as compared with other classes, such an office should, I think, use a year of birth grouping." It did not seem to him, however, that much would be gained, from the point of view of mortality, by such a grouping, unless Select Tables were used. Indeed, he thought it might lead them to lay too much stress on the results. The effect of selection on the reserves differed very considerably as between class and class, and particularly between whole-life and endowment assurances. He had tabulated for age at entry 20, and duration 5 years, the Aggregate and Select Reserves, by the O^{M} and $O^{[M]}$ 3 per-cent Tables respectively, for (1) a whole-life policy, (2) an endowment assurance maturing at age 45, and (3) an endowment assurance maturing at age 70, and had calculated the increase per-cent of aggregate reserve required to provide a select reserve. The results were 4·1 per-cent in the case of the whole-life policy, 1·3 per-cent in the case of the endowment assurance maturing at age 45, and 3·7 per-cent in the case of the endowment assurance maturing at age 70. The trend of the results was, of course, to be expected, but he thought the figures illustrated the impossibility of making a proper comparison of mortality profits as between class and class on the basis of aggregate tables.

It was clear, therefore, that the conclusions which they were able to draw from aggregate tables were only of the most general nature, and an office would hardly be justified in changing its method of valuing endowment assurances merely for the purpose of working on true ages, when it continued to work on what he might, for the purpose, describe as "untrue" mortality tables.

Mr. JAMES BACON said that in his Paper Mr. Elderton had asked whether the method of grouping suggested by Mr. Thomson had been used in practice, and had further raised the question whether the modification of that method would be applicable to other tables and rates of interest than the O^{M} Table with 3 per-cent interest. In that connection the results of a valuation for which he (Mr. Bacon) had recently been responsible might be of interest. The data were of quite a different type from those adopted by Mr. Elderton, consisting of Industrial Endowment Assurances maturing at the

six quinquennial ages 45 to 70 inclusive, and the valuation was by the English Life Table No. 3, Males, with interest at $3\frac{1}{4}$ per-cent. At each of the last four quinquennial valuations the policies had been valued as though they matured at ages 60 and 65, and whoever was responsible for the introduction of the method had such faith in it that no records were kept of the true entry and maturing ages, but each policy was recorded as though it matured at one or other of the ages adopted for the valuation, the entry age being suitably modified. The result was that the great bulk of the business, approximately £84,000 of the sums assured, was assumed to mature at age 50, and approximately £5,500 at age 65. These had been re-valued as though they matured at age 55, the age adopted by Mr. Elderton, the ages and net premiums being suitably adjusted, and the result was that a total reserve of £31,695 was reduced to £31,646, a difference of £49 only in defect, or 1·5 per mille. The effect of valuing at the one age was in this particular case, which was something of an extreme case owing to the distribution of the data, to increase the value of the sums assured by 4·6 per mille, and the value of the net premiums by 9·5 per mille, the net result being 1·5 per mille of the net liability in defect, as previously stated. It was, therefore, fairly evident that although the method might possibly not be sufficiently accurate for adoption without modification where a table following a fixed law was used, where the mortality table adopted did not follow any fixed law (as was the case with the English Life Table No. 3) it would probably produce results that were for all practical purposes as good as those produced by what was termed an exact method, and since extended tables of endowment assurance functions were not available for the valuation of industrial assurance policies, and questions of a bonus did not usually arise, the use of the method seemed to be fully justified for such valuations.

Mr. A. LEVINE desired to make two general remarks in reference to the Paper, the first being in relation to the author's statement that the Z method involved extra labour. He thought the extra labour was so small as to be negligible. He had had experience for some years of the annual valuation of a comparatively large endowment assurance business, involving between 15,000 and 20,000 policies, and, using Mr. Coote's valuable little table, and the moveable slide devised by Mr. Brown, he found that the labour of calculating the average maturing age for each group of endowment assurances at the end of the year, and of calculating the proper valuation factors, was very trifling. After all, the great labour incidental to a valuation was not the actual valuation but the grouping. The agreement of the data for valuation took considerably more time than the actual valuation. The second remark he wished to make was, that his great objection to the author's suggested plan was that it took them still further away from the realities of the situation, by using net premiums which had no relation to the office premiums that were charged. In the discussion on Mr. Warner's Paper of some years ago on net premium methods, several members

expressed a desire to bring their net premiums into some relation with the office premiums, and he was not sure that there might not eventually come a time when they would frankly value office premiums, less a certain percentage for loading, in order that they might make their valuation correspond with the actual experience, the actual premiums charged, the actual expense incurred, and the actual interest earned. The author's method took them still further away from actual facts, and as a result it would disturb any analysis of mortality or loading profit that might be made. It would be unsuitable for any system of bonus distribution based on the sources of profit; and, much as he admired its ingenuity, he should hesitate long before adopting it, for the reasons he had stated.

Mr. R. TODHUNTER said that in discussing the merits of a new method of valuation, such as that suggested by the author, the two principal questions for consideration seemed to him to be, first: Was the method sufficiently accurate for practical purposes? and, secondly: Was it better than existing methods if regard be had both to accuracy and to simplicity of application? With respect to the first question, he felt personally little doubt, in view of the illustrations given in the Paper, and of Mr. Fraser's testimony to the satisfactory working of the method in actual practice. It seemed highly improbable that the error could ever be so large as to affect materially either the practical soundness of the valuation or the profit disclosed in an inter-valuation period; in fact, the latter would probably be more accurate than the valuation itself, because the error would change very slowly from one valuation to the next. Moreover, it seemed to him possible that somewhat more accurate results might be obtained by a slightly different method of procedure. The author proceeded on the assumption that the net premiums to be valued must correspond exactly to the assurance and annuity-values used in the valuation; but was that assumption really necessary? As the sole object was to obtain policy-values as near as possible to the true values, while using a single table of assurance and annuity-values, it did not seem to him that there would be any serious objection to using an artificially constructed table of net premiums not agreeing with the assurance and annuity-values, if such a table would in fact give better results.

The author, in effect, made his policy-values correct for duration 0 and duration t, and he left the intermediate values to take care of themselves. It would be admitted that they did it remarkably well, but might they not do it better if the net premiums were so chosen as to give a correct value for some intermediate duration, or on the average, instead of for duration 0? Supposing, for example, that the annuity and assurance-values to be used in the valuation were those for policies maturing at age 60. Then, for policies maturing at other ages, a table of net premiums could be constructed which would give correct policy-values at the end of five years or at the end of $\frac{1}{2}\ t$ years, or alternatively policy-values of which the average would be the same as the average of the true policy-values. The effect of that would be that absolutely correct

policy-values would be obtained for the important group of policies maturing at age 60, and correct values for some intermediate duration —or on the average for all durations—for all other policies. This modification of the author's method would entail the calculation of a table of net premiums for all ages and durations, but the labour involved in calculating such a table would be done once and for all, and would not be very serious. To take a single instance, supposing the valuation to be made by the author's assurance and annuity-values given in Table A, then for a 30-year policy maturing at 70 the net premium required to give policy-values having the same average as the true policy-values would be 2·581 per-cent, as against the author's 2·591. He could not see any necessity for having an average error which might easily be avoided. By using the net premium of 2·591 the author made his average policy-value too high throughout.

With regard to the second and more important question—whether the method was really better than existing methods when account was taken of simplicity of application as well as accuracy—he thought that the Paper conveyed, rather by implication than by any direct statement, an incorrect impression as to the amount of work entailed by the Z method. In that respect he associated himself entirely with what Mr. Levine had said. The extra work involved in writing-up the Z's in the classification was hardly worth considering. After a little experience the Z Table became as familiar as the multiplication table, and in the great majority of cases the values were written up in practice without the pen being lifted from the page; then at valuation time there was a single two or three figure column to be cast, and a few simple divisions to two, or at the most three, figures to be made. For that small amount of extra work the Z method gave them practical certainty of being on the safe side if the maturing ages were taken to the next highest integral age, and it also gave an automatic adjustment from year to year for the varying incidence of the maturing ages. He thought on the whole the balance of advantage rested with the Z method.

Mr. H. E. RAYNES said that a good deal of attention had been directed to the question of the accuracy of the method. He thought that a valuation depended on so many contributory factors which could not be accurately determined that the point of exact mathematical accuracy was comparatively unimportant. It seemed to him that the result of valuation by the method advocated by the author departed so minutely from the rigid accuracy of an individual valuation, that there was no reason why the method should not be used as a proper one for the ordinary quiquennial valuation.

A valuation had two objects in view, namely, to ascertain and compare the financial position of the office from time to time, and to determine the amount of surplus to be divided. For comparative purposes he thought it was advisable to adhere to any particular process of group valuation when once it had been

introduced. At least, it should not be changed unless a valuation on the old basis and one on the new were made as at the same date, so that the results of the two methods could be compared. The small deviation in the reserves by one method of group valuation was, as compared with the rigid accuracy of an individual valuation, always constant and in one direction: but when a new process of group valuation was introduced a slightly different standard was set up, and for comparative purposes it was not such a good instrument as the old. It had occurred to him that under the system before them it would be necessary to exclude endowment assurances by limited premiums and perhaps also single-premium and paid-up endowment assurances; but as to the latter possibly as the method of Section IV was correct for bonuses it would also serve for fully-paid policies.

During the discussion allusion had been made to valuations excluding the element of mortality. Though for ordinary office valuations that might be impossible, yet it could be applied in connection with office calculations for surrender-values. As offices grew the work involved in taking out surrender-values under endowment assurances became very heavy, and he saw no reason why an endowment assurance should not be treated as a sinking fund policy. A percentage of the reserve might be quoted as the surrender-value, provided a slightly higher rate of interest were employed in calculating the reserve. For most terms he thought this course would be quite safe.

Mr. NORMAN BLANCHARD said that reference had been made earlier in the discussion to the author's suggestion that an office desirous of paying special attention to the mortality profit from endowment assurances as compared with other classes, should use a year of birth grouping. He had always been under the impression that the mortality profit and expected claims could be found accurately enough by using the Z method, and treating each maturity-year-group as one policy at the age given by that method. The author's remark led him to test the accuracy of that practice, and he accordingly examined several groups of actual policies with sums assured and bonuses amounting to £602,556, the nearest unexpired terms being 3.8.13.18. &c., up to 38 years. The results showed that the use of the Z method gave what he would consider to be a sufficiently close approximation. It brought out an expected strain of £3,571 against the true figure of £3,552, and expected claims of £8,603 against £8,659. Both estimates were within two-thirds of 1 per-cent of the correct figures, or, to put it in another way, the age obtained by the Z method merely needed to be written down about 1-10th of a year for calculating expected death strain, and written up a similar amount for calculating expected claims. He would not, of course, suggest the general adoption of such a rule-of-thumb without further tests.

Mr. H. J. RIETSCHEL, referring to the point discussed by a preceding speaker, said that a net premium valuation would not enable a comparison to be made of the mortality profit earned

from different classes of business. For example, if an office valued by the O^M Table and made a mortality loss on its non-profit whole-life business, such loss would at the valuation be made good from the profit obtained from, say, the endowment assurances, and both classes would start the new quinquennium with O^M reserves. If on the basis of these reserves the mortality profit or loss in the two classes were compared during the next quinquennium the mortality profit in respect of the whole life non-profit business would be over-estimated, and that in respect of the endowment assurances under-estimated, because the latter class would have contributed to the general reserves a greater amount than was shown by the O^M reserves. Only a valuation which took into account the office premiums and the manner in which those premiums had contributed to build up the reserves would enable an office to obtain a true comparative statement of its mortality profit or loss from the various classes of assurance. The author mentioned that the objections to his proposed method seemed to be that (1) the expected mortality could not be accurately estimated, (2) the loadings might be distorted. He thought that these objections might be urged with even greater force against the net premium method of valuation generally. The net premium method of valuation meant to him only a rough approximation to an office premium valuation, with an additional reserve for future expenses and bonuses, and any analysis of the profit on the basis of the net premium valuation into mortality, loading, and interest profit was misleading. The objections to the suggested new method on the grounds that the expected mortality could not be accurately estimated, and that the loadings might be distorted, were consequently of little importance.

Mr. H. J. BAKER, in closing the discussion, said that the valuation of endowment assurances had been greatly simplified by Mr. Lidstone's discovery of the Z method, and now the author of the Paper went further and described a plan by which, to use his own words, the valuation of endowment assurances became as short and simple as if they were whole-life assurances. The Paper was, therefore, a very practical one and called for most careful consideration. The new method was based upon the theory that the endowment assurances policy-value, while varying with the original term of the endowment and the unexpired term, was approximately independent of the age of the life. An estimate of the accuracy of that assumption could be made by means of Table I. Reference to that table showed that in the case of short-term endowment assurances the O^M 3 per-cent policy-value generally decreased as the age at entry increased, while the reverse was the case for the longer term endowments. In some instances, however (*e.g.*, 25-year endowment assurances—5 years in force), the reserve first decreased with the entry age and then increased. The differences were, however, small, and the error introduced in a valuation by disregarding those changes in the policy-value consequent on a change in the age was insignificant, at least for non-profit business. For example: The sum of all the values in Table I for ages at

maturity 45–75 inclusive, and for endowment terms of 10–35 years inclusive, was 6,237·24. If all the values were taken on the assumption that the maturity age was 55 the total was 6,240·99, a difference in excess of only 3·75 or ·6 per mille. If 60 were taken as the maturity age the difference was 14·03 in defect or 2·25 per mille. These figures also confirmed the author's conclusion that for the O^{M} Table, age 55 was the best maturity age to adopt in using the method for the valuation of non-profit endowment assurances.

To test the applicability of the suggested method to other tables of mortality, he had made a valuation by the $O^{M(5)}$ Table at 3 per-cent of the policies set out in Table III (excluding the bonuses), and the results were as follows: The exact valuation gave a total reserve of 300,747—the valuation as at maturity age 55 required a reserve of 299,821, and as at maturity age 60 a reserve of 300,303. The valuation as at maturity age 55 showed, therefore, a deficit of 926 or 3·1 per mille, and as at maturity age 60 a deficit of 444 or 1·5 per mille. These differences, although small, were larger than those shown in Table III for the O^{M} Table, and, instead of the error being less "as at age 55" than "as at age 60", the reverse was the case for the $O^{M(5)}$ Table. It might, therefore, be found necessary to vary the assumed maturity age if a change were made in the mortality table used in the valuation. In that case the method would be less elastic than the Z method, for he believed it had been found unnecessary in practice to change the Z's on passing from a H^{M} to an O^{M} or $O^{M(5)}$ valuation.

The successful application of the new method depended largely upon the choice of a suitable maturity age, and that age would probably vary considerably according to the peculiar circumstances of individual offices. From the examples given by Mr. Elderton, it would seem probable that the same maturity age could be used in successive valuations so long, at least, as the valuation basis remained unaltered, in spite of changes in the incidence of new business, &c. That was an important point, for if it were found necessary to alter the maturity age the net premiums would also require to be re-calculated, and the chief advantage possessed by the method over the Z method, namely, saving of time and labour, would then disappear. Mr. Elderton alluded to the labour involved under the Z method in calculating the average maturity ages and the assurance and annuity-values, but offices using that method possessed tables which enabled this work to be completed in a very few hours. It was, unfortunately, impossible from the tables in the Paper to form any estimate of the error in the value of the sums assured and the error in value of the net premiums, only the error in the reserve being given. It was, of course, quite possible that although the error in the reserve might be small yet the values of the sums assured and of the net premiums might each differ considerably from the true values. If such were the case, having regard also to the fact that the net premiums were not the true net premiums, he did not think the suggested method should be adopted except for a check valuation or for the purpose of making Foreign and Colonial

Government returns. Perhaps the author could give some information on that very important point.

He had recently made a rough check valuation of the endowment assurance business of two companies by simply taking the Board of Trade returns. assuming all the policies would mature at age 60 and using the actual premiums. In the first case the sums assured and reversionary bonuses amounted to £1,379,248. the accurate valuation of which by the O^{M} 3 per-cent Table was £879,013. and the value of the net premiums £546,494. The rough valuation made by the arithmometer in less than two hours gave the following results :—Value of sums assured and bonuses, £879,981 ; value of net premiums, £552,986 ; reserve, £326,995 ; showing a deficit of £5,524, or 1·7 per cent. of the true reserve. He was not in possession of information enabling him to make any adjustment for special policies.

In the second case the sums assured amounted to £199,348, and the office premiums to £9,633—none of the policies having been in force more than 10 years. A gross premium valuation by the $O^{M(5)}$ Table at 3 per-cent (each policy being valued separately) gave the value of the sums assured as £128,212, and the value of the office premiums as £106,748. The check valuation produced £128,104 as the value of the sums assured, £106,673 as the value of the office premiums, and £21,431 as the reserve. The check valuation. therefore, differed from the accurate valuation by only £33 or ·15 per-cent of the true reserve. In that case he was able to make allowance for special policies such as double endowments and increasing assurances. It would. therefore, seem possible to make a sufficiently accurate check valuation by choosing a suitable maturity age and valuing all the policies on the assumption that they matured at that age, using the premiums already adopted for the official valuation. The suggested method was artificial amd empirical, as compared with the Z method, and its adoption would therefore, he thought, depend largely upon the results of its practical application, of which no example was given in the Paper.

The PRESIDENT, in proposing a hearty vote of thanks to the author for his interesting and instructive Paper, said the Institute was very much indebted to Mr. Elderton in many ways, and the members always welcomed an essay from him with great enthusiasm. So far as the Paper itself was concerned, he was not quite clear, if the suggested method were used, whether it would be necessary for an office to make an investigation to see whether their endowment assurance policies or experience were in the same proportion as those in Dr. Buchanan's Endowment Assurance Model Office, because if such an investigation had to be made the labour involved would surely be greater than the labour involved under the Z method. The question of grouping endowment assurances according to the unexpired term was a very old one. More than twenty years ago he used that method in valuing endowment assurances, but when Mr. Lidstone published his Z method, he discarded the unexpired term method and adopted the Z method. Mr. Levine, in his remarks,

had referred to the net premium method of valuation in terms which revived a very old controversy. It was doubtful, he supposed, whether the author's method was, strictly speaking, a net premium method at all. In modern days the reasons for making net premium valuations had perhaps disappeared, but in former times it was most important that the net premium valuation should be insisted upon, because it provided a definite basis. If office premiums less a deduction were used, that deduction could vary according to the conscience of the actuary or manager using it.

The resolution of thanks was then put, and carried with acclamation.

Mr. W. PALIN ELDERTON, in reply, dealt in the first place with the references that had been made to the question of mortality investigations. It was suggested that he had asserted that if policies were grouped according to the year of birth a perfectly accurate comparison could be made between various classes by an aggregate mortality table. He thought that on reference to his Paper it would be found that he had been very careful to avoid saying that. All he said was that if policies were grouped according to the year of birth it was easier to make a comparison. Policies were entered according to year of entry in their year of birth group, which enabled the actuary much more easily to compare the real mortality according to the year of duration than if they were grouped according to the unexpired term. Mr. Penman had asked how he proposed to value sinking fund policies without any grouping. It was comparatively simple, although it was a little hard to explain verbally. Supposing that one started, say, on 31 December with ascertained values, added on a year's interest, added on the premiums received during the year, and half-a-year's interest, which could be done from the valuation schedules, and deducted the value of the policies going off, and half-a-year's interest, which could be obtained without any grouping at all and was necessarily required for other purposes, the valuation could be obtained automatically without any grouping. He might be told that that meant practically an individual valuation, but if such a valuation meant less work than a group valuation he saw no objection to adopting it.

Mr. Bacon and Mr. Fraser had referred to practical trials which had been made either with the method as described or with very similar methods, and they both seemed to find the results quite satisfactory. Mr. Levine remarked, if he understood him correctly, that there was practically no extra labour in the Z method as compared with the method described in the Paper, or rather that it was so little that it did not matter: he also added that the labour was not in the actual valuation but in balancing the valuation books, &c. He quite agreed. But the valuation books had to be balanced whatever the method of valuation adopted, and in his opinion that ought to be done by the date on which the books were closed. The saving of labour occurred mainly after that. Mr. Levine also remarked—and he thought Mr. Baker implied it too—that the method he described took them further from realities,

or that it was artificial and empirical. With that he disagreed. He could not see that it was departing further from realities to use a method that gave a nearly accurate result for every individual policy. It seemed to him that that was departing less from facts than to use a method which merely gave the right result for the group, and might be very far out for the individual policies. It did not seem to him any more empirical to modify a net premium than it did to use a constant which, however accurate results it might lead to, was a more or less empirical constant. Mr. Baker had suggested that the method might be useful in connection with Foreign and Colonial Government returns, not perhaps intending the suggestion to be taken seriously. But there were some offices which had a large amount of business in the Colonies or in foreign countries, and this meant that they had to make a large number of valuations at each 31 December, and many of the returns had to be made before fixed dates. In those circumstances every minute that could be saved in a valuation ought to be saved, and it seemed to him the method he had described in the Paper helped to save those minutes. For offices which need not get out a valuation until several months after closing their books, it did not seem to him to be very important what method was used, but such offices were, he thought, becoming fewer in number. He felt, therefore, that even if there was only a small saving of time in making a valuation by a new method it was worth considering.

Another point which had been mentioned was the question of the adjustment of net premiums. He pointed out in the Paper that that could quite easily be done, because the net premiums could be adjusted as required. Mr. Todhunter had stated that he did not see why one should not adopt a modified method similar to that of Section IV, but with net premiums which did not correspond exactly to the annuity-values. He agreed: he did not see any inherent objection to it; but he had made one or two trials, and came to the conclusion—probably he did not make a sufficient number of trials—that there was very little gain in accuracy, and that there were certain objections owing to the reserves at the beginning of the period being somewhat distorted. Also, it seemed to him that the method described in the Paper was simpler and quicker, and from his point of view preferable. He thought that Mr. Todhunter's remarks were the best evidence that could be given in support of the method described in the Paper. He said that the method was sufficiently accurate, and then went on to observe that, although it saved a little time, it did not save very much, and that the Z method was better. He (the author) did not say for a moment that the Z method was not better; he merely said the present method was shorter, and was sufficiently accurate; and it seemed to him that Mr. Todhunter entirely agreed with him in that respect. He had been very much interested in Mr. Baker's remarks about the $O^{M(5)}$ Table. He agreed with him that it was quite likely that the age of maturity which ought to be used for the $O^{M(5)}$ Table might be different from that for the O^{M} Table. But Mr. Baker urged—

wrongly, in his opinion—that it was an objection to the method, because if the basis had to be changed then it would be necessary to alter the age that had been assumed. He did not think that was of any consequence, because in any case the net premiums had to be altered, and all the functions could be obtained quite as easily whether 65, 60, or any other age was used; in fact, it seemed to him that this indicated the flexibility of the method. Mr. Baker also remarked that different offices might have to use different maturity ages. He did not think that was an objection.

[Mr. Elderton has sent us the following communication with reference to the foregoing Paper and the discussion thereon.—Eds. *J.I.A.*]

Approximate Valuation of Endowment Assurances.

In the discussion that followed the reading of my Paper, Mr. Fraser referred to certain results that he had obtained. He has kindly furnished me with the following abstract of his valuation:

Comparison of results of true and approximate valuations, the approximate valuation being made on the assumption that all the policies matured at death or 60.

With Profits:

Sums assured 6,628,273
Bonus additions 619,734

	True	Assumed Death or 60	Excess percentage of Assumed over True
Net premiums	228,849	233,911	2·21
P.V. sums assured ...	4,497,410	4,517,153	·44
,, bonuses	467,181	468,201	·22
,, net premiums ...	2,194,367	2,216,801	1·02
Liability	2,770,224	2,768,553	−·06

Without Profits:

Sums assured 216,118

	True	Assumed	Excess percentage of Assumed over True
Net premiums	9,013	9,229	2·40
P.V. sums assured ...	158,182	158,679	·32
,, net premiums ...	71,640	72,312	·94
Liability	86,542	86,367	−·20

I may perhaps be allowed to take this opportunity of referring to three other little points that may be of interest :

1. It is quite easy to extend the approximations that can be manufactured more or less on the lines of section IV of the Paper, and one method would be by calculating net premiums based on a certain distribution of business, using the average net premiums so found for each policy, and thus giving effect to the distribution of business. The annuity and assurance values for subsequent valuations would, of course, be calculated from the premiums so found.

2. An alternative method to the sinking fund method suggested at the end of the Paper is to assume that the value of an endowment assurance is equal to

$$k'\mathrm{P}'_{\overline{n}|}(s'_{\overline{n+1}|}-1)+k''\mathrm{P}''_{\overline{n}|}(s''_{\overline{n+1}|}-1)$$

where $k'+k''=$ sum assured and the other functions have their usual significance, the dashes indicating different rates of interest. In other words, the value of an endowment assurance is equal to the sum or difference of two sinking fund policies calculated at different rates of interest. It would seem possible to select these rates of interest so that the average values of endowment assurances were very closely reproduced.

3. With regard to Mr. Penman's remarks it may be mentioned that the values of double endowments vary rather more than those of endowment assurances. Judging from a fairly large class it would seem that a sufficiently accurate valuation can, however, be made by the method, but it is advisable to assume a fairly early age at maturity if it is considered necessary to avoid even a small understatement of the reserve. In endowment assurances the premiums vary more than the reserves: in double endowments the reserves more than the premiums.

W. P. E.

A Novel Superannuation Scheme.

SUPERANNUATION schemes and pension funds, and the economic advantages accruing to the employer, the employed and the State, by their establishment on a sound financial basis, are subjects of growing interest and importance. The State, as a large employer of labour, especially in those countries where

the Government enters into commercial undertakings which, in the old countries, have been left to private enterprise, such as Railways, Gas, Water, and Electric supply, &c., is bound, in the interest of the welfare of its citizens, to make a provision, in some shape or other, for its employees when they are no longer able to perform their duties efficiently. And whatever standard is fixed by the State becomes, to a large extent, a model for private and joint stock employers.

Whether the State should create and accumulate a fund or pay superannuation benefits out of current revenue; whether the employee should be required to contribute or not out of his current salary or wages; whether superannuation should take the form of the payment of a capital sum down or an annuity for the remainder of life, or a combination of both, are economic questions which have not yet been definitely settled, and opinions on which may vary according to the conditions of life and predilections of the citizens in the various States.

As the actuary is now generally consulted in the preparation of these schemes, it is desirable that he should make himself familiar with the various plans as they are formulated, and be able to discuss their merits and demerits.

A novel scheme has recently been formulated by a Departmental Committee appointed on 17 February 1911, by the Premier of the New South Wales Government, to consider the whole question of providing a pension scheme for public servants, and the practicability of including municipal and shire employees in such scheme, and to advise the Government thereon. The Report is dated 9 January 1912, and the Actuarial Report 12 April 1912.

There are already three funds in existence in the Colony, namely:

(i) One created under the Civil Service Act, 1884, which now only applies to persons appointed prior to 1896,* and provides a pension of 1-60th of average of last 3 years' salary for every year of service, to which the members contribute 4 per-cent of salary;

(ii) Another created by the Railway Superannuation Act, 1910;

(iii) And a third, created by the Police Superannuation Act, 1889 (amended 1906).

* Persons appointed since 1895 are compelled to take out Endowment Assurance Policies maturing at age 60 in an approved Society for amounts approximating to one year's salary.

These are all contributory schemes.

The immediate cause of the appointment of the Committee appears to have been a Bill promoted by a body called the Public Service Association. No particulars of this Bill are given, but it appears to have provided for a pension based on salary and years of service and a pension of £26 to widows.

Of the need of a pension scheme for the Service the Committee speak with the emphasis of conviction. They say, "A "service without a pension scheme will, in the long run, fail to "attract the kind of ability it most urgently needs. The normal "scale of official emoluments, particularly in responsible positions, "is not in itself sufficient to induce able and energetic men to "give up the chance of larger earnings that are possible in the "field of private enterprise." "The corporation that can "ensure its employees a reasonable permanency of employment, "promotion in order of precedence and fitness, and a satisfactory "provision for old age, will inevitably attract the highest grade "of men, and obtain from them the most efficient work." "Those "members of the Committee who have experience of the service "are unanimously of opinion that the absence of a pension "scheme has, for some years past, been making it increasingly "difficult to attract able men into the service."

The case thus presented seems clear and unanswerable. But granted there should be a pension scheme, what type of scheme should be adopted? This question cannot very well be discussed until it is settled whether the State should finance the whole scheme, or whether a fund should be created to be financed wholly by the employees themselves, or partly by the Government and partly by the men.

So long as the Civil Service pensions are confined to the salaried Officers and select classes of wage earners, the writer has always contended that the cheapest and most efficient service is that where satisfactory pensions based on salary and years of service are given without requiring contributions from the employees; and that the Government receives full value for what it pays in salaries and pensions. When, however, pensions are to be granted to ordinary labourers, it must be confessed that these views may require modification on account of the interference of Trades Unions, and, perhaps, more particularly in the Australian Colonies, on account of the influence of the Labour party in the legislatures.

The Committee go on to say, "Whilst it can be shown that

"no State does actually pay its highest Officers anything like "what men of similar ability with similar responsibility might "earn in private business, it can be shown with equal con-"clusiveness that the theory and practice of most Governments "are now to pay the average public servant the full value of the "work performed." This appears to be a *non sequitur*, but it all turns on the meaning to be attached to "average public servant." They continue, "And if the bulk of civil servants "be paid the full value of their services, what justification can "the State find for providing them with a full pension, whilst "it leaves other citizens to make their own provision for old "age, &c.? If it actually did so, the public would have good "reason to exclaim against the creation of a privileged class." If by "the bulk of civil servants" is meant the great army of labour, which, through its organizations, is not allowed freedom of contract, we find it difficult to resist the conclusion; on the other hand, we think there is little doubt that, in the long run, a service including a non-contributory pension for faithful performance of duties would attract the best and most efficient class of labour, and therefore the cheapest; and that personal interest and the fear of losing the pension would outweigh the irresponsible orders of the labour leaders, as actually happened in some cases during the Railway strike in England in 1911.

The Committee reject the contention that the Fund should be wholly financed by the Government, and summarily dismiss the opposite view, that the employees should pay the whole of the contributions, as being no more tenable than the first. Then, after referring to the pension schemes of many railway companies in Europe and America, many banking and insurance institutions as well as commercial undertakings, they consider the State is under an obligation to share the expenses of providing pensions for its servants, and propose that the employer and employee shall contribute in equal proportions. The arguments used are rather weak; but when it is proposed that outside benefits, costing more than half as much as the pensions themselves, are to be introduced into the scheme, there certainly is justification for the suggested equal division of the contributions.

As regards the type of pension scheme to be adopted, the Committee reject the usual form of pension based on a percentage of salary and years of service, with contributions of a percentage of salary according to age at entry to the fund, on the familiar objection that "Under such a scheme, as a man rose

"in the service, though his prospective pension and his con-"tributions would also increase, the increase of pension would "be out of proportion to the increased contribution, and indeed "would bear no relation to the amount of contribution." As to this, their own remarks in another place, to which we shall refer again, are the best comment. "Too much importance is "attached to this argument. . . . The higher pensions . . . "will form an insignificant part of the total expenditure. This "will be readily understood from the statement that in a "service of 16,000 persons (exclusive of the railways, shires, "and municipalities) there are only 409 (or about 2½ per-cent) "who receive salaries of £420 and over."

Having disposed of the ordinary type of superannuation scheme, they proceed to propound one of their own. This scheme, which is intended to embrace every public service, is named the "Unitary system", and is thus described:

A.—Benefits and Beneficiaries.

The family is the basis of the scheme of insurance adopted. In other schemes the pension has usually lapsed with the death of the contributor or pensioner. In this, provision is made for widow and children. The benefits proposed are:

1. Full pension on retirement at age 60 after ten years' service in case of a man, and in case of a woman at 55 or 50, according to the Table she elects to contribute under; and an actuarially-reduced pension to an employee who elects to retire after thirty-five years' service before reaching age 60.
2. On death of a pensioner one-half the pension to his widow, and £13 per annum in respect of each child under 16.
3. On death of contributor before age of retirement, one-half the pension, for which he was contributing, to his widow and £13 per annum in respect of each child under 16.
4. On retirement, after 10 years' service, of a contributor through invalidity or incapacity not due to his own fault, a full pension, and on his death one-half the pension to his widow and £13 per annum to each child under 16.
5. On retirement, after 10 years' service, of contributor through invalidity or incapacity due to his own

fault, a pension, the actuarial equivalent of the contributions made by himself and by his employer in respect of him up to the time of his retirement. On death of such contributor one-half of such pension or £26 per annum (whichever is greater) to his widow, and £13 per annum to each child under 16.

6. On resignation, dismissal, or discharge before age 60, a refund of the employee's contributions.
7. On termination of service through retrenchment,
   (*a*) a pension the actuarial equivalent of the contributions made by the employee and in respect of him; or
   (*b*) a payment equivalent to twice the amount of his contributions.

B.—Scale of Pensions.

The unit of pension is £26 per annum.

Minimum pension for which an employee may contribute, two units or £52 per annum.

The amount of pension for which an employee *must* contribute will automatically increase with salary until the *maximum* pension of £312 per annum is reached, in accordance with the following Table:

Salary	Pension units to be provided	
£		£
Up to 130	2 units	52
131 " 156	2½ "	65
157 " 208	3 "	78
209 " 260	4 "	104
261 " 312	5 "	130
313 " 364	6 "	156
365 " 416	7 "	182
417 " 468	8 "	208
469 " 520	9 "	234
521 " 572	10 "	260
573 " 624	11 "	286
625 and over.	12 "	312

If an employee's salary be increased after age 40, it will not be compulsory for him to contribute to additional units.

Any employee may elect to contribute for such number of units of pension as is prescribed for the salary group next higher than that to which according to the above Table he belongs.

the employer in such case contributing in respect of the like number of units.

C.—CONTRIBUTIONS.

The employer (*i.e.*, a Department of State, the Chief Railway Commissioner, a Municipality, &c.), and the employee, except in certain special cases, are required to contribute in equal proportions to the cost of providing the prescribed pension.

D.—RATES OF CONTRIBUTION.

The employee's bi-monthly contributions for a pension of £52 per annum, carrying rights of pension to widow and children under 16, are shown in the following Tables at various ages. The difference between the rates in Table 1 is due to the fact that the contributions for the first £52 per annum carry orphan's benefits.

This is ingenious, but can it strictly be called a Superannuation scheme?

A superannuation scheme, as generally understood, may be described as a scheme to provide a servant, in consideration of his faithful service, with a pension or allowance on his retirement from old age or infirmity; the allowance, as a rule, having relation to the salary received and length of service. The proposed scheme is purely and simply compulsory assurance, every member being compelled, according to the amount of his salary, to insure a deferred annuity for himself, a survivorship annuity for his widow and a fixed temporary survivorship annuity for each of his children until they are 16 years of age; and for which the employer, in consideration of being relieved of a moral liability to superannuate a faithful servant, undertakes to pay half the premiums.

The employer cannot morally be expected to provide for the widows and orphans of deceased servants; but there is no doubt that it would be a wise provision for a service, as well for the community at large, that the employer, especially when it happens to be the State, should have a scheme of compulsory insurance for widows and orphans. This is frequently carried out in this country by voluntary associations of employees, assisted often by employers; but we consider it would be much better if employers would recognize the need of such a fund and make membership of it compulsory.

Extract from Table I.—Men.

Age	Bi-monthly contributions *	
	1st £52 per annum	2nd and subsequent £52 per annum
	£ s. d.	£ s. d.
16	0 2 6	0 2 2
20	0 3 2	0 2 10
25	0 4 5	0 3 11
30	0 5 9	0 5 2
35	0 7 6	0 6 10
40	0 9 11	0 9 2
45	0 13 11	0 13 2
50	1 7 1	1 1 3
55	2 5 11	2 4 10

Extract from Table II.—Women.

Age	Bi-monthly contributions *	
	For retirement at age 60	For retirement at age 55
	Each £52 per annum	Each £52 per annum
	£ s. d.	£ s. d.
16	0 1 1	0 1 11
20	0 1 6	0 2 7
25	0 2 3	0 3 10
30	0 3 2	0 5 8
35	0 4 6	0 8 5
40	0 6 6	0 12 10
45	0 9 10	1 1 11
50	0 16 10	2 9 9
55	1 17 6	...

The pension, it will be noticed, has no relation to length of service, being dependent absolutely on salary, and subject to a maximum of £312 a year.

The rates of contribution were based on the experiences of the various services, so far as they could be relied upon, and

*When first we saw these Tables of contributions we began seriously to think. Why should the Committee select such a strange interval as two months for the periodic contributions? And why were the contributions so small? But when we came to the illustrations we discovered there were 24 bi-monthly contributions in a year! We trust our Australian cousins are not going to disfigure the English language with such a distortion as bi-monthly for half-monthly. We are glad to note that in the Report of the Actuarial Sub-Committee the interval is always described as half-monthly.—H. W. M.

where the data were considered of little or no value, reference was made to the experiences of other services. The rate of interest used in the calculations was $3\frac{1}{2}$ per-cent, which is an exceedingly poor rate for a State or municipality to offer its servants when it has to pay at least 4 per-cent for any money it wishes to borrow. As nearly all the great Railways in this country guarantee 4 per-cent interest on their Superannuation Funds, and retirement is not compulsory before age 65, it is not possible to make any comparison of rates.

To show the working of the proposed scheme the best plan will be to take an example.

"A man, General Division, enters the service at age 20 on a salary of £70, the subsequent increases of salary being as shown":

Age	Salary	Total Pension Obligation	Increment of Pension	Increment of Half-monthly Contribution	Total Half-monthly Contribution	Annual Contribution	Percentage of Contribution to Salary	Pension, as Percentage of Last Salary	Pension, as Percentage of Last Salary and Years of Service	Pension, as Percentage of Average Salary and Years of Service
	£	£	£	s. d.	s. d.	£ s. d.				
20	70	52	...	...	3 2	3 16 0	5·4	...	...	...
21	90	52	...	...	3 2	3 16 0	4·2	...	...	...
22	110	52	...	...	3 2	3 16 0	3·5	...	...	...
26	120	52	...	..	3 2	3 16 0	3·2	...	...	...
30	140	65	13	1 4	4 6	5 8 0	3·9	46·4	4·6	5·3
32	150	65	...	...	4 6	5 8 0	3·6	43·3	3·6	4·3
35	165	78	13	1 9	6 3	7 10 0	4·5	47·3	3·2	3·9
40	175	78	...	...	6 3	7 10 0	4·3	44·6	2·2	2·8
50	195	78	...	...	6 3	7 10 0	3·8	40·0	1·3	1·7
60	200	78	...	...	6 3	7 10 0	3·5	39·0	1·0	1·3

Here it will be seen that the scale of pension decreases with length of service, when measured by any of the usual standards, and the rate of contribution is generally heaviest when the salary is small. A better idea, however, of the true working of the scheme will be obtained by setting out the benefits and showing how much is paid for each benefit; thus:

Age	Salary	Annual Contributions for: Member's Benefit	Widow's Benefit	Orphans' Benefits	Benefits: Member's Pension on breakdown after 10 years or at age 60	Pension to Widow on Death of Member	Orphans' Benefits
	£	£ s. d.	£ s. d.	s. d.	£ s. d.	£ s. d.	
20	70	2 4 0	1 4 0	8 0	...	26 0 0	A pension of £13 to each until age 16.
21	90	2 4 0	1 4 0	8 0	...	26 0 0	
22	110	2 4 0	1 4 0	8 0	...	26 0 0	
26	120	2 4 0	1 4 0	3 0	...	26 0 0	
30	140	3 4 0	1 16 0	8 0	65 0 0	32 10 0	
32	150	3 4 0	1 16 0	8 0	65 0 0	32 10 0	
35	165	4 10 0	2 12 0	8 0	78 0 0	39 0 0	
40	175	4 10 0	2 12 0	8 0	78 0 0	39 0 0	
50	195	4 10 0	2 12 0	8 0	78 0 0	39 0 0	
60	200	4 10 0	2 12 0	8 0	78 0 0	39 0 0	

It will be seen that the cost of providing half pension to the widow is from 55 to 60 per-cent of the cost of full pension to the member. The cost of the orphans' benefit is a fixed amount per annum.

After studying these Tables we think it will be admitted that the pension after 40 years' service is a very poor one, considering the rate of contribution demanded, for it can never, except in rare cases, exceed half the final salary, it will mostly be between one-third and two-fifths, and where the salary is high it may be as low as one-fourth or less. And this is all they propose to offer by way of pension when, after referring to the fact that only about $2\frac{1}{2}$ per-cent of persons in the service (exclusive of railways, shires, and municipalities), receive salaries of £420 and over, the Committee say, "It is on this small band, or perhaps one "should say on only a small part of it, that Governments rely "for the efficient management of undertakings and the efficient "administration of laws that every year increase in variety and "importance. It would seem, therefore, to be not only an act "of justice, but a wise policy to secure for such men a decent "standard of comfort in old age." "As additional evidence in "favour of a reasonable scale of pensions, it may be mentioned "that the pension schemes of several local banking companies "and other private firms provide for pensions up to two-thirds "of the average salary; in some cases more. In competition "with such firms the State, as employer, so long as it refuses to "assist in the provision of higher pensions, will be at serious "disadvantage. If it wishes to get the best business ability, it

"must offer the inducements which private firms find it necessary "to offer." We have seldom seen so much wisdom produce such poor results.

Let us consider for a moment the employee's side of the question. He does not want to be made virtuous against his will. Argue as much as you will on the advantages of the other benefits, he will tell you that he wants a satisfactory pension so that he shall be comfortable in his old age. He may never marry; and if he does, he may not leave a widow and children under the age of 16 years of age. Why should he be made to pay for other people's widows and children, and in consequence have his own pension reduced? No! he will say, give me a satisfactory pension and make me feel happy for the future, and then I can look after my wife and family myself. It may be selfish, but it is human nature.

Is it not possible then to combine the two benefits—pension and family endowment? We think it is, particularly when the employer is the State. The State is a permanent institution, and when properly governed will always be solvent. So far as security is concerned the employees will be satisfied with the undertaking of the State, and, therefore, there is no reason why a fund should be created, except for the purpose of equalizing the charges on revenue from year to year.

Now, the service of the State has a certain attraction on account of its permanence and security, and the dignity attached to the appointment; but it also wants to attract the best talent and best business ability, which can always command its value, and it has to pay for it. At what price can it secure such men? We are now speaking of the salaried class. There is always some relation between salary without pension and salary plus a satisfactory pension; and it is the writer's opinion, derived from a long acquaintance with and study of the question, that the difference in the salaries generally is as much as 10 per-cent; it may be less at the young ages, say about 5 per-cent, and greater in the case of the high salaries, probably from 15 to 20 per-cent, and more.

Assuming that all enter from 18 to 20 years of age, and retire on the average at 62 or previous incapacity, a non-contributory pension of 1-60th of average of last three years' salary for every year of service, the pension not to exceed two-thirds of such average, would cost $7\frac{1}{2}$ per-cent of salaries, reckoning interest at $3\frac{1}{2}$ per-cent, or $6\frac{1}{2}$ per-cent of salaries reckoning interest at 4 per-cent. Say

the average salary without pension would be £200 a year; how much less would a State employee be willing to take if he had such a pension guaranteed to him for faithful service? We should be inclined to say £180 to £185 a year. Let us say £185; then the cost of the pension to the State would be $7\frac{1}{2}$ per-cent of £185 = £13. 17*s*. 6*d*. at $3\frac{1}{2}$ per-cent interest, or £12. 0*s*. 6*d*. at 4 per-cent interest. In the first case the State would make a profit of £1. 2*s*. 6*d*., and in the second case a profit of £2. 19*s*. 6*d*. Even if the State had to pay a salary of £190, the loss would only be £4. 5*s*. at $3\frac{1}{2}$ per-cent or £2. 7*s*. at 4 per-cent. But taking the salaried service throughout we believe that the cost of salaries and pensions together would not be more than for salaries without pension; and the State would have a contented, able, and efficient staff without any extra cost to itself.

There is a view, we believe a mistaken one, that the men would appreciate the pension more if they were made to contribute to its cost. We never came across anyone who did not fully appreciate the value of a non-contributory pension. But suppose the State required that the employee should contribute towards the cost, then the whole case assumes a different appearance. As the State actually takes from the employee money which belongs to him, an agreement will have to be made by which the money deducted will have to be returned on resignation, dismissal, or death, and if the pension payments do not amount to such deductions the difference will have to be returned, otherwise the State will be charged with robbery. Again, since the employee has to make a contribution out of his salary, he will require to have the salary he would otherwise be content with increased by the amount of contribution he will have to make. If the cost of the above pension were equally divided between the State and employee, with a return of the employee's contributions on resignation, dismissal, or death, &c., each would have to contribute $4\frac{1}{4}$ per-cent of salary (reckoning interest at $3\frac{1}{2}$ per-cent), and if the employee were content with a salary of £185 with a straight out pension instead of £200 without pension, he would, under the altered circumstances, want a salary of £193. 4*s*. because out of that he would have to pay £8. 4*s*. towards pension; and as the State would also have to contribute £8. 4*s*., the cost of salary and pension together would be £201. 8*s*., and the State would lose £1. 8*s*. instead of gaining £1. 2*s*. 6*d*. If the equivalent of £200 without pension were £190 with pension, then, if the employee had to contribute half

the cost of pension, he would want a salary of £198. 8*s*. 8*d*., because his contribution of $4\frac{1}{4}$ per-cent on that would be £8. 8*s*. 8*d*. and as the State would have to contribute a like sum, it follows that the State would have to pay £206. 17*s*. 4*d*. instead of £204. 5*s*., in the case of a straight out pension.

With regard to the wages class, assuming they all entered between the ages of 20 and 30, a straight out pension of 1-60th of average of last 3 or 5 years' wages for every year of service not exceeding 40, would be covered by 5 per-cent of wages, reckoning interest at $3\frac{1}{2}$ per-cent. Owing, however, to the interference of Trades Unions it is doubtful if the men would take less than the Trade Union wages; but, at the same time, it must be remembered that this satisfactory provision for old age and infirmity would attract the steadiest and most efficient men, and, for the extra 5 per-cent, the State would have the pick of the labour market; and if the pension were dependent on continuous and faithful service, the power of the irresponsible agitator to foment a strike would be reduced, and the men would have recourse to constitutional means, through the legislature, for the consideration of any increase in wages.

We now come to the consideration of what is grandiloquently called "The Endowment of Motherhood", which put in plain language means the provision of pensions for the widows and children of deceased employees and pensioners. It is very difficult to take seriously the proposition that an employer has not only a moral duty to provide for his employees in old age and infirmity, but to provide for their widows and families after death whether happening early or late. It may be "demonstrably a good investment on the part of the State", but that "The extension of the principle to the whole nation is only a question of time" we beg leave to doubt. To us it seems to be the endowment of reckless and improvident marriages.

We consider it to be the moral duty of every man to provide, according to his ability, for his wife and young children on his death, and, further, that such provision is most cheaply and effectually secured by the members of a staff adopting some form of compulsory insurance. We know of several cases where the employees have voluntarily started Widows' and Orphans' Funds among themselves, and made membership practically compulsory, and the establishment of such a Fund has generally been encouraged by the employer, who has helped with donations and subscriptions, and sometimes made membership of the

Fund a condition of service. All such schemes are to be highly commended, and a good employer will always encourage them; but many things are done by voluntary effort and sympathetic assistance which would be strongly resented if put in the form of a moral duty.

Reviewing the whole scheme, we are bound to say that it is very disappointing; and, from internal evidence, we do not think that the members of the Committee are very enamoured with it. As a State scheme we believe it to be based on wrong principles. No amount of arguments or formulas will ever persuade an employee that his pension should not be based on his salary and years of service. We have given reason why we believe that, in the case of a State, which guarantees a proper pension, it is able to obtain the best and most efficient service, at a cost, including pension, not much, if any, greater than a service without pension. If the authorities are very much concerned by the smallness of the pension allowed to those who are compelled by ill-health to retire at an early age, provision could be made that the pension shall never be less than one-fourth or one-third of salary at a very small additional cost.

As to the "Family endowment" or "Endowment of Motherhood", we repeat that we consider that to be a matter for the employees themselves. Probably, if they had not to contribute to their pensions they would be willing to agree that contribution to a Widows' and Orphans' Fund should be compulsory. But what the scheme should be would be a matter of serious consideration after consultation with the men themselves. If the widow's pension were to be based on the employee's pension, say, one-half or one-third, the contributions would be high; at the same time, it would seem only reasonable, if contributions are based on salary, that the pension should in some way be based on salary also. Perhaps the system of units of pension might be conveniently worked in here.

H. W. M.

Whole Life Non-Profit Assurances, by Finlay James Cameron, F.I.A., F.F.A., *of the Friends' Provident Institution.*

[Read before the Institute, 22 December 1913.]

THE subject of this Paper is Whole Life Non-Profit Assurances with uniform Premiums payable throughout life.

In choosing this subject for investigation, I have been influenced mainly by the following reasons :

(1) This type of policy represents life assurance in one of its simplest and earliest forms. The premium and the sum assured are both fixed throughout life, save for the very slight chance of insolvency of the office causing the contract to be reduced. Also nowadays it is not uncommon to find that the surrender values and equivalent paid-up policies are guaranteed from the outset.

(2) Other tables such as Whole Life by Limited Premiums, Half Premiums for first "*n*" years, &c., are as a rule founded on the basis of the simple whole life ordinary rates.

(3) A considerable body of data derived directly from the experience of British Offices under ordinary whole life non-profit policies is available as a result of the British Offices Mortality Investigation, 1863–93.

(4) In spite of the apparent simplicity of the contract, there are marked divergencies in the rates of premium charged by first-class offices and in the surrender values and equivalent paid-up policies offered by them.

The paper is divided into four sections, viz. :

(1) Office Premiums.
(2) Reserve Values.
(3) Surrender Values.
(4) Paid-up Policies.

I.—OFFICE PREMIUMS.

Although on first thoughts it might be imagined that the calculation of the premiums for non-participating whole life assurances would present less difficulty than in the case of participating assurances, it will be readily seen that, whereas under the latter class the assumptions that have been made as to rates of interest, expense, and mortality can to a great extent be rectified in the periodical divisions of profits, provided that there is some consistency between the premiums and the method of dividing the profits, there is very little room for re-adjustment under the former class. It therefore behoves us to be

particularly careful in making our estimates in the case of premiums for non-participating assurances.

It is unusual for offices to show separately the results of the working of the non-profit assurances, and any insufficiency in the premiums charged for whole life non-profit assurances may therefore not be apparent, but it cannot be argued on equitable grounds that it is just to expect the participating classes to make up such deficiency in the non-participating classes except in the case of an emergency.

In the formation of the premiums the principal factors are

(*a*) Mortality,
(*b*) Interest,
(*c*) Expenses.
(*d*) Competition.

Mortality.

The first element in the composition of an office premium is the table of mortality on which the net premium is to be based. It seems hardly necessary to make this statement, but in these days of keen competition there is sometimes a tendency to overlook this fundamental feature and to speak only of "commercial" rates of premium, which may or may not bear any relation to the net premium.

In regard to the form of mortality table, I think it will be agreed that select tables are generally the most suitable for the purpose of premium calculations, for the net premiums derived from an aggregate table are too small at the younger ages and too large at the higher ages as compared with select net premiums. The ultimate table has received a considerable amount of attention in recent years. In the United States, for the calculation of office premiums, and indeed for practically all calculations in life assurance, this form of table is in great vogue. By using ultimate premiums the light mortality experienced in the early years of a policy (which is taken into account by the select premiums) is ignored and set off against the heavy expenses incurred in these early years. In this country, however, it is generally agreed that, as the select table corresponds most nearly with the actual mortality experienced, it is the most appropriate basis for the construction of office premiums, and that the question of expense is better dealt with separately.

Seeing that the British Offices' $O^{[NM]}$ Table (1863–1893) was derived from the experience of first-class lives assured under

whole life non-profit policies, one would have expected that the premium tables now in use would be based for the most part on this experience. It would appear, however, that in a number of offices the premiums have been constructed from the lighter mortality data of the $O^{[M]}$ experience.

The following figures show the extent of the statistics on which the $O^{[NM]}$ table was based, as compared with the corresponding figures for the H^M and $O^{[M]}$ tables.

——	H^M	$O^{[NM]}$	$O^{[M]}$
Entered	130,243	62,186	672,893
Exposed to risk	1,200,401	659,111	8,647,246
Average years of exposure	9·2	10·6	12·8

It is argued by a number of actuaries that the $O^{[NM]}$ experience, containing as it does a number of lives assured in connection with financial transactions, does not truly represent the mortality experienced in the past or likely to be experienced in the future in the case of the majority of offices. It is also argued that, as the passing of the Finance Act, 1894, has led to the transaction of whole life non-profit business in connection with provision for Estate Duty, and that as this class of business comes under the head of "provident" rather than "financial," it tends to leaven, as it were, the heavier mortality of the "financial" business. (In this connection I should like to remark that I have come across several "Estate Duty" policies which have been subsequently used in connection with loan transactions.) There is no doubt that many very large whole life non-profit policies continue to be effected for "financial" purposes, and while it is the case that several offices specialize in "financial" business, yet a number of their policies are so large that re-assurance is required and other offices are affected. Furthermore, apart from Estate Duty business, it is doubtful if many people deliberately effect non-profit policies for provident purposes, and it would appear that the amount assured by these "provident" policies is comparatively small.

It is possible to proceed too rapidly to conclusions in regard to assumed changes of mortality. There is a good rule that an office should be on the safe side where there is any doubt as to the proper basis. Acting on this rule I think that, until we

have some fairly definite knowledge that whole life non-profit mortality has appreciably improved since 1893, or that "financial" business is alone responsible for the heavy mortality shown by the $O^{[NM]}$ experience as compared with the $O^{[M]}$, it is desirable for most offices to base their whole life non-profit rates on the $O^{[NM]}$ table.

In connection with the question of mortality I should like to quote the following remarks.

Mr. G. F. Hardy, at the commencement of his Memorandum on the Graduation of the $O^{[NM]}$ experience (*J.I.A.*, vol. xxxviii, p. 501), said :

"In the graduation of this experience, it has been borne in "mind that the main practical purpose to which the table will "be put will be the calculation of premiums, rather than the "determination of reserves. It is most important therefore, "that the graduation should not under-estimate the mortality, "and, consequently, the premiums."

Mr. Gordon Douglas, in his Presidential Address to the Faculty of Actuaries, delivered 7 November 1910, on the subject of "Life Assurance under the 1870 Act", discussing the question of the reduction in Whole Life Non-Profit rates which had taken place between the First and Last Valuations of a large number of Offices, said (T.F.A. V, p. 219) :

"The New Experience Tables clearly showed that Without "Profit business was subject to heavier rates of mortality than "With Profit Assurances, but this fact seems in process of being "ignored, and if the experience of the future, in respect of "mortality, corresponds with that of the past, it will, I am afraid, "mean a considerable eating into of the Profit from other sources "apparently reckoned upon in the calculation of certain of the "Non-Profit rates now current."

Messrs. J. Burn and W. C. Sharman, in their joint paper on "Changes in the Rates of Mortality amongst Assured Lives during the past Century", read before the International Congress at Amsterdam in 1912, said :

"So keen is the competition nowadays, that it may very "possibly happen, more particularly in the case of the premiums "charged on some whole life non-profit assurances, that a "comparatively small variation in the mortality rates may "change an apparently profitable business into an actual and "dangerous loss."

These observations are, I think, significant in view of the

general tendency in recent years to reduction in whole life non-profit rates.

Comparisons of select Net Premiums.—In the table given below (Table 1) are shown the values of 100 $\pi_{[x]}$* by three select mortality tables at $3\frac{1}{2}$ per-cent interest.

TABLE I.

Select Net Premiums. *Interest $3\frac{1}{2}$ per cent.*

Age at Entry $[x]$	$100\pi_{[x]}$			Ratios per-cent			Age at Entry $[x]$
	$H^{[M]}$ (Sprague's Select)	$O^{[NM]}$	$O^{[M]}$	$\frac{O^{[NM]}}{H^{[M]}}$	$\frac{O^{[M]}}{H^{[M]}}$	$\frac{O^{[NM]}}{O^{[M]}}$	
20	1·471	1·364	1·265	92·7	86·0	107·8	20
25	1·600	1·551	1·444	96·9	90·3	107·4	25
30	1·814	1·788	1·671	98·6	92·1	107·0	30
35	2·100	2·093	1·959	99·7	93·3	106·9	35
40	2·477	2·486	2·329	100·4	94·0	106·7	40
45	2·974	2·997	2·806	100·8	94·4	106·8	45
50	3·618	3·669	3·424	101·4	94·6	107·2	50
55	4·493	4·557	4·232	101·4	94·2	107·7	55
60	5·681	5·743	5·295	101·1	93·2	108·4	60
65	7·283	7·339	6·700	100·8	92·0	109·5	65

It would appear from this table that an Office in 1893 whose whole life non-profit premium rates were founded on Sprague's Select $H^{[M]}$ experience would be justified, other factors being the same, on the assumption of $O^{[NM]}$ mortality, in reducing its rates, under a new table, up to say age 35, and increasing them very slightly thereafter. If, however, $O^{[M]}$ mortality were assumed for the new table, the new rates would be considerably less than the old rates.

Actual Changes in Whole Life Non-Profit Rates.—The following Table (Table II) is, I think, interesting as showing the change in rates of premium which has actually taken place in twenty British Offices in the period from the last Valuation before 1893 to April 1913. I should explain in regard to these rates :

(1) The twenty offices all contributed to the British Offices' experience.

(2) All the Offices pay Commission.

(3) The premiums are the ordinary non-profit rates for assurances of £100, taken from the Board of Trade

* Throughout this paper π is used for the net, and P for the office, premium.

Returns in the case of the "1893" rates and from the latest prospectuses (April 1913) in the case of the "1913" rates.

TABLE II.

Age at Entry	Average Office Premiums (20 offices)		Ratios per-cent			Age at Entry
	Last Valuation before 1893	1913	1913 : 1893 (Office)	$O^{[NM]} : H^{[M]}$ (Net)	$O^{[M]} : H^{[M]}$ (Net)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)
	£ s. d.	£ s. d.				
20	1 13 3	1 11 9	95·5	92·7	86·0	20
30	2 2 1	2 0 3	95·6	98·6	92·1	30
40	2 16 5	2 14 9	97·0	100·4	94·0	40
50	4 0 9	3 19 0	97·8	101·4	94·6	50
60	6 7 0	6 2 9	96·7	101·1	93·2	60

It will be noticed that the present average rates are in all cases less than those in force at the last Valuation before 1893, the difference appearing to be fairly constant up to age 50 but considerably greater at age 60.

In columns (5) and (6) of the above Table are repeated the ratios taken from columns (5) and (6) of Table I.

The figures in Table II show that the twenty offices have not on the average decreased their premiums in a ratio so great as $O^{[M]} : H^{[M]}$. In the period, however, from the last Valuation before 1893 to 1913, there has been a decided increase in the cost of obtaining new business, for, although the general average rate of expenses to premium income is not much different from that experienced twenty years ago, the average premium per £100 assured has risen considerably owing to the rapid growth of endowment assurances. It would follow, therefore, if we assume the same rate of interest now as was earned in 1893 (as we may reasonably do), that the effective decrease in premium rates is greater than appears from a comparison of those now charged with those in operation in 1893.

INTEREST.

For a number of years prior to 1902 the trend of the average net rate of interest was steadily downwards, but since that date, and especially during the last two or three years, there has been a decidedly upward tendency in the average rate. This

tendency may be traced partly to the heavy amounts written off securities in recent years, and partly to an extension of the field of investment to first-class Colonial and Foreign Securities, but in a large degree to a general rise in the rate of interest obtained on investments both at home and abroad.

In February 1908 Mr. P. L. Newman* said that the manner in which the average rate of interest would be raised (or not allowed to fall) in the future, would probably arise not so much from a general increase in the rate of all or most offices but from a levelling up of the offices earning lower rates to the position of those which had for many years been earning rates above the average, and there are several recent instances of offices which formerly earned somewhat low rates of interest, having produced a substantial increase in the rate earned.

Most of the ordinary offices have been able for some time to obtain at least $3\frac{3}{4}$ per-cent net on their funds (in 1912 nearly 4 per-cent), and it might therefore be considered reasonable to adopt a rate of interest of say $3\frac{3}{4}$ per-cent in the calculation of premiums. It is, however, highly desirable to leave a margin in the calculation of premiums for profit and fluctuations, and I therefore propose $3\frac{1}{2}$ per-cent as a suitable rate for those offices adopting the $O^{[M]}$ table as their basis, while in the case of offices which assume the safer basis of $O^{[NM]}$ mortality and are able to earn at least $3\frac{3}{4}$ per-cent on their funds, I think the higher rate of $3\frac{3}{4}$ per-cent might be used.

EXPENSES.

The first division of Expenses is into—

(*a*) Commission,

(*b*) General Expenses.

These two items are always given separately in the Returns made by offices to the Board of Trade, but various sub-divisions require to be made before loadings can be obtained therefrom which will be suitable as a basis for office premiums.

Commission.—It has been the practice for a number of years to pay commission in the form of a poundage on the sum assured followed by a percentage on the renewal premiums. The normal amounts of these commissions are 1 per-cent and $2\frac{1}{2}$ per-cent respectively, but it is common knowledge that these rates are frequently exceeded in practice. In a rough investi-

* J.I.A., vol. xlii, p. 309.

gation which I have made into the Annual Accounts of 47 representative British offices given in the Board of Trade Returns for the last two years (dealing in most cases with the accounts for the years 1910 and 1911, respectively), I have estimated the average initial commission paid per office at about 23*s.* per-cent on the new sums assured. In obtaining this rate advantage was taken of the much more elaborate sub-divisions of new business, premiums, &c., shown by the Government Returns since the passing of the 1909 Act. Only figures relating to business transacted within the United Kingdom were used, and all the figures are net—after deducting re-assurances. The renewal premium income was obtained by deducting the single premiums and new premiums from the total premium income, and $2\frac{1}{2}$ per-cent thereon was assumed to represent renewal commission. In the case of single premiums the commission was taken as 3 per-cent on the amount of the premiums, and in the case of annuities 1 per-cent on the consideration or the actual commission shown where separate accounts are given. The balance of commission after deducting these three items was treated as the initial commission on yearly premium cases and the ratio per-cent thereof to the new business (adjusted by deducting twice the amount of the single premiums as an approximation to the sums assured thereby) was taken as the initial commission per-cent on assurances by yearly premiums.

As, however, it appears to be less usual to pay large poundage commissions in the case of whole life non-profit business than in the case of classes subject to higher rates of premium, and as I am desirous of obtaining loadings on a moderate scale, I have adopted the normal 1 per-cent on sums assured and $2\frac{1}{2}$ per-cent on renewal premiums.

General Expenses.—The allocation of general expenses is a more difficult problem than the apportionment of commission, for the latter in the case of individual policies bears a fixed proportion to the new sums assured and the premiums, while the former, for the most part, are not increased in proportion to either of these factors. In fact, apart from commission and policy stamps, each policy effected causes about the same amount of expense to the office. It has, therefore, been argued that these general expenses should be divided equally between all the policies, and while there is something to say for this argument on the "wholesale principle", it is clear that such an arrangement logically would render it necessary for different

rates of premiums to be calculated in each class according to the amount of the sum assured, and would also make the expense loading for a small policy very heavy. The plan I have adopted is the usual one of assuming that the expenses charged against each policy are in proportion to the sum assured, and as I intend to compare the premiums loaded in this way with average office premiums per £100 assured (irrespective of the amount of the policy) the method seems appropriate for such a purpose.

Initial General Expenses consist of specific items such as medical fees and policy stamps which relate to individual policies, general items such as advertising, and such portion of stationery, salaries, directors' fees, &c., as may be considered to have been spent on the acquisition of new business. The first question then arises as to the apportionment of these expenses, and the next as to their treatment in connection with office premium loadings. Mr. H. J. Rietschel, in his paper on the "Analysis and Apportionment of the Expenses of Management of a Life Office, with a view to ascertaining the Office Premium Loadings" (*J.I.A.*, vol. xliv, p. 415), estimated in the case of an actual office with an expense ratio (including commission) of 15 per-cent to the premium income, that (excluding commission) £3. 3*s*. per-cent of the new sums assured was spent in the acquisition of new business. He then proposed that the whole of this £3. 3*s*. (together with £1 as initial commission) should be charged in the office premium loading against each new policy as an amount to be repaid gradually out of the premiums. As Mr. Rietschel himself pointed out, his method of apportionment was "to some extent empirical", and on examination of the table given on page 417 (*J.I.A.*, vol. xliv) it will be seen that the whole of such items as the salaries of the new business and agency departments, as well as advertising and travelling expenses, have been included in the initial charge of £3. 3*s*. per-cent.

Whilst I am in agreement with Mr. Rietschel, where non-profit premiums are concerned, that the loadings in the premiums should reflect the actual expenditure—provided such expenditure is not excessive—I do not think that it is equitable to charge every item of expense concerned with new business against each new policy. Although in most offices a large proportion of the general expenses of management is incurred on account of new business, yet without a reasonable amount of new business an office would fall into a state of decay. It is, therefore, I think,

to the interests of the old policyholders, if they wish to share in the profits of a going concern, that they should contribute towards the indirect expenses of acquisition of new business.

The direct general expenses of new business are the medical fee and the policy stamp, and these may be adequately provided for by charging an initial loading of 10*s*. per-cent on the sum assured. For a policy of £250, the stamp of 2*s*. 6*d*. and the medical fee of 21*s*. would be covered by an initial loading of 9*s*. 5*d*., but for policies of greater amounts the initial loading required would be less and a margin would therefore be provided in all cases of £250 and over. As to what further amount should be charged against the new sums assured in respect of the indirect expenses of advertising, salaries, &c., it is difficult to say, but I should be inclined to consider that a further 10*s*. per-cent would be sufficient for a "minimum" loading. The initial loading for expenses of new business (apart from commission) has therefore been taken as 20*s*. per-cent.

Annuities.—In a few offices a separate account is given for the Annuity section, but in the great majority the annuities are included in the accounts with the Life Assurance section. It is, therefore, perhaps advisable to make some allowance for the general expenses of conducting the Annuity business. Mr. H. J. P. Oakley (*J.I.A.*, vol. xliii, p. 302) arrived at £1 per-cent on the purchase money, $1\frac{1}{2}$ per-cent on the annuity payments, and 5 per-cent on the annual premiums for deferred and other annuities as an average charge for general expenses (excluding commission). By the passing of the Finance Act of (1909–1910) 1910 the rate of Stamp Duty has been increased to £1 per-cent where the consideration is over £500, and I have therefore taken $1\frac{1}{2}$ per-cent on the consideration as approximating to the initial expenditure. For the charge on the annuity payments I have followed Mr. Oakley's estimate of $1\frac{1}{2}$ per-cent. As regards deferred annuities the amount of the premiums is so small, relatively either to the total premium income or to the current annuity payments, that I have not attempted to make any estimate of the expenses due to this business.

Balance of Expenses.—The balance of expenses remaining after deduction of the 20*s*. per-cent charge on the sums assured on account of new business, and in the case of annuities, $1\frac{1}{2}$ per-cent on the consideration together with $1\frac{1}{2}$ per-cent on the annuities paid, would consist in most offices partly of expenses in connection with new business and partly of renewal and fixed

expenses due to business already on the books. There are a number of ways in which this balance may be allocated, but I think that fair treatment will be given by apportioning it equally according to the periodical premium income and the total sums assured in force. The bonus additions should, I think, be included in the last mentioned item, seeing that a certain amount of expenses is due to dividing and looking after profits which it is hardly fair to charge to the non-profit business.

Investigation of General Expenses.—In order to obtain average loadings I have investigated the accounts of 10 old-established British Offices given in the Government Returns issued in the years 1912 and 1913 respectively. It should be explained that these offices employ agents and transact life business only—also that the business is almost entirely obtained within the limits of the United Kingdom. As regards the ratio of their total expenses (including commission) to premium income, the arithmetical mean of these ratios for the latest valuation periods is about 14 per-cent, which may be considered a fair average rate. In accordance with the preceding paragraphs the expenses were allocated as follows:

Annuities—

(*a*) $1\frac{1}{2}$ per-cent on Consideration } or actual expenses
(*b*) $1\frac{1}{2}$ per-cent on Annuities paid } where shown.

Assurances—

(*c*) 1 per-cent on New Business.
(*d*) Balance of Expenses equally between Periodical Premium Income and estimated Total Sums Assured (including Bonuses) in force.

The result of this investigation was to show for the item (*d*) an average per office of about $3\frac{3}{4}$ per-cent on the Periodical Premiums and 2*s*. 3*d*. per-cent on the Total Sums Assured, but with a view to keeping the loadings as moderate as possible I have assumed that $3\frac{1}{2}$ per-cent of the Periodical Premiums and 2*s*. per-cent on the Total Sums Assured will be sufficient for the purpose.

The various loadings I have derived for Commission and General Expenses are combined in the following table:

	Percentage of		
	New Business	Periodical Premiums	Total Sums Assured
Commission	1·0	2·5	...
General Expenses ...	1·0	3·5	·1
	2·0	6·0	·1

This allocation of Commission and Expenses, although derived to a certain extent arbitrarily—as all such allocations must be—reflects in my opinion an expenditure somewhat less than the average, and is, I think, a suitable foundation on which to base the loadings for moderate whole life non-profit rates of premium. The percentage of the Periodical Premiums, namely, 6 per-cent, is probably on the small side, seeing that the average premium per-cent receivable by the 10 offices (about £3 per-cent of Sums Assured plus Bonuses) is in excess of the average whole life non-profit rate, but on the other hand the allocation of a portion of the general expenses in proportion to the Sums Assured in force weighs more heavily on whole life than on endowment assurance policies.

Formula for Premiums.—The formula I would suggest as giving effect to the various factors mentioned above, is

$$\frac{1}{\cdot 94}\left\{\pi_{[x]}+\frac{\cdot 02}{\mathbf{a}_{x}}+\cdot 001\right\} \mathrm{O}^{[\mathrm{NM}]}\, 3\tfrac{3}{4}\%$$

which assumes for expenses :

(1) Initial—2 per-cent on Sum Assured.

(2) Percentage—6 per-cent of each Annual Premium.

(3) Constant—2*s*. per-cent per annum on Sum Assured.

Examples of Premiums.

In Table III are given the average of the published premium rates of 25 British Offices which

(1) contributed to the British Offices Experience,

(2) pay commission,

(3) appear to charge full expenses against the life business; also the highest and lowest premiums at each age and the

average premiums of the five offices publishing the lowest rates at age 40.

Examples of premiums are also given calculated by the following formulas:

(A) Dr. T. B. Sprague (*J.I.A.*, vol. xxii, p. 396):

$$1{\cdot}075\left\{\pi_{[x]}+\frac{{\cdot}01}{\mathbf{a}_{[x]}}+{\cdot}00125\right\}$$

(B) Mr. H. J. Rietschel (*J.I.A.*, vol. xliv, p. 420):

$$1{\cdot}0474\pi_{[x]}+\frac{{\cdot}043}{\mathbf{a}_{[x]}}+{\cdot}00068$$

(C) Mr. H. W. Andras (Trans. 7th Cong. I, p. 770):

$$1{\cdot}025\left\{\pi_{[x]}+\frac{{\cdot}02}{\mathbf{a}_{[x]}}+{\cdot}00125\right\}$$

(D) New formula:

$$\frac{1}{{\cdot}94}\left\{\pi_{[x]}+\frac{{\cdot}02}{\mathbf{a}_{[x]}}+{\cdot}001\right\}$$

It should be explained that formula (A) was put forward for general use, while formula (B) was derived—so far as the loadings are concerned—from the actual experience of a particular office and was not suggested by its author as a "model" premium, and formula (C) was framed to provide "the minimum rate of office premium consistent with actuarial safety."

For the sake of comparison the premiums have been calculated by all four formulas on the basis of $O^{[NM]}$ $3\frac{3}{4}$ per-cent, $O^{[M]}$ $3\frac{1}{2}$ per-cent and $O^{[M]}$ $3\frac{3}{4}$ per-cent respectively, although the bases originally advocated for the various formulas are:

Formula.		Date.		Bases.
(A)	...	1881	...	$H^{[M]}$ (Sprague's Select) 4 per-cent
(B)	...	1910	...	$O^{[M]}$ $3\frac{1}{2}$ per-cent
(C)	...	1912	...	$O^{[M]}$ $3\frac{1}{2}$ per-cent
(D)	...	1913	...	$O^{[NM]}$ $3\frac{3}{4}$ per-cent

A curious coincidence may be noted in the fact that, for the same bases of mortality and interest, formulas (A) and (D) give practically identical results, although their composition is quite different. Yet it must be borne in mind that while the initial

loading of 1 per-cent on the sum assured in Dr. Sprague's formula is not sufficient for present day requirements, the percentage loading is probably on the large side, including as it does $2\frac{1}{2}$ per-cent for fluctuations.

TABLE III.

Office Annual Premiums for £100 Assured.

Bases	Age at Entry														
	20			30			40			50			60		
	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*	£	*s.*	*d.*
Average Premiums of 25 Offices	1	11	7	2	0	0	2	14	4	3	18	8	6	2	2
Highest Premium at each age	1	13	8	2	2	2	2	16	3	4	1	2	6	6	5
Lowest Premium at each age	1	8	2	1	17	2	2	11	11	3	15	6	5	15	3
Average Premiums of 5 Offices publishing lowest rates at age 40	1	10	1	1	18	2	2	12	3	3	16	2	5	18	1
$O^{[NM]}$ $3\frac{3}{4}$ per-cent															
Formula (A)	1	12	2	2	1	2	2	16	2	4	1	11	6	6	11
,, (B)	1	13	4	2	2	4	2	17	5	4	3	2	6	8	5
,, (C)	1	11	9	2	0	4	2	14	10	3	19	7	6	2	11
,, (D)	1	12	5	2	1	4	2	16	5	4	2	1	6	7	1
$O^{[M]}$ $3\frac{1}{2}$ per-cent															
Formula (A)	1	10	11	1	19	8	2	14	0	3	17	9	5	18	5
,, (B)	1	11	10	2	0	8	2	15	1	3	18	11	5	19	9
,, (C)	1	10	5	1	18	11	2	12	8	3	15	7	5	14	8
,, (D)	1	11	0	1	19	10	2	14	1	3	17	11	5	18	6
$O^{[M]}$ $3\frac{3}{4}$ per-cent—															
Formula (A)	1	9	11	1	18	7	2	12	9	3	16	4	5	16	11
,, (B)	1	11	0	1	19	9	2	13	11	3	17	8	5	18	4
,, (C)	1	9	6	1	17	11	2	11	6	3	14	3	5	13	3
,, (D)	1	10	1	1	18	9	2	12	11	3	16	6	5	17	0

It will be seen that in every case the formulas (A), (B), (C) and (D) by $O^{[NM]}$ $3\frac{3}{4}$ per-cent produce larger premiums than the average of the 25 offices, but that the differences in the case of formula (C) are slight.

In comparing with the premiums calculated by $O^{[M]}$ $3\frac{1}{2}$ per-cent, it will be noticed that the average rates of the 25 offices lie between those produced by formulas (A) and (B), except at age 60, where both formulas give much lower rates. Formula (C) produces rates fairly close to the average of the five low-rate

offices except at age 60. As regards formula (B) it was only to be expected that it would bring out rates slightly above the average for most ages, seeing that the loadings were derived from the apportionment of the expenses of an office whose ratio of Commission and Expenses to Premium Income was 15 per-cent, which is rather above the average, and that all the new business expenses were charged against the new business in the initial loading. On the other hand, Mr. Andras's formula (C) is based on the idea of producing the lowest rate which could be used with safety, and with this end in view he only provided a percentage loading of "the bare renewal commission of $2\frac{1}{2}$ per-cent", leaving a portion of the expenses to be met out of profits.

The premiums based on $O^{[M]}$ $3\frac{3}{4}$ per-cent have been inserted for comparative purposes, but it is doubtful if many British Offices would be justified in calculating premiums on so favourable a basis as to mortality and interest. It will be noticed, however, that formulas (A) and (D) give results fairly close to the average premiums of the five low-rate offices, that formula (B) gives rates at ages 20, 30 and 40 closely resembling the "Average of the 25 Offices", and that formula (C) produces rates which are higher at ages 20 and 30, but lower at ages 40, 50 and 60, than the lowest rates of the 25 offices.

In the next Table are given premiums calculated by various hypothetical formulas so as to correspond as nearly as possible with the average rates of the 25 offices, and the five low-rate offices, respectively.

It is seen that on the basis of $O^{[NM]}$ $3\frac{3}{4}$ per-cent the "Average of the 25 Offices" may be very closely represented by a formula which is similar to formula (C) save that the percentage factor is 1·02 instead of 1·025, while the "Average of the five Offices" may be almost exactly reproduced by a formula which is identical with formula (A) without the $7\frac{1}{2}$ per-cent addition. In the former case the loadings are on the small side and could only be defended on the ground that a portion of the expenses would be met out of profits. In the latter case I think it would be agreed that the loadings, namely, 1 per-cent on the sum assured and a constant of 2*s*. 6*d*., are altogether inadequate for the expenses of a commission paying office.

Assuming $O^{[M]}$ $3\frac{1}{2}$ per-cent the loadings would appear to be adequate in the formula which represents the "Average of the 25 Offices" except at age 60, where some allowance would appear to have been made by the majority of the 25 offices for the

TABLE IV.

Average Office Premiums for £100 Assured.

Bases	Age at Entry 20			30			40			50			60		
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
Average Premiums of 25 Offices	**1**	**11**	**7**	**2**	**0**	**0**	**2**	**14**	**4**	**3**	**18**	**8**	**6**	**2**	**2**
Formula:															
$1{\cdot}02\left\{\pi_{[x]}+\frac{\cdot 02}{a_{[x]}}+\cdot 00125\right\}(O^{[NM]}3\frac{3}{4}\%)$	1	11	7	2	0	2	2	14	7	3	19	2	6	2	4
$1{\cdot}08\left\{\pi_{[x]}+\frac{\cdot 02}{a_{[x]}}+\cdot 001\right\}(O^{[M]}3\frac{1}{2}\%)$ ...	1	11	6	2	0	5	2	14	11	3	19	1	6	0	3
$1{\cdot}075\left\{\pi_{[x]}+\frac{\cdot 02}{a_{[x]}}+\cdot 0015\right\}(O^{[M]}3\frac{3}{4}\%)$ ...	1	11	6	2	0	3	2	14	7	3	18	5	5	19	4
Average Premiums of 5 low-rate Offices ..	**1**	**10**	**1**	**1**	**18**	**2**	**2**	**12**	**3**	**3**	**16**	**2**	**5**	**18**	**1**
Formula:															
$\pi_{[x]}+\frac{\cdot 01}{a_{[x]}}+\cdot 00125(O^{[NM]}3\frac{3}{4}\%)$	1	9	11	1	18	4	2	12	3	3	16	2	5	18	1
$1{\cdot}06\left\{\pi_{[x]}+\frac{\cdot 02}{a_{[x]}}+\cdot 0005\right\}(O^{[M]}3\frac{1}{2}\%)$ ...	1	9	10	1	18	7	2	12	10	3	16	6	5	17	0
$\frac{1}{\cdot 94}\left\{\pi_{[x]}+\frac{\cdot 02}{a_{[x]}}+\cdot 001\right\}(O^{[M]}3\frac{3}{4}\%)$ ...	1	10	1	1	18	9	2	12	11	3	16	6	5	17	0

heavier mortality of the $O^{[NM]}$ table. The loadings in the formula to represent the "Average of the five Offices" appear to be sufficient for an office with expenses well below the average and earning at least $3\frac{3}{4}$ per-cent on its funds.

In the case of the formulas based on $O^{[M]}$ $3\frac{3}{4}$ per-cent, the loadings in the formula representing approximately (except at age 60) the "Average of the 25 Offices" are on the large side. It will be observed that the formula used to approximate to the "Average of the five Offices" is formula (D) of Table III.

It is clear, I think, from a consideration of Tables III and IV that the premium rates now in force in many offices either are based on a mortality decidedly more favourable than $O^{[NM]}$ select, or are founded on loadings which would require to be supplemented by profits.

The argument has been used that an office is justified in

quoting rates of premium for whole life non-profit assurances with less loading than is strictly speaking required for the expenses incurred, on the ground that the balance of the expenses will be recouped from profits. But profits (apart from "loading") arise mainly from mortality and interest. As regards the first source, it is quite possible that some of the offices which have adopted the $O^{[M]}$ table in the calculation of non-profit premiums may meet with loss instead of profit from mortality. A certain amount of profit will arise from interest in the usual case of offices calculating their premiums at a rate not higher than $3\frac{1}{2}$ per-cent, and earning about $3\frac{3}{4}$ per-cent; but it is evident from a consideration of Table III that a margin of much more than $\frac{1}{4}$ per-cent in the rate of interest is required to balance the difference of mortality between $O^{[NM]}$ and $O^{[M]}$. If offices "back their skill in selecting lives by using the $O^{[M]}$ table",* earn $3\frac{3}{4}$ per-cent net on their funds and calculate their premiums on the usual basis of $3\frac{1}{2}$ per-cent, a constant margin in each office premium is thereby provided of from 1*s.* to 1*s.* 6*d.* per-cent. By discounting this margin. *i.e.*, by basing the calculations of the premiums on $O^{[M]}$ $3\frac{3}{4}$ per-cent, the resulting premiums will be those given in Table III (for the various formulas) under $O^{[M]}$ $3\frac{3}{4}$ per-cent, and, as previously mentioned, formulas (A) and (D) give results fairly close to the "Average of the five low-rate offices." At the same time I think it is advisable for most offices using the $O^{[M]}$ table to leave a margin for adverse mortality, and if they do not provide such a margin in the loading then I think they should do so in the rate of interest assumed.

THE FACTOR OF COMPETITION.

After the various bases have been decided on for the calculation of office premiums, there remains the question of competition. This question is particularly acute in the case of whole life non-profit rates, as there is no compensating element of bonus to hold before the public. It is evident from the divergency in the actual rates quoted that there are some offices which can obtain business at relatively high rates of premium for reasons such as:

(*a*) The standing of the office,
(*b*) The persuasive powers of its representatives.
(*c*) The compulsory nature of the assurance (in most "financial" cases),

* Mr. G. W. Richmond, T.F.A. vi, p. 117.

but the tendency in recent years has been to bring out "minimum" rates of premium.

Competition between offices is a good thing in so far as it prevents them from being over-cautious in their estimates in the calculation of premium rates, and it may be that in the past there was a tendency to charge rather higher premiums for whole life non-profit policies than the circumstances required. Naturally enough, when the public began to find that in many offices the participating policies were on the average more profitable to the assured than non-profit policies, the offices commenced to reduce their non-profit rates, and in effect to allow those assured under the revised rates a portion of the profits. The system of discounted bonus policies has also helped to reduce the non-profit rates. So long as the assured took the risk of having his sum assured reduced or his premium increased in the event of the profits falling below the amount discounted, it was possible to have the discounted bonus rates in some cases actually lower than the non-profit rates. At the same time it has been considered that it would seriously injure the reputation of an office to enforce its rights in the event of the bonus not being sufficient. To make the contract more secure, however, the further development arose of discounted bonus policies with sums assured and premiums both guaranteed, the premiums for such policies being of necessity higher than the non-profit rates. All these influences have tended to reduce the rates for non-profit policies, so much so that in some cases the designation "non-profit" is really a misnomer. Instead of the non-profit policies in such cases paying a special extra premium for the right to have their contracts paid in full (at the expense if necessary of the surplus arising from the participating policies) it would appear that they are being given a considerable portion of profit in advance by way of reduction of premiums. It is to be hoped that these offices will be able to realize sufficient profit from the non-profit policies to enable them to pay their way, without assistance from other sources, but it appears doubtful in some cases if they will be successful in this.

It has been suggested, I believe, that offices should agree to a uniform scale of premiums for whole life non-profit assurances, but I do not think that this would be a practicable step in view of the varying rates of interest, mortality and expenses, in different offices, and of the interference with freedom which such a plan involves. There is the further objection that, if every

office were to adopt the same rates of premium for whole life non-profit business, many anomalies would arise between that section and the other sections of the life business. I think, therefore, that we may dismiss this suggestion as impracticable under present conditions.

Of more importance at the present time than a uniform scale of premiums is a uniform scale of commission. Apart from the ethics of the question there can be no doubt that the growing custom of paying inordinate commissions for the introduction of business is detrimental to life assurance business. It is quite possible for an office to be in such a position that it can only compete with others by outstripping them in the matter of commission, but in very few cases can it be argued that such a practice is for the benefit of the assured, who are after all the chief parties to be considered in the question.

On the somewhat delicate question of "unearned commission", under which title might be placed all rebates and commissions to "own-case agents", there is a considerable difference of opinion. On the other side of the Atlantic, both in Canada and the United States, the reaction against rebating was so strong that the legislatures of these two great countries saw fit to enact laws forbidding the giving or taking of rebates. In this country we do not seek or desire prohibitive legislation of this nature, but it seems to me that the system of rebating:

(1) is opposed to the mutual or co-operative basis of life assurance, seeing that it operates in favour of those who take advantage of their knowledge in obtaining rebates from the offices, as against those who either are not aware of the practice, or who consider that they are not entitled to any rebate.

(2) is detrimental to the genuine agent.

II.—RESERVE VALUES.

Whatever bases of mortality, loading and interest have been used in the construction of the office premiums, the points for each office to consider, directly or indirectly, in the valuation of a whole life non-profit business are:

(1) The amount of the office premiums receivable.

(2) The basis of mortality which may safely be adopted, irrespective of the mortality bases of the premiums receivable.

(3) The rate of interest which it may be safely assumed will be received while the policies are on the books.

(4) The annual amount required to meet the expenses.

In connection with (3) and (4) a complication arises through offices valuing at a rate of interest considerably below the average rate actually earned by them. Especially is the process complicated with offices valuing at $2\frac{1}{2}$ per-cent interest, and earning from $3\frac{3}{4}$ per-cent to 4 per-cent on their funds; the result being in such cases that, unless the offices make an additional reserve for expenses, beyond the ordinary Net Premium reserve, the margin between the office premiums and the net premiums is very small and at some ages even negative under certain scales of premiums now in use, although these offices in fact are providing a considerable proportion of their expenses from excess interest. It is, however, the practice of several such offices to reserve a specific percentage of their whole life non-profit office premiums to provide for expenses, thereby departing from the principles of a net premium valuation.

According to the Board of Trade Returns, most offices value their whole life non-profit business on the same bases as to mortality, interest, &c., as the participating business, although, as I have already mentioned, several offices reserve a fixed deduction from their office premiums to meet future expenses, irrespective of the amounts of the net premiums.

As the majority of offices at present are earning a rate of interest of rather over $3\frac{3}{4}$ per-cent net on their funds, and are likely to continue to earn at least that rate for many years to come, it would appear that $3\frac{1}{2}$ per-cent is a sufficiently low rate to be used by such offices in the valuation of their whole life non-profit business.

The $O^{[NM]}$ table was not intended to be used for the purpose of ascertaining the reserves of policies, but it is of interest to compare the values of 100 ${}_{n}V_{[x]}$ by the $O^{[NM]}$ table with the corresponding values by the $O^{[M]}$ table.

It is seen that for durations up to about 20 years the $O^{[NM]}$ table gives smaller values, and for longer durations the two tables give almost identical results. But it must be remembered that even in a net premium valuation—if a rate of interest not much less than that actually earned be assumed—it is necessary to have a sufficient difference between the office premium and the valuation net premium to provide for future expenses. Now,

TABLE V.

Reserve Values (select at entry) of £100 Assured. Interest $3\frac{1}{2}$%.

Age at Entry (x)	Duration (n)	$O^{[NM]}$	$O^{[M]}$	Ratio $\frac{O^{[M]}}{O^{[NM]}}$
20	5	4·30	4·64	108
	10	8·75	9·24	106
	20	19·76	20·03	101
	30	33·52	33·53	100
	40	49·18	48·96	100
	50	64·81	64·46	99
	60	78·03	77·70	100
30	5	6·19	6·53	105
	10	12·58	13·05	104
	20	27·58	27·73	101
	30	44·63	44·50	100
	40	61·67	61·36	99
	50	76·07	75·75	100
40	5	8·90	9·26	104
	10	17·80	18·31	103
	20	37·16	37·27	100
	30	56·49	56·32	100
	40	72·84	72·59	100
50	5	12·65	13·14	104
	10	24·49	25·24	103
	20	47·72	47·94	100
	30	67·36	67·33	100
60	5	17·60	18·38	104
	10	32·34	33·63	104
	20	57·76	58·36	101

as I have already attemped to show, the premiums charged by a number of offices at the present time are only sufficient to provide for $O^{[M]}$ mortality. Assuming that these offices wished to leave the original loading in their premiums intact for future expenses, and that the $O^{[NM]}$ table corresponded with their actual mortality, the select net premium reserve required to be made for their recent policies would be,

$$A^{O^{[NM]}}_{[x]+n} - \pi^{O^{[M]}}_{[x]}(1 + a^{O^{[NM]}}_{[x]+n})$$

which is equal to

$$_nV^{O^{[NM]}}_{[x]} + (\pi^{O^{[NM]}}_{[x]} - \pi^{O^{[M]}}_{[x]})(1 + a^{O^{[NM]}}_{[x]+n})$$

In fact the $O^{[NM]}$ net premium reserve $_nV_{[x]}$ would require to be increased by the commuted value of the deficiency in

the original office premium. This demonstration of the fact that an adequate reserve cannot be built out of an inadequate premium seems obvious enough from first principles, but I think that the net premium method of valuation sometimes serves to obscure the actual facts as to loading, mortality and interest, and that it can only be justified where it produces reserves at least as strong as those by a method of valuation which takes directly into account all the various factors.

Assuming then that the office premiums have been constructed on a correct mortality basis, $O^{[NM]}$ or $O^{[M]}$ as the case may be, a select net premium valuation by the $O^{[M]}$ table could safely be used in either case, the lighter mortality by the $O^{[M]}$ table as compared with the $O^{[NM]}$ table being counteracted by the larger loading margin provided by the $O^{[M]}$ table. Most offices, however, are reluctant to go to the trouble and expense of conducting a select valuation and rely instead on the broad averages of an aggregate table for valuation purposes.

A net premium valuation by an aggregate table ignores both selection and unequal incidence of expenditure, and Mr. Jas. Sorley in 1878 (*J.I.A.*, vol. xxi, p. 192) showed that, on the basis of the H^M experience, the one element counteracted the other at the end of five years. His general conclusion was that in the case of an office obtaining new business "at a moderate cost," a valuation by an aggregate table could be considered "as a "compromise or *viâ mediâ*, where selection, expenses, occupation, "residence and other influences—some affecting the result in "one way and some in another—are all alike ignored, and whose "defence takes its stand on the broad principle of averages, "which lies at the foundation of all actuarial questions."

As however Mr. Sorley had assumed the (now) very moderate rate of expense of 50 per-cent of the new premiums and 7 per-cent of the renewals, and as the conditions in regard to mortality had also changed, Dr. Jas. Buchanan in 1904 (T.F.A. II, p. 195) investigated the question again on the basis of Mr. King's Model Office figures, and demonstrated that in an office valuing its whole life participating policies by O^M 3 per-cent and experiencing $O^{[M]}$ mortality, with an expense ratio of 75 per-cent of the new premiums and $7\frac{1}{2}$ per-cent of the renewals, and with average rates of premium, a balance was established in about seven years between the mortality profit shown by using the aggregate table and the extra expenses actually incurred over a level expenditure of $12\frac{1}{2}$ per-cent of the office premiums.

In regard to whole life non-profit policies, Dr. Buchanan said that a longer time than seven years would require to elapse before the office could write off its initial expenses in this way. "But," he adds, "if the class taken as a whole can, after providing for its own claims and expenses, show a margin, there is no injustice in throwing part of the cost of new business on the older policies, for no bonus rights of these policies have to be considered."

An alternative and more direct method of writing off initial expenses is by making an addition to the net premium of the annual instalment required to redeem these expenses (or a portion thereof) during the remainder of life. It has been urged against this method that if generally adopted it might possibly be used to cover up extravagant expenditure, and that, inasmuch as the method virtually takes credit as an asset for expenses incurred in the past on the assumption that these expenses will be repaid out of the loadings of future premiums, there would be a danger of serious loss to the office in the case of withdrawals in the early years. If, however, the amount of the initial expenses thus taken credit for be limited to about 1 per-cent on the sum assured, and if negative values be excluded, the method would appear to be both safe and reasonable.

In Table VI a comparison is given between the Reserve Values deduced from the $O^{[M]}$ and $O^{[NM]}$ tables respectively, by the formula

$$ {}_nV_{[x]} - \cdot 01 \times \frac{\mathbf{a}_{[x]+n}}{\mathbf{a}_{[x]}} \text{ which is equal to } 1\cdot 01\, {}_nV_{[x]} - \cdot 01 $$

(allowance being made for selection at entry, and for the redemption of initial expenses of 1 per-cent), and the values by the aggregate formula ${}_nV_x$. Interest has been taken at $3\frac{1}{2}$ per-cent for the reasons already stated.

It is seen that for ages at entry 20 and 30, at all the durations given above, the O^M values are higher than those by the select formulas, but for ages at entry 50 and 60 the O^M values are slightly less. On the whole the O^M values would appear to give the highest reserves in the aggregate. In order, however, to have a complete comparison between policy-values on different bases, it is necessary to make a full valuation by the various bases. Mr. King's Model Office provides us with the total reserves for offices of various ages on the bases of both O^M and $O^{[M]}$ (select at entry). For the latter table the reserves are only given at 3 per-cent interest, and I have therefore taken the reserves in both cases at that rate. In the first place the O^M and the $O^{[M]}$

Table VI.

Reserve Values (aggregate) of £100 Assured, compared with Select Values with allowance for Initial Expenses of 1 per-cent on Sum Assured.

Interest $3\frac{1}{2}$ per-cent.

Age at Entry (x)	Duration (n)	$100_n V_x$ O^M	$100\{1{\cdot}01_n V_{[x]} - {\cdot}01\}$ $O^{[NM]}$	$O^{[M]}$
20	1	·85	·00	·06
	2	1·71	·89	·98
	3	2·60	1·72	1·89
	5	4·43	3·34	3·69
	10	9·35	7·84	8·33
	20	20·68	18·96	19·23
	30	34·29	32·86	32·87
	40	49·69	48·67	48·45
	50	65·08	64·46	64·10
30	1	1·15	·38	·43
	2	2·32	1·66	1·74
	3	3·51	2·85	3·02
	5	5·96	5·25	5·60
	10	12·50	11·71	12·18
	20	27·51	26·86	27·01
	30	44·50	44·08	43·94
	40	61·48	61·29	60·97
40	1	1·58	·96	1·00
	2	3·20	2·79	2·87
	3	4·84	4·55	4·70
	5	8·21	7·98	8·35
	10	17·16	16·98	17·49
	20	36·57	36·53	36·64
	30	55·97	56·05	55·88
50	1	2·25	1·80	1·85
	2	4·53	4·45	4·52
	3	6·83	6·98	7·15
	5	11·51	11·78	12·27
	10	23·43	23·73	24·49
	20	46·85	47·20	47·42
60	1	3·14	3·03	3·07
	2	6·27	6·83	6·91
	3	9·39	10·38	10·57
	5	15·60	16·77	17·56
	10	30·59	31·66	32·97
	20	56·52	57·34	57·94

reserves were taken from the tables given by Mr. King (*J.I.A.*, vol. xl, p. 12), and the reserves by the latter table were then adjusted in accordance with the formula $1{\cdot}01_n V_{[x]} - {\cdot}01$ as shown below in Table VII.

TABLE VII.

Mr. George King's Model Office. Interest 3 *per-cent. Comparison between Aggregate Net Premium Reserves by the* O^M *Table and Reserves (allowing for Initial Expenses of* 1 *per-cent on the Sum Assured) by the* $O^{[M]}$ *Table.*

Age of Office (years)	Number of Policies	ΣV $O^{[M]}$	$1{\cdot}01 \times (3)$	$\cdot 01 \times (2)$	$(4)-(5)$ $\Sigma V'$ $O^{[M]}$	ΣV O^M	Ratio per-cent $\frac{(6)}{(7)}$	Age of Office (years)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
5	543,574	28,430	28,714	5,436	23,278	24,918	93·42	5
10	973,117	87,756	88,634	9,731	78,903	80,278	98·29	10
15	1,334,534	167,361	169,035	13,345	155,690	157,397	98·92	15
20	1,640,264	259,725	262,322	16,403	245,919	248,435	98·99	20
25	1,895,857	357,582	361,158	18,959	342,199	345,693	98·99	25
30	2,103,303	452,991	457,521	21,033	436,488	440,973	98·98	30
35	2,264,464	538,894	544,283	22,645	521,638	526,969	98·99	35
40	2,383,021	610,043	616,143	23,830	592,313	598,306	99·00	40
45	2,464,773	664,048	670,688	24,648	646,040	652,496	99·01	45
50	2,515,624	700,420	707,424	25,156	682,268	688,999	99·02	50

It is seen that the O^M 3 per-cent valuation is stronger throughout, and this would appear to be a further vindication of the use of an aggregate net premium valuation as giving a reserve on the safe side. It will perhaps put the matter in a clearer light if we consider the inverse problem, namely, as to what amount of initial expenses is allowed for by using an O^M 3 per-cent valuation as an approximation to an $O^{[M]}$ 3 per-cent valuation with allowance for such expenses. Of late years the use of an ultimate table for the purposes of valuation has been recommended as setting off mortality gain against initial expenses, and I have therefore made calculations in addition on the basis of an $O^{M(5)}$ 3 per-cent valuation, similar to those in the case of the O^M 3 per-cent valuation. *i.e.*, with a view to ascertaining the initial expenses taken credit for on an $O^{[M]}$ 3 per-cent basis. As compared with an $O^{[M]}$ select 3 per-cent net premium valuation the maximum amount of initial expenses allowed for by an O^M 3 per-cent net premium valuation is 17*s.* per-cent, and by an $O^{M(5)}$ 3 per-cent premium valuation 25*s.* 2*d.* per-cent. Both these sums are well within the average initial expenses allowed for in the calculation of the office premium and actually incurred, but many offices in arriving at their reserves would probably hesitate to take credit for a larger initial expense than 20*s.* per-cent on the sum assured, and would therefore value their policies on the ordinary O^M net premium basis.

TABLE VIII.

Mr. George King's Model Office.

Age of Office (years)	Comparative Reserves O^{M} 3 % = 10,000		Initial Expenses per-cent allowed for on $O^{[M]}$ 3 per-cent basis		Age of Office (years)
	$O^{[M]}$ 3 % Valuation	$O^{M(5)}$ 3 % Valuation	O^{M} 3 % Valuation	$O^{M(5)}$ 3 % Valuation	
5	11,410	9,613	·68	·87	5
10	10,932	9,672	·84	1·14	10
15	10,633	9,719	·85	1·23	15
20	10,454	9,755	·82	1·26	20
25	10,344	9,783	·77	1·26	25
30	10,273	9,804	·73	1·25	30
35	10,226	9,819	·69	1·24	35
40	10,196	9,830	·66	1·23	40
45	10,177	9,838	·64	1·23	45
50	10,166	9,844	·63	1·22	50

The preceding calculations on the basis of the Model Office assume 3 per-cent interest, but the comparative results in all probability would not be greatly affected if the rate were taken at $3\frac{1}{2}$ per-cent.

III.—SURRENDER VALUES.

The question of the amount to be paid on the surrender of a whole life non-profit policy is one of considerable importance at the present time, when so many offices guarantee to pay certain amounts as surrender values in the case of each policy issued under this class.

The principal points to be considered in arriving at the surrender values which may be allowed are :

(1) Mortality,
(2) Expenses,
(3) Adverse selection,
(4) Interest.

Mortality.—It is desirable, I think, to use the same mortality experience in the calculation of surrender values as is used by the office for the calculation of its reserve values. I therefore propose to base the calculations on the $O^{[M]}$ or O^{M} experience.

Initial Expenses.—The question of the initial expenses incurred in the acquisition of new business is very important, especially in the early years of the policy. Unless the office

recoups itself for the outlay which it has made to secure the business, a burden is thrown on the remaining policies. Probably the most equitable method of giving effect to this item is to spread it over the term of the policy by dividing it by a whole life annuity and to deduct the value of the outstanding annual charges in arriving at the surrender value.

Renewal Expenses.—The renewal expenses consist of renewal commission and such general office administration charges as can be set against the premium income. The former, of course, ceases on surrender. In regard to the latter, Mr. F. W. Fulford (*J.I.A.*, vol. xxxv, p. 199), advocated a deduction of $2\frac{1}{2}$ per-cent from each of the future office premiums as representing the contribution due from the person surrendering for future general expenses, but this deduction seems to penalise him too heavily. If he repays the outstanding balance of the initial expenses, that is, I think, as much as can reasonably be recovered on account of expenses.

Adverse Selection.—It is a much debated question as to what selection is exercised against the office in the case of a surrender. Mr. A. D. Besant (*J.I.A.*, vol. xxxv, p. 234) gave an analysis of surrenders in a commission-paying office in which 77 per-cent of the surrenders were due to necessity and not to deliberate selection against the office. It may, however, be confidently stated that a policy as a rule would not be surrendered where the life assured was in very bad health, especially when there are now so many facilities for maintaining an assurance in force, either automatically under a non-forfeitable system or by borrowing thereon. Statistics are conflicting on the general question, but, it seems to me that if there is even a slight presumption that selection is exercised against the office, the calculation of the surrender value should be made on the safe side by making some deduction on that account, especially for the longer durations, where a person surrendering is more likely to be a select life than one who has been assured only for a few years. After the policy has been in force for some time, it is not infrequent that the reason for the surrender is that the policy has served its purpose and that the person surrendering feels that he can put his money to more profitable use in other directions, while on the other hand a withdrawal in the early years (at any rate at the younger ages at entry) is very often found to be due to pecuniary difficulties which are not as a rule conducive to longevity, or merely to over-persuasion at the outset. There

is the further point that it is not unusual in cases where the policy has been in force for a number of years for the office to do its best to dissuade the policyholder from surrendering. In my own experience I have found that a letter from the office pointing out the alternatives of loan or paid-up policy or still better a personal visit from a representative of the office will frequently prevent the surrender from taking place. It would seem therefore that an office has grounds for assuming those who withdraw after a number of years to be select lives.

Rate of Interest.—Looking at the question of the surrender of a policy from the point of view of a sale in the open market, the office is entitled to assume a good investment rate of interest in calculating surrender values, especially for policies such as we are considering, where there is no question of future bonus to be brought into account. It might, I think, reasonably be stated that whereas at the present time it is comparatively easy for offices to obtain a net rate of nearly $4\frac{1}{2}$ per-cent on new investments, yet in the course of say, 20 or 30 years, there is a considerable prospect of such a rate being difficult to obtain, with the general increase in wealth which it is natural to assume. On this basis, I think, it would be argued that $4\frac{1}{2}$ per-cent is a suitable rate of interest on which to base the surrender values for the shorter durations, while for the longer durations a lower rate, say 4 per-cent, might be used, but that in practice the higher rate of $4\frac{1}{2}$ per-cent would be safer to use throughout, in arriving at a formula for future use.

But is it a correct standpoint for an office to take that it is in competition with buyers in the open market? No doubt there are cases where such competition does take place, but in these cases there are frequently special conditions which entitle the policy to a higher market value than the amount offered by the office for its surrender. Is it not the better point of view that an office in framing a table of surrender values—especially where such values are to be a guaranteed part of the contract—should consider what amounts it can equitably pay out to those who exercise their right of withdrawal, irrespective of the prices which might or might not be realized in the open market? On this basis, I think, a rate of interest of $3\frac{3}{4}$ per-cent or even $3\frac{1}{2}$ per-cent might be used, and the necessary deductions made from the values derived in that way. But, apart from the investment point of view, there is a practical advantage in using a high rate of interest in the calculation of surrender values, namely,

that a deduction is thereby made automatically from the reserve value.

I think, therefore, that while most offices would not lose by basing their surrender values on a rate of interest of, say $3\frac{3}{4}$ per-cent, they would be well advised to leave a margin of about $\frac{1}{4}$ per-cent for the disturbance and trouble caused by the surrender, and value at 4 per-cent.

FORMULAS.

In arriving at a formula which will give effect to the various points above mentioned, we are confronted with numerous difficulties. Assuming that the surrender values should be based on net premium reserve values, and that a rate of interest of 4 per-cent is sufficient to provide an adequate margin for contingencies, we have still to deal with the problems of Initial Expenses and Selection.

In order to give effect to the Initial Expenses outstanding, the plan has been frequently used of ignoring the first year's premium in the calculation, or, in other words, of assuming that the policy is taken out one year later than the actual date, the formula therefore being (for an aggregate table) ${}_{n-1}V_{x+1}$. The deduction of the first year's premium assumes that the first premium is absorbed by the initial expenses plus the mortality risk run in the first year, and it is interesting to see what amount of initial expenses this plan allows for on the basis of aggregate and select tables respectively:

Let $\quad \mathrm{I}=$ Initial Expenses per unit allowed for by above plan

then

$\dfrac{\mathrm{I}}{\mathbf{a}_x}\times\mathbf{a}_{x+n}=$ Outstanding Initial Expenses on the assumption that an equal amount is written off each year

$$={}_nV_x-{}_{n-1}V_{x+1}=\frac{\mathbf{a}_{x+n}}{\mathbf{a}_x}\times\frac{a_x-a_{x+1}}{\mathbf{a}_{x+1}}$$

whence $\quad \mathrm{I}=\dfrac{a_x-a_{x+1}}{\mathbf{a}_{x+1}}$

and similarly,

$$=\frac{a_{[x]}-a_{[x+1]}}{\mathbf{a}_{[x+1]}}$$ if the life be taken as select both at age $[x]$ and age $[x+1]$.

In Table IX below are given the values of I as derived in the above way for the O^{M} and $O^{[M]}$ tables respectively, assuming nterest at 4 per-cent per annum.

TABLE IX.

Initial Expenses allowed for (per £100 assured) by ignoring first year's premium.

Age at Entry (x)	Initial Expenses		Age at Entry (x)
	O^{M}4%	$O^{[M]}$4%	
(1)	(2)	(3)	(4)
20	·77	·62	20
30	1·06	·93	30
40	1·49	1·40	40
50	2·18	2·00	50
60	3·12	2·76	60

It is apparent from this Table that the amount of the initial expenses equivalent to ignoring the first year's premium is a quantity increasing with the age at entry. It is assumed, for instance, that the amount of the initial expenses incurred in the case of a policy on a life aged 20 at entry is rather less than one quarter of the expenses incurred in the acquisition of a policy on a life aged 60 at entry. The method therefore charges too little for the younger ages at entry and too much for the older, and can only be justified on the grounds of practical expediency. A special feature may be noted in column (3), viz.: that on the basis of $O^{[M]}$ 4 per-cent the initial expenses come to exactly 2 per-cent on the sum assured at age 50. If, therefore, it be granted that 2 per-cent is an equitable amount to charge for initial expenses at all ages at entry, and that the life is still select a year after entry, it follows that for all ages under 50 at entry the ignoring of the first premium makes too small a deduction, and that for ages above 50 at entry the deduction is too large.

Mr. G. S. Crisford.

Mr. G. S. Crisford (*J.I.A.*, vol. xxi, p. 301) advocated the following formula:

$$\cdot 95\,{}_{n}V_{x} - \cdot 025\,P_{x}(1 + a_{x+n})$$

where ·95 was taken as representing the average ratio which a policy value in the case of a life withdrawing would bear to

an aggregate reserve value, and ·025 as the proportion of the office premium equivalent to the annual amount due from the policyholder for future expenses.

Mr. F. W. Fulford.

Mr. F. W. Fulford (*J.I.A.*, vol. xxxv, p. 199) derived the formula :

$$_n(hV)_x-(1+a_{[x+n]})(r_{[x]}+s_x)$$

where $r_{[x]}=\dfrac{\cdot 021}{\mathbf{a}_{[x]}}$, *i.e.*, the annual equivalent of initial expenses of 2·1 per-cent.

$s_x=\cdot 025\text{P}_x$, *i.e.*, $2\frac{1}{2}$ per-cent of the office premium (for future general expenses, as in Mr. Crisford's formula).

New Formulas.—Apart from allowing a margin for disturbance which may be provided suitably in the rate of interest used, it would appear that the only two elements to be allowed for are :

(A) Initial expenses outstanding,

(B) Adverse selection by withdrawal.

In regard to the former, Mr. Crisford did not make any specific provision, but he provided for the deduction of an annuity sufficient to meet future expenses. Mr. Fulford, on the other hand, allowed both for initial expenses and for future expenses, and thus (as was pointed out in the discussion) charged expenses twice over against the person surrendering, for, if the initial expenses be recouped, the office will have enough in hand to pay the expenses of a fresh policy in place of the one surrendered.

If it be assumed that a life withdrawing is as select as one which has just been accepted as first-class after medical examination, and that the initial expenses (or such portion of them as was charged to the policy) were ·02 in respect of each unit assured, then the formula for the surrender value of a policy of £1 is

$$_n(hV)_x-\cdot 02\times\frac{\mathbf{a}_{[x+n]}}{\mathbf{a}_{[x]}}.$$

Now, it may be proved easily that this formula is equal to

$$1\cdot 02\,_n(hV)_x-\cdot 02$$

and we thus have a compact and simple formula.

The values of ${}_n(h\mathrm{V})_x$ are not tabulated, but the values of $a_{[x]}$ are, of course, given, and the formula may be put in terms of the annuities thus

$$1-k_{[x]}(1+a_{[x+n]}) \text{ where } k_{[x]}=\frac{1{\cdot}02}{\mathbf{a}_{[x]}}$$

in a form suitable for rapid calculation either by logarithms or the arithmometer.

There is, however, an objection to the above formula, on the score of the assumptions made by it as to adverse selection being too stringent, at any rate for the early years in the case of policies issued at the younger ages.

With a view to finding what deduction should be made from the O^{M} aggregate reserve values in order to provide for the assumption of a life being still select, the following table has been prepared showing the percentages of the select value ${}_n(h\mathrm{V})_x$ by the $\mathrm{O}^{[\mathrm{M}]}$ table to the aggregate value ${}_n\mathrm{V}_x$ by the O^{M} table, interest being taken at 4 per-cent.

TABLE X.

Ratios per-cent of Select Policy Values ${}_n(h\mathrm{V})_x$ to Aggregate Values ${}_n\mathrm{V}_x$
O^{M} experience. Interest 4%.

Duration	Age at Entry 20	30	40	50	60	Duration
1	80·3	88·6	93·9	91·6	89·1	1
2	81·7	88·7	93·6	91·9	89·1	2
3	82·0	89·4	93·6	92·0	89·0	3
5	82·7	90·1	93·5	91·9	89·0	5
10	84·8	91·7	93·5	91·6	89·2	10
20	89·4	93·3	93·1	91·5	...	20
30	92·0	93·5	92·9	...	...	30
40	93·0	93·5	...	...	...	40
50	93·5	...	...	...	...	50

It is seen that the maximum percentage in the examples given above is 93·9, and that for many durations the percentage does not vary greatly from this amount. For the younger ages at entry the percentage is much smaller for the shorter durations, but it is reasonable to assume that the selection really exercised against the office in these cases is comparatively small. I would suggest therefore. on the lines put forward by Mr. Crisford in his remarks in the discussion following Mr. Fulford's paper

(*see J.I.A.*, vol. xxxv, p. 231), that a uniform ratio of say 93 per-cent be taken as giving a practical and reasonable deduction for selection. In the case of the comparatively few entrants at ages 50 and upwards this percentage appears to be too high, but against this may be set the fact that the average policy effected at the older ages is for a larger amount, and that, as it has been charged with expenses in proportion to the sum assured, the deduction for selection need not be so stringent.

The formula $1{\cdot}02\,{}_n(hV)_x - {\cdot}02$ would thus become

$$\cdot 93 \times 1{\cdot}02\,{}_nV_x - \cdot 02$$

which may be taken as approximately equal to

$$\cdot 95\,{}_nV_x - \cdot 02^*$$

This formula has the following advantages:

(1) It is very simple to apply in practice, the values being obtained almost by inspection.

(2) It allows for the return of the balance of the level initial expenses assumed in the premium formula.

(3) It allows to a certain extent for adverse selection.

(4) It is elastic in form, as the deductions for initial expenses and selection may be varied according to the practice of different offices.

In the following table (Table XI) are shown, for a policy of 1,000, the surrender values by the two new formulas now proposed, the values produced by various formulas previously suggested, and the average of the surrender values given by first-class offices. Columns have also been added showing the Reserve Values by O^M $3\frac{1}{2}$ per-cent, and the amounts of $33{\cdot}\dot{3}$ per-cent and 25 per-cent respectively of the premiums (the premiums used being the average of the 25 offices), namely:

Column. *Formula, &c.*

(2) ${}_nV_x$ (O^M $3\frac{1}{2}$ per-cent).

(3) ${}_{n-1}(hV)_{x+1}$ ($O^{[M]}$ 4 per-cent).

(4) ${}_{n-1}V_{x+1}$ (O^M 4 per-cent).

(5) $A_{[x+n]}^{O^{[M]}4\%} \times \left(1 - \frac{P_{x+1}}{P_{x+n}}\right)$ where P_{x+1} and P_{x+n} are

* Some modification of the percentage would be advisable at very old ages. The select data in the B.O. Experience do not go beyond age 75, and I would suggest that, for each year of age attained after 75, the percentage of 95 be increased by 1 up to a limit of 100 for age attained 80 onwards. On this basis the formula for age attained 80 and upwards would be ${}_nV_x - {\cdot}02$

the office premiums derived from the formula $P_x = 1{\cdot}1\pi_{[x]} + {\cdot}001$ ($O^{[NM]}$ $3\frac{1}{2}$ per-cent). This method was put forward by Dr. A. E. Sprague (*T.F.A.* III, p. 201), but it may be added that he recommended the use of $4\frac{1}{2}$ per-cent interest for $A_{[x+n]}$.

(6) $_n(hV)_x - {\cdot}02 \times \frac{\mathbf{a}_{[x+n]}}{\mathbf{a}_{[x]}} = 1{\cdot}02_n(hV)_x - {\cdot}02$ ($O^{[M]}$ 4 per-cent).

(7) ${\cdot}95_nV_x - {\cdot}02$ (O^M 4 per-cent).

(8) Average of surrender-values allowed by five first-class offices, the average of whose premium and expense rates does not differ greatly from the general average of all the offices.

(9) $33{\cdot}\dot{3}$ per-cent of the premiums paid (average of 25 offices).

(10) 25 per-cent of the premiums paid (average of 25 offices).

It is apparent from a consideration of this table that for ages at entry 20, 30 and 40, the values by the two new formulas proposed (columns (6) and (7)) are considerably less, and for age 60 at entry are greater, at the short durations than those produced by the formulas in columns (3) and (5). This was only to be expected, seeing that the new formulas deduct a larger amount for initial expenses at ages at entry under, say, 50 and a smaller amount thereafter, than is provided for by ignoring the first premium. At age at entry 50 the values in column (3) and column (6) are identical, the omission of the first premium being in this case exactly equal to the deduction of initial expenses of 2 per-cent on the sum assured (*see* Table IX). Apart from the difference in the values at the short durations, there is considerable similarity between columns (3), (5), (6), (7) and (8). It will be noticed that, after age at entry 20 and the early years for ages 30 and 40, as between column (7) (${\cdot}95_nV_x - {\cdot}02$) and column (8) (average of five offices), there is very little difference throughout the rest of the examples given.

As regards a minimum surrender value of a fixed percentage of the office premiums paid, it is apparent that the percentage of $33{\cdot}\dot{3}$ (guaranteed by a number of offices after payment of *three* premiums) gives values which are somewhat high in the early years, especially at the young ages, but low thereafter.* If a minimum surrender value of say 25 per-cent of

* See reply to Discussion.

Table XI.

Surrender Values of Policies for 1000.

Duration	Reserve Value O^M $3\frac{1}{2}\%$	Surrender Values: ${}_{n-1}(hV)_{x+1}$ $O^{[M]}$ 4%	${}_{n-1}V_{x+1}$ O^M 4%	A. E. Sprague 4%	New Formulas: $1{\cdot}02\,{}_n(hV)_x - {\cdot}02$ $O^{[M]}$ 4%	New Formulas: ${\cdot}95\,{}_nV_x - {\cdot}02$ O^M 4%	Average of 5 Offices	$33{\cdot}\dot{3}\%$ of Premiums	25% of Premiums
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Age at Entry 20. Annual Premium 15·792									
2	17	6	8	6	(−7)	(−5)	6	11	8
3	26	13	16	12	(−1)	2	12	16	12
5	44	27	33	24	14	18	24	26	20
10	94	66	78	60	54	61	58	53	39
20	207	166	185	154	154	161	148	105	79
30	343	292	317	279	282	286	269	158	118
40	497	438	471	427	431	431	419	211	158
50	651	588	629	580	582	580	580	263	197
Age at Entry 30. Annual Premium 20·000									
2	23	10	11	9	(−1)	0	9	13	10
3	35	20	22	18	9	11	18	20	15
5	60	41	45	37	30	32	37	33	25
10	125	98	106	91	88	90	89	67	50
20	275	235	251	224	227	226	219	133	100
30	445	393	420	383	386	385	379	200	150
40	615	554	593	548	550	547	551	267	200
Age at Entry 40. Annual Premium 27·167									
2	32	14	15	13	9	8	13	18	14
3	48	29	31	27	23	23	28	27	20
5	82	59	63	56	53	53	56	45	34
10	172	140	150	134	135	134	133	91	68
20	366	318	341	310	313	313	311	181	136
30	560	499	538	495	496	497	502	272	204
Age at Entry 50. Annual Premium 39·333									
2	45	20	22	20	20	21	21	26	20
3	68	41	45	40	41	42	43	39	29
5	115	83	91	81	83	84	84	66	49
10	234	191	209	188	191	194	193	131	98
20	469	406	444	406	406	413	418	262	197
Age at Entry 60. Annual Premium 61·083									
2	63	28	31	28	35	38	31	41	31
3	94	55	62	56	62	66	61	61	46
5	156	111	125	112	117	124	118	102	76
10	306	246	276	248	251	263	263	204	153
15	444	372	417	378	377	393	397	305	229

all the premiums paid were adopted until the values by the proposed methods, columns (6) and (7), became greater, the values in the early years for the younger ages at entry would be more in consonance with general practice, and would therefore be more suitable in competition. But it must be borne in mind that the lapse rate is heaviest in the first year or two after the issue of a policy, and that, by giving minimum surrender values as above for short terms in force, an office would have either to draw on other sources of profit or to reduce its expenses, say, by limiting the commission on premiums under £2 per-cent to one-half of the premium.

With regard to the number of premiums which should require to be paid before a policy becomes entitled to a surrender value, the great majority of offices do not allow any surrender value under whole life non-profit policies until three years' premiums have been paid. This practice would appear to be quite justified for ages at entry, say under 40, but it seems to deal rather hardly with policies effected at ages 40 and upwards, at normal rates of expense.

IV.—PAID-UP POLICIES.

There are a number of different formulas by which paid-up policies may be calculated, but for practical purposes I think the two most useful types of formula are those which are based on

(1) The actual office premiums.

(2) The surrender values.

In his paper on "Paid-up Policies and Surrender Values", read before the Faculty of Actuaries in January 1907 (T.F.A. III, p. 201), Dr. A. E. Sprague, in order to illustrate the construction of his paid-up policies, used a table of office premiums derived by the formula

$$P_x = 1{\cdot}1\pi_{[x]} + {\cdot}001 \ (\mathrm{O}^{[NM]} \ 3\tfrac{1}{2} \text{ per-cent}) \quad . \qquad . \quad (1)$$

so as to approximate to the average rates then charged by the British Offices. The premiums produced by this formula are given below together with those calculated by my formula—

$$P_x = \frac{1}{{\cdot}94}\left\{\pi_{[x]} + \frac{{\cdot}02}{\mathrm{a}_{[x]}} + {\cdot}001\right\} \mathrm{O}^{[NM]} \ 3\tfrac{3}{4} \text{ per-cent} \quad . \quad (2)$$

and it will be noted that they bear a fairly close resemblance to each other:

Age at Entry (x)	Office Premiums for 100			
	Formula (1)	Formula (2)	Formula (3)	Average of 25 Offices
20	1·600	1·621	1·556	1·579
30	2·067	2·067	1·997	2·000
40	2·835	2·821	2·723	2·717
50	4·136	4·104	3·953	3·933
60	6·417	6·354	6·110	6·108

In view of the similarity between the rates produced by the two formulas (1) and (2), and of the opportunity which is thereby afforded of making comparisons with Dr. Sprague's resulting paid-up policies, I purpose to use his premiums as the basis of the calculation of paid-up policies. As, however, these premiums are greater than the average of the 25 offices, I have also used the premiums obtained by formula (3), namely:

$$P_x = 1{\cdot}04\ \pi_{[x]} + {\cdot}001375\ (O^{[NM]}\ 3\tfrac{1}{2} \text{ per cent}) \quad \ldots\ldots \quad (3)$$

which, as will be seen above, very closely represents the average rates of the 25 offices (*see* Table III).

Dr. Sprague's formula for the paid-up policy was

$$1 - \frac{P_{x+1}}{P_{x+n}}$$

There are two modifications, however, which I would propose, namely:

(1) that the initial expenses be taken as a constant of say £2 per-cent on the sum assured, and not, as in the above formula, an amount increasing with the age at entry.

(2) that the initial expenses to be recouped on conversion to a paid-up policy, be calculated on the sum assured relinquished, and not, as in the above formula, on the whole sum assured, seeing that the use of office premiums allows to a certain extent for expenses.

Both of these modifications are given effect to in the following formula—

$$1 - \frac{P_x}{P_{x+n} - \frac{\cdot 02}{\mathbf{a}_{[x]}}}$$

for, let F $=$ the Free or Paid-up Policy.

P_x $=$ the office premium actually payable.

P_{x+n} $=$ the office premium at age $(x+n)$.

$\cdot 02$ $=$ the initial expenses.

(taking the functions as per unit),
then $P_{x+n}\ (1-F) = P_x$, but only on the assumption that the assured will, immediately after conversion, replace the assurance relinquished. If the assured cancels $1-F$ of his original sum assured, he must return to the office the equivalent of $\frac{\cdot 02}{\mathbf{a}_{[x]}}(1-F)$ per annum, thereafter. The modified equation therefore becomes $P_{x+n}(1-F) = P_x + \frac{\cdot 02}{\mathbf{a}_{[x]}}(1-F)$

$$\text{whence } F = 1 - \frac{P_x}{P_{x+n} - \frac{\cdot 02}{\mathbf{a}_{[x]}}}$$

The paid-up policies calculated by the last mentioned formula, using the rates of premium under formulas (1) and (3) respectively, and taking $\mathbf{a}_{[x]}$ by $O^{[NM]}$ $3\frac{1}{2}$ per-cent, are given below, together with Dr. Sprague's figures on the basis of $1 - \frac{P_{x+1}}{P_{x+n}}$ and premiums according to formula (1). I also give for comparison the paid-up policies produced by dividing the surrender values derived from the formula $\cdot 95\,{}_nV_x - \cdot 02$ (O^M 4 per-cent) by $A_{[x+n]}$ ($O^{[M]}$ 4 per-cent), the average paid-up policies of the five offices whose surrender values were used in column (8) of Table XI, and the total of the premiums received (using in this case the average premiums of the 25 offices). In column (2) the values of $\frac{{}_nV_x}{A_{x+n}}$ (O^M $3\frac{1}{2}$ per-cent) are shown as representing the full theoretical Paid-up Policy without deduction for expenses or selection.

TABLE XII.

Paid-up Policies in lieu of Original Policy of 1000.

Duration	$\frac{_nV_x}{A_{x+n}}$ (O^M $3\frac{1}{2}\%$)	A. E. Sprague $1-\frac{P_{x+1}}{P_{x+n}}$ P by Form: (1)	$1-\frac{P_x}{P_{x+n}-\frac{\cdot 02}{a_{[x]}}}$ P by Form: (1)	$1-\frac{P_x}{P_{x+n}-\frac{\cdot 02}{a_{[x]}}}$ P by Form: (3)	Surrender Value $A_{[x+n]}$ $O^{[M]}$ 4%: S.V. $=\cdot 95{}_nV_x-\cdot 02$ by O^M 4%	Surrender Value $A_{[x+n]}$ $O^{[M]}$ 4%: S.V. =25% of Prems. paid	Average Paid-up Policy of 5 Offices	Total Premiums Paid	Duration
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	Age at Entry 20.				Annual Premium 15·792				
2	62	23	...	...	...	33	28	32	2
3	92	47	14	10	8	48	50	47	3
5	150	94	65	60	69	77	91	79	5
10	282	208	189	182	210	...	194	158	10
20	498	423	416	408	442	...	391	316	20
30	665	604	604	597	620	...	570	474	30
40	790	745	747	741	753	...	715	632	40
50	877	845	847	843	845	...	824	790	50
	Age at Entry 30.				Annual Premium 20·000				
2	67	29	10	7	...	33	34	40	2
3	99	58	41	38	34	48	63	60	3
5	161	114	101	97	99	77	116	100	5
10	301	250	243	238	247	...	240	200	10
20	534	486	488	481	490	...	466	400	20
30	708	669	673	668	672	...	649	600	30
40	828	798	802	799	798	...	784	800	40
	Age at Entry 40.				Annual Premium 27·167				
2	74	35	30	28	22	37	42	54	2
3	109	69	67	64	58	51	78	81	3
5	177	136	136	132	129	83	144	136	5
10	333	290	295	290	291	...	290	272	10
20	581	542	550	546	547	...	536	543	20
30	754	722	728	725	725	...	717	815	30
	Age at Entry 50.				Annual Premium 39·333				
2	84	41	50	47	43	41	50	79	2
3	125	81	90	88	85	59	94	118	3
5	202	157	168	165	164	95	171	197	5
10	373	328	341	338	339	...	338	393	10
20	631	591	603	600	603	...	599	787	20
	Age at Entry 60.				Annual Premium 61·083				
2	96	46	67	65	63	52	59	122	2
3	141	92	112	110	109	76	110	183	3
5	227	177	197	195	196	121	197	305	5
10	412	362	381	379	383	...	383	611	10
15	559	511	528	526	532	...	531	916	15

It is seen that,

(*a*) as between Dr. Sprague's Paid-up Policies—col. (3)—and those in column (4), there is fairly close agreement for many of the durations given, and throughout for age at entry 40, but that there are large differences at the shorter durations for ages at entry 20 and 30, and for all durations at age at entry 60.

(*b*) the Paid-up Policies by the new formula on the basis of the two different scales of office premiums, columns (4) and (5), show very little difference throughout in spite of the considerable difference between the scales of premium used.

(*c*) The Paid-up Policies produced by dividing the surrender values according to the formula $\cdot 95\,{}_nV_x - \cdot 02$ (O^M 4 per-cent) by $A_{[x+n]}$ $O^{[M]}$ 4 per-cent, are in remarkably close agreement with those in columns (4) and (5), except for age at entry 20 and for the shorter durations at other ages at entry.

It is highly desirable that the surrender values and paid-up policies allowed by an office should be consistent with each other, so that, in the event of a person who has converted his assurance into a paid-up policy enquiring as to the surrender value, a quotation may be given him which will show, for each year elapsed since the conversion, a surrender value increasing from the basis of the surrender value of the policy immediately before conversion. It is often the case that the surrender value calculated in the ordinary way after a policy has been altered to a paid-up assurance is less than the former surrender value, and while various theoretical reasons may be given for this reduction, such reasons will be found, I think, invariably to be beyond the comprehension of "the man in the street." Dr. A. E. Sprague in the paper already referred to, with a view to overcoming this difficulty, calculated the paid-up policy and then obtained the surrender value therefrom by multiplying by the single premium $A_{[x+n]}$ $O^{[M]}$ 4 per-cent or $4\frac{1}{2}$ per-cent. It seems, however, preferable to obtain in the first place the formula for the surrender value, seeing that an office is much more frequently called upon to quote surrender values than paid-up policies, and that a formula as simple as possible is therefore desirable. If, after calculating the surrender value and the paid-up policy by independent methods which can each be justified on their own

merits, the relationship of the surrender value of a single premium, or paid-up, policy is found approximately to exist between the two calculations, then, as a practical and convenient method, there is much to say for the plan of obtaining the paid-up policy direct from the surrender value. As to the basis of the single premium for the conversion, or inversely for the surrender of the paid-up policy after conversion, I think that $A_{[x+n]}$ by $O^{[M]}$ 4 per-cent is quite suitable, as selection is thereby given effect to, and the rate of interest is the same as that used in the calculation of the original surrender value.

If a minimum surrender value of say 25 per-cent of the premiums paid be allowed, the paid-up policy should be consistent therewith. I have therefore calculated for durations 2, 3, and 5, "minimum" paid-up policies by dividing 25 per-cent of the premiums paid by $A_{[x+n]}$ $O^{[M]}$ 4 per-cent. These "minimum" paid-up policies represent, for the short durations at the younger ages at entry, a much larger amount of assurance than, strictly speaking, would be available, but it is arguable that it might be advisable to allow such values even at the cost of taking profits from other sources for this purpose.

SUMMARY.

In the preceding pages an attempt has been made to discuss the principles underlying the calculation of Office Premiums, Reserve Values, Surrender Values and Paid-up Policies, in the case of Whole Life Non-Profit Assurances, and to demonstrate formulas for each of these. It is not claimed that the particular formulas given above are suitable for general use without modification. The special circumstances of each office must be taken into account. The object of the paper is rather to show that in a class such as whole life non-profit assurances there is not much margin for readjustments, and that therefore it is essential for equitable treatment between this and other classes, and between entrants of different ages, that the calculations should take into account the most reliable data as to mortality, interest, expenses, and selection.

The main conclusions of the paper may be summarized as follows:

Office Premiums—

(1) There has been a decided decrease in the rates during the last 20 years or so.

(2) The British Offices $O^{[NM]}$ Table is the safest basis on which to calculate premiums and is not necessarily applicable only to "financial" lives.

(3) If the $O^{[NM]}$ Table be used in the calculation of premiums an office earning at least $3\frac{3}{4}$ per-cent interest on its funds might, with reasonable safety, use $3\frac{3}{4}$ per-cent as the interest basis, seeing that there is not the same need for a margin for fluctuations as there would be in using a table of mortality not derived from the experience of whole life non-profit lives.

(4) If offices "back their skill in selecting lives by using the $O^{[M]}$ Table" as the basis of their premiums, then it is advisable for them to leave, for adverse mortality, a margin of at least $\frac{1}{4}$ per-cent between the rate of interest earned and that used in the calculation of the premiums.

(5) The method of deriving the expense loadings should take into account the actual expenditure of the office, and not leave any appreciable portion of the expenses to be recouped out of profits, unless, of course, the office has good reason to anticipate that sufficient profits will arise *in the same class* to repay any expenditure not provided for in the loading.

(6) As regards "new business" expenditure, it may reasonably be argued that a portion thereof is for the general good of the office, and may therefore be provided by the "old" business.

(7) The premiums charged by a number of offices appear to have been calculated on a basis of mortality considerably lighter than $O^{[NM]}$.

(8) The influence of competition, while good so far as it helps to modify over-cautious estimates, seems to be producing premium rates which in some cases leave too much to the uncertain element of profits —especially when $O^{[M]}$ mortality is assumed.

(9) A fixed scale of premiums for all offices is impracticable under present conditions, but an agreement between offices operating in the United Kingdom as to the maximum amounts of commission to be paid seems

within the bounds of practical politics, and would have a beneficial influence on life assurance business.

Reserves—

(1) Provided the premiums are on an adequate basis as regards mortality, interest and loading, there would appear to be very little difference between select net premium valuations by the $O^{[NM]}$ and $O^{[M]}$ Tables respectively, the latter table in most cases showing slightly larger reserves.

(2) If it be granted that an office may reasonably consider that initial expenses to the extent of 1 per-cent on the sum assured can be allowed for in the reserves, then an ordinary aggregate O^M net premium valuation may be safely used, as it provides stronger reserves than a select valuation which takes these expenses into account.

(3) For the majority of offices a valuation on a $3\frac{1}{2}$ per-cent basis would appear to be sufficiently stringent in the case of non-profit assurances.

Surrender Values—

(1) The surrender values should be on the same basis as the reserve values in regard to mortality.

(2) The balance outstanding of such portion of the initial expenses as was directly due to the acquisition of the new business should be recouped on surrender.

(3) As between entrants at different ages it seems equitable to make the deduction for initial expenses on the same basis, and not to charge an increasing amount according to the age at entry by ignoring the whole of the first premium in the calculation of the surrender value.

(4) Adverse selection should be allowed for in the calculation, but the allowance should be modified to a certain extent by the age at entry and the duration.

(5) The rate of interest should be very little if anything in excess of the average rate earned by the office on its funds.

(6) The surrender values offered in the early years of a policy appear as a general rule to be too high for most ages at entry, and especially at the younger ages.

(7) For purposes of competition a larger value than is strictly available might be allowed during the early years of a policy, but it would appear inadvisable to guarantee more than 25 per-cent of the office premiums paid.

Paid-up Policies—

(1) It is desirable, if possible, to have the amounts of the paid-up policies consistent with both the office premiums and the surrender values.

(2) Initial expenses should be allowed for on a similar basis to that adopted in the calculation of the office premiums and the surrender values.

Concluding Remarks.

In conclusion I should like to say that I am conscious of having raised many very controversial subjects in this paper, but I trust that the discussion thereon will prove an excuse for my rashness, by throwing fresh light on these subjects.

I wish to express my gratitude to officials of offices and others who have kindly supplied me with useful information, also to Mr. H. G. Sharp, A.I.A., for his assistance in the calculation of the numerical tables, and for several valuable suggestions made during the preparation of the paper.

Abstract of the Discussion.

Mr. H. J. RIETSCHEL said that the author had pointed out at the beginning of the Paper that, whereas in the case of participating premiums, errors in the assumptions as to interest, expense and mortality could be rectified in the periodical divisions of profits, there was very little room for re-adjustment in the case of non-participating premiums, and further that it could not be argued on equitable grounds that it was just for the participating classes to make up any deficiency, except in case of emergency. He thought that in those words the author had laid down the principles which should guide them in considering the Paper which he had presented to the Institute. With regard to the Mortality

Tables to be employed, all British actuaries would agree that Select Tables of Mortality should form the basis of premiums, but considerable difference of opinion would probably disclose itself as to whether the $O^{[M]}$ or the $O^{[NM]}$ tables should be adopted. The author referred to the fairly general opinion that the higher rate of mortality shown by the $O^{[NM]}$ tables might be ascribed to the heavier mortality amongst the lives assured in connection with financial transactions, which would be found mainly among the non-participating policyholders. He confessed, however, that he had grave doubts whether that was the main cause of the difference, and he was confirmed in his doubts by experiences he had recently seen and by the paper read before the Institute in April last by Messrs. Rusher and Kenchington (*J.I.A.* xlvii, p. 433). In that paper the authors remarked: "The opinion has been expressed that the light rate of mortality shown by the British Offices' Experience under endowment assurances would not be a permanent feature of the experience of this class of assurance, as it was thought that the self-selection which had undoubtedly been operative in former times would not be maintained, seeing that endowment assurances have to a large extent replaced whole life assurances in modern practice. The same feature is, however, observed in such a marked degree in the present investigation that we are inclined to think that it is of a more permanent character than has been generally believed to be the case." Mr. Rusher, in the discussion, said that the same feature had been observed in the general experience of the Prudential. It was difficult to believe in self-selection, but if that were not the cause of the difference in the mortality always observed between whole-life and endowment assurances then the only alternative explanation would seem to be found in the two sections of policies not being composed of lives of the same social and financial standing. Non-participating policies, as a class, would certainly be more "financial" than participating policies, and if class affected the mortality the non-participating policies must produce the worse mortality. He was, therefore, driven to the conclusion that if the $O^{[M]}$ table represented the with-profit mortality, then it should not be used for non-participating premiums. In Table I the author showed that in an office experiencing $O^{[NM]}$ mortality, the employment of $O^{[M]}$ premiums would entail a loss to the office on account of adverse mortality equivalent to a loading of about 8 per-cent of the net premiums.

As regards the rate of interest to be employed in the calculations, the author said that it was highly desirable to leave a margin in the calculation of premiums for profit and fluctuations, and proposed $3\frac{1}{2}$ per-cent as a suitable basis for those offices adopting the $O^{[M]}$ tables as their mortality basis, whereas in the case of those offices adopting the safer $O^{[NM]}$ table and able to earn at least $3\frac{3}{4}$ per-cent on their funds the higher rate of $3\frac{3}{4}$ per-cent might be used. He presumed that the gain from the employment of a lower rate of interest was intended to compensate the office for the very possible higher rate of mortality. In his opinion it was extremely un-

desirable to confuse the items of interest and mortality, as such a process might easily prove misleading. As a matter of fact, $\frac{1}{4}$ per-cent difference in the rate of interest would by no means compensate for the realization of $O^{[NM]}$ instead of $O^{[M]}$ mortality, and he could see no reason why a table of mortality as near to the actual facts as possible should not be employed together with a rate of interest somewhat less than that likely to be earned, perhaps $\frac{1}{4}$ per-cent below.

The author dealt next with the question of loading for expenses, and from an investigation of the accounts of 47 British offices he arrived at an estimated initial commission of 23*s.* per-cent of the new sums assured. As, however, the new business was not analyzed according to classes of assurance, probably the author had included all the temporary survivorship assurances and the other classes upon which the initial commission was usually smaller than for whole life and endowment assurances. If that were the case, the 23*s.* per-cent was probably an under-estimate. The author pointed out that some offices which granted higher initial commissions than £1 per-cent on participating business refused to pay more than that rate for non-participating business, but for the other offices a loading for £1 per-cent initial commission might very possibly be insufficient. In any case the initial commission allowed for by the author was certainly a minimum. In discussing the initial general expenses, the author stated that it was not equitable to charge every item of expense incurred for new business against each new policy, and the reason he gave was that it was to the interest of the old policyholders to share in the profits of a going concern. The old policyholders had, however, already defrayed the initial expenses incurred in placing the past business on the books, and he failed to see how new non-profit business could prove of advantage to them if they were obliged to defray not only their own initial expenditure but in addition part of that of the future non-profit new business. Surely the only sound principle to adopt was, as far as possible, to make the non-participating business self-supporting and to rely on the extraneous sources of profit only for accidental, and therefore unforeseen, losses. As the author did not give his reasons for considering 10*s.* per-cent of the sum assured sufficient loading for the initial expenses other than the stamp duty and medical fees, it was impossible to discuss the figure, but he would put in a plea that any loadings should certainly be based upon actual results and not solely upon the preconceived opinion of the actuary.

He gathered that the author had been intent on obtaining very low premiums; he referred to his desire of obtaining loadings "on a moderate scale," and again of obtaining a "minimum" loading. He had reduced his average renewal expenses "with a view to keeping the loadings as moderate as possible," and he stated that the loadings reflected in his opinion an expenditure somewhat less than the average, and were, he thought, a suitable foundation on which to base loadings for moderate whole life non-participating

rates of premium. He (the speaker) therefore concluded that the premiums brought out by the author were smaller than he considered advisable. Table III showed that even those premiums, when based on $O^{[M]}$ mortality with $3\frac{3}{4}$ per-cent interest, were in excess of some of the published rates of premium, and on the basis of $O^{[NM]}$ mortality the average of the office rates would appear to be considerably below the author's. Table IV brought out in a very striking manner the inadequacy of some of the published rates of premium, if $O^{[NM]}$ mortality was experienced.

Before leaving the question of office premiums he might mention that in several public papers the gradual supplanting of with-profit by without-profit policies had been advocated. Such a change might be unlikely but the experience of life offices proved that the companies and the policyholders alike had benefited by premiums in excess of those required having been charged and that the surpluses so produced had enabled life offices to weather many storms which institutions trading on the narrow non-profit margin would have been unable to withstand. That stability could not have been attained by the issue of non-profit policies alone, and it must be considered most impolitic to charge non-participating rates, which besides being perhaps inadequate might tend to alter the public taste for participating policies. The present depreciation of securities would have had very unpleasant consequences for some offices had it not been for the strong reserves created partly out of past profit margins.

In the next three sections of the paper the author discussed the question of Reserves, Surrender Values and Paid-up Policies. It was quite impossible to deal adequately in one discussion with all the various points which arose from a survey of such a vast field, and he therefore took the liberty of suggesting that the discussion should be devoted principally to throwing further light upon the difficult question of office premiums, upon which the reserves, surrender values and paid-up policies mainly depended.

Mr. C. R. V. COUTTS said that most of the companies appeared to avoid the $O^{[NM]}$ table, and he thought they showed a sound instinct in so doing. He had seen some of the cards which went to make up that experience, and he knew of one society where the great majority of non-members' cards sent in were in respect of under-average lives that had been rejected for the members' classes. He did not think that the author had given enough weight in his Paper to the fact that a large amount of provident business which used to be taken on a with-profit table was now transacted under non-profit tables with fixed surrender values. The proposer who had intended to take a with-profit policy—an $O^{[M]}$ life—did not become a worse life because at the last moment he changed his mind and took up a more attractive non-profit policy. With regard to the question of rebates and undercutting, the author gave it as his opinion that they did not desire legislation similar to that which was in force in New York, but he (the speaker) could not conceive on what grounds they should deprecate such legislation, for they

were all at one in wanting to put down dishonesty in the business. Only the other day he heard of a society that had three prices for the same article. It had its published rates in its prospectus, which it got if it could ; it had a second scale—appreciably lower than the first—which it put into the hands of its agents and inspectors ; and then it had a third and still lower scale, which could be used if the agent was willing to forego some of his commission. That would not be so dishonest a method of doing business if the public were made aware that there were three scales of prices, but he suggested that it was dishonesty for a society to represent to the public that it charged a fixed rate when as a matter of fact it did not. He hoped that if the question came up in Parliament—as he was convinced it would some day—some legislation would result. With regard to surrender values and the whole question of selection, it was a very plausible argument to say that the man who surrendered would only do so if he were a first-class life. As a matter of fact, they knew that nothing of the sort happened in practice. Surrenders during the first four or five years of assurance had no financial effect to speak of upon the societies. The important surrenders were those where the policy had been in force thirty or forty years, and in those cases he was convinced that the companies gained ; the selection was not against them but in their favour.

Mr. E. C. THOMAS said that the question of rates of premium for non-participating assurances was a subject for each individual office ; there were no hard and fast lines that could be laid down for all cases. If they made their rates too cheap there would be no profit; if, on the other hand, they made them too dear there would be no business. The question for each office was to find the point of equilibrium, or in other words, the scale of premiums which would pay best in the long run. As to fixing that point, if the mortality in the non-profit section were as favourable as that experienced in the participating class there was one definite test : Were the bonuses greater in value than the loadings charged for participation in profits ? If so, their rates for the non-profits class were not unduly low. In cases where the mortality in the non-profit class was worse than in the other class, it remained to be seen whether the loss on mortality as compared with the participating class was greater or less than the extra value given in bonuses. It was worth some trouble for a company to investigate and settle that matter, because it was important for each company to secure a proper share of both classes of business. As Mr. Rietschel had said, the profit margins on the participating class were very important in bad times ; they constituted the first line of defence, and therefore it was obvious that they required a sufficient proportion of with-profit business. On the other hand, in times of prosperity it was a great advantage to have on their books a reasonable amount of non-participating business as their contributions to profits were all to the good.

Again, it was a problem for each office to decide what proportion of with-profit and non-participating policies it required.

With regard to the mortality of the non-participating class, which entered very largely into the question, that depended upon three considerations: First, the rates charged; secondly, the bonus results; and, thirdly, the system of bonus distribution. If the rates were very low or very high it seemed to him, although it might sound paradoxical, that the result was the same in both cases, namely, a high rate of mortality. If the rates were too low there was a great inducement to under-average lives, which required the greatest amount of cover at the smallest outlay, to enter in large numbers; if, on the other hand, the rates were unduly high the business would probably be restricted to financial lives. If the bonus results were not attractive, then the proportion of non-profit business would probably be greater than the other class and the mortality of the former class would consequently be improved, because they would have a larger proportion of average lives. In that case, however, it would probably be better for the office to do more with-profit business. Again, if they had a system of profit distribution which gave a bonus increasing with the duration of the policy, that would tend to produce a high rate of mortality among the non-participating class, and they might expect that feature to be more or less marked according to the steepness of the curve of increase. In settling their rates, competition must play a large part in the matter, but it was necessary that the other considerations he had mentioned should be brought into account by each individual office in settling its own internal economy.

He thought there was some danger of paying too much regard to the question of competition, and it certainly ought never to blind them to the necessity of securing consistency between their own various offers to the public. Some offices had a heritage of past accumulations of undivided profit which constituted a fund from which bonuses could be paid without their being earned. In those fortunately placed offices it was by no means necessary that non-profit business should produce a profit of its own, or even pay its way. It was quite possible to give this class a share of the surplus constantly being produced from those undivided accumulations and yet for the position of the participating policyholders and of the office generally to be stronger for their introduction. Such a company might publish a scale of non-participating premiums obviously insufficient to cover the risk and expenses, and as long as that scale was designed and used simply for the purpose of re-adjusting the proportions of with-profit and non-profit business in the particular office concerned, it would be a quite legitimate and possibly profitable thing for it to do. As soon, however, as it endeavoured by those means to obtain a huge volume of new business at the expense of other companies the effort would recoil upon it and produce a smaller bonus-giving power and consequently less attractiveness to the public. Closely associated with the question of rates was that of reserves. It seemed an anomaly that though the business might be obtained at reasonable rates and at a moderate expense, yet the immediate effect of a great influx of new business

was in most cases a fall in the profits and the amount available for distribution, owing to the method of valuation usually employed. There was a great deal to be said for the author's suggestion that some sort of allowance for the cost of new business should be made in their valuations. He was not suggesting that they should lower their reserves. Should those companies that had been making a net premium valuation in the past decide to adopt another formula they might carry the difference to a special fund and use it as a reserve for equalizing bonuses in the future.

Mr. C. C. MONKHOUSE referred to the effect which surrender values had on the mortality. There was a great number of without-profit policies effected solely because of the guaranteed surrender value, and the fact that a man could surrender a policy at a certain value was a great inducement to continue. Such policies would be more likely to be kept up when the reason for their being effected had no longer existed.

Mr. VYVYAN MARR said he should like to join with Mr. Rietschel in emphasizing the author's remarks, that particular care was necessary in regard to the calculation of non-profit premiums. There were no rectifying influences such as subsisted in participating policies; and the differences in the rates of premium quoted for without-profit whole-life policies were as remarkable as the apparent difficulty in fixing a one year short term rate. In Table I there was a very interesting set of ratios showing the relation between $H^{[M]}$, $O^{[NM]}$, and $O^{[M]}$ premiums. He would like to know the reason why the ratios of the $O^{[NM]}$ and $O^{[M]}$ premiums to the $H^{[M]}$ were higher at ages 45 to 55 than at later ages. For example, at 55 the ratio $O^{[NM]}: H^{[M]}$ was 101·4, whilst at age 65 it fell to 100·8. He regarded it as significant, and in that connection he was in hearty agreement with the opinions expressed by Messrs. Burn and Sharman, in their paper before the 1912 Congress. In speaking of mortality in old age they said: "Although we are unable to find direct evidence of an increase in mortality rates amongst assured lives, yet we feel there is sufficient indirect evidence to give cause for some apprehension on the subject." Personally he preferred the $O^{[NM]}$ tables to the $O^{[M]}$ for some calculations, because they gave an increasing percentage as the age at entry increased. The $O^{[NM]}$ table gave an extra loading, increasing to 2 per-cent at ages 60 and 65. as compared with $O^{[M]}$, and he thought for entrants at late ages offices would in future have to select their assured from a set of men of lower average vitality than that of former generations.

The author had analyzed the Expenses Account with the view of obtaining loadings, but he presumed that his motive was to get comparative rates rather than to advocate the use of those rates by every office, and that each office should fix its loadings according to its individual circumstances. He thought the author was a bold man to have touched upon the subject of rebates and commission. Personally he considered a solution of those difficulties was as far off now as ever. The question of rebates had recently taken a very interesting turn in America. Some offices had been

granting what were called blanket policies to the staffs of companies. So far as he could make out, those policies were issued without medical examination, and only covered death provided it occurred whilst the man was in the employment, and in consequence the rates were low. The Agents' Associations had been so incensed at this practice of the companies that they had actually gone the length of drafting Bills for submission to the Legislatures of the different States, with a view to prohibiting the offices undertaking that class of business.

Mr. J. MAYHEW ALLEN agreed with the author that it was easy to overdo the idea as to the non-applicability of the $O^{[NM]}$ table to a particular office. Of course, every individual office had its own peculiar circumstances. But the fact remained that the $O^{[NM]}$ table represented the collective experience of the offices which contributed to it, and therefore an individual office should not cast that table aside, except after coming to the conclusion—as the result of very careful consideration of the facts—that its own special circumstances justified the expectation that its own mortality experience on ordinary whole life non-participating business would be distinctly better than the collective experience of the British offices. In this connection it was interesting to quote a short passage from Mr. Levine's review of "The New Mortality Experience" (*J.I.A.*, xxxv, 486) :—" . . . under modern competitive conditions, when the heavy initial expenses and commission are provided for, and a 3 per-cent, or even $2\frac{1}{2}$ per-cent, net premium reserve made for new as well as for old policies, it is difficult for the non-participating class to pay its own way, . . . and if the mortality experience is heavy, its funds may require to be supplemented from the with-profit class . . ." It was certainly a matter for serious reflection when one considered those remarks in connection with Table II of the Paper, more particularly in view of the three clauses explaining the construction of that table. The figures given in it certainly inclined one to wonder whether there was so much use after all in analyzing mortality experience, if average premiums were to go in the direction which was the very reverse of that indicated by the signpost afforded by the analysis. A few years ago he was confronted with the task of calculating office premiums for a company embarking on the transaction of life assurance business, and he found the rates for ordinary whole life non-participating business the most troublesome of all those required for the prospectus. The trouble was largely due to the desire to arrive at rates which would be satisfactory both to himself as actuary to his company and to the outside staff as well. The author had pointed out certain circumstances which might justify an individual office in disregarding the $O^{[NM]}$ table, but those circumstances did not apply to the case of an office starting life business. The basis of calculation which he ultimately adopted differed from all the formulas given in the present Paper, but it was very interesting to note that the rates brought out by it were, for ages at entry 20 to 50 inclusive, almost identical with those of

formula D, differing nowhere (in this range of ages) by more than 3*d*. in the rate per-cent.

The principle of basing non-profit rates on a formula which in one or more respects cut matters too fine, and of relying on making up any deficit in one respect by means of a surplus in another, was in his opinion a dangerous one. The non-profit policyholder ought to understand his position to be in a way analogous to that of a preference shareholder; he paid a rate which made his position perfectly secure against all contingencies, except those of the insolvency of the office, *i.e.*, he was not to be affected by whatever was done by the office in the way of earning or failing to earn profits. He ought, therefore, to be content to pay for his privileges, and he ought not to expect to be provided with a compromise between a non-participating and a participating assurance. High and low were relative terms, and the author's paper would have served one useful purpose if it, and the discussion upon it, succeeded in stopping the further reduction of ordinary whole life non-participating rates, and it would serve a still more useful purpose if it could result in the disappearance of some of the extremely low rates which were already quoted.

Mr. N. J. CARTER said that the author recommended $O^{[NM]}$ instead of $O^{[M]}$, but suggested that with the $O^{[NM]}$ table $3\frac{3}{4}$ per-cent might be used. If one substituted $O^{[NM]}$ $3\frac{3}{4}$ for $O^{[M]}$ $3\frac{1}{2}$, in the formula for non-profit premiums, in the case of an office employing $O^{[M]}$ $3\frac{1}{2}$ for both profit and non-profit premiums, and loaded the with-profit premium to provide a simple bonus of 1 per-cent per annum, the differences between profit and non-profit premiums were anomalous, and at age 70 the non-profit premium would be greater than the with-profit. With and without-profit premiums calculated by the author's formula D—$O^{[NM]}$ $3\frac{3}{4}$ per-cent being used for the latter and $O^{[M]}$ $3\frac{1}{2}$ per-cent with a 1 per-cent bonus loading for the former—would be found to exhibit the same anomaly.

Mr. N. BLANCHARD said that in connection with the mortality of so-called "financial" lives it was sometimes overlooked that the people who had substantial reversionary life interests to mortgage became life tenants in their declining years, and they never heard much complaint about the excessive mortality among that class. He had just been examining the mortality among the small group of non-profit policies over the past eight years and comparing it with that of the $O^{[M]}$ table. There were only about 5,000 years of exposure and the sum assured averaged £1,544 (after deducting re-assurances). The actual deaths numbered 80 against 125·4 expected, and the claims £131,455 against £195,030. That was to say the deaths were 63·8 per-cent of the expected and the claims 67·4 of the expected, showing, he thought, that the mortality among large policies was little inferior to the mortality among small policies. But the most interesting point brought out was the lightness of the mortality after the first ten years' exposure, especially on the larger policies —the claims being 55·8 of the expected. In the first ten years

(owing to one bad claim under an Estate Duty policy) it was slightly in excess of the expected. The author described several methods of calculating paid-up policies. There was a useful and rather practical method that was not mentioned, viz.: that of surrendering a sufficient part of the sum assured to commute the premiums on the remainder. Commutation on an $O^{[M]}$ 4 per-cent basis brought out values for age 30 at entry below those given in Table XII as the average of five offices, and it had other advantages. It paid initial expenses on the portion lapsed if surrender values made the necessary allowance for them; it could not produce negative values; it could be applied to all classes of policies, especially with-profits; it could be explained to the policyholder.

Mr. CHAS. H. ASHLEY, discussing the paper from an industrial office point of view, said that in industrial companies the average sum assured under non-profit whole life policies was very small —only £43, according to the latest Board of Trade Returns—and policies to cover death duties or financial transactions were extremely rare. The mortality experienced was decidedly heavier than under with-profit whole life assurances. Policies were effected mainly by persons who wanted the maximum immediate death benefit which a given premium would procure, and the element of speculation was more likely to creep into that class than any other. In consequence of that, and the general heavier commission paid, the premiums charged were much higher than the purely ordinary offices' rates. The average and the lowest ordinary branch premiums of the 13 principal industrial offices were:

Age.	Average.	Lowest.
20	£1 12 10	£1 10 8
30	2 2 1	2 0 3
40	2 17 0	2 14 8
50	4 2 0	3 19 10
60	6 9 8	6 4 5

He agreed with the author that it was in the interests of the old policyholders that they should contribute towards the indirect expenses of acquisition of new business; and that was perhaps more particularly true in the case of industrial offices. At the same time, it was hardly equitable that the non-profit premiums should be fixed so high as to yield substantial additions to the profits divided among the with-profit policyholders, which would appear to be the case in some industrial offices.

Amongst industrial offices the agent's fee for procuring a new case was frequently paid in the form of a percentage on the first year's premium, 30 per-cent being a moderate fee; in addition, the superintendent would generally get the same percentage, or each might be paid £1 per £100 assured. Then frequently an assistant would get a fee. From a general knowledge of such offices' terms, he considered that the initial expenses were best provided for by a percentage of the sum assured and a percentage of the premium. For that class of business he had recently had

occasion to calculate a table of office premiums, and the following formula was found to produce a satisfactory competitive table:

$$\frac{\bar{A}_{[x]} + \cdot 0125}{\cdot 92a_{[x]} + \cdot 58667} \; O^{[NM]} \, 3\tfrac{3}{4}\%$$

The expenses provided for were—(1) Initial—£1 5*s*. per-cent on sum assured and $33\frac{1}{3}$ per-cent of the annual office premium. (2) General —8 per-cent of all the office premiums. The absence of a constant favoured the young ages at the expense of the old—a desirable feature in that connection.

Mr. E. J. HANCOCK said there was one point which he thought had not been sufficiently emphasized in the debate, and that was the possibility of the $O^{[M]}$ and $O^{[NM]}$ being to a certain extent out of date. The offices which had made their own investigations had no doubt found that the mortality they were experiencing in their with-profit class, and in endowment assurances, was much better than $O^{[M]}$. The intimation in the annual reports that the mortality had been very much lower than the expected was not, however, conclusive, because the expected mortality was generally based on aggregate tables. He thought it had been found that the mortality among the non-profit lives at the present time was something approximating to $O^{[M]}$. In with-profit business the assured themselves gained by higher bonuses, but in the non-profit business the policyholder did not get any return for the better mortality experienced.

Mr. S. G. WARNER said the interesting and practical paper they had heard that night was divided into four sections, which were intimately associated with one another; three referred expressly to the relations between an office and the public, while the other (the question of reserve values) was more a domestic concern of the office itself. One of the most important features of the paper was the inclusion of competition among the principal factors in the determination of premiums, and this was emphasized by the suggestion that in endeavouring to obtain a higher rate of premium for an identical benefit, an office would employ as one of its instruments for the purpose "the persuasive powers of its representatives." Such allusions brought them into contact with things as they were, and it was refreshing to find them given a prominent place in an actuarial paper. Although this element was a most vital one, it was one they could not possibly gauge or measure. It was easy to see the capital importance to such questions as they were considering, of competition and its consequences. It was possible to devise an admirable mathematical formula for a premium, one which afforded absolute safety, and then to pay a commission which destroyed all the excellent work thus done. For that reason and one or two others he thought a great deal of the work in the Paper, admirable and instructive as it was—he did not wish to depreciate it for a moment—did not lend itself profitably to practical discussion. The elaborate calculations with regard to expenses

were built upon a large number of assumptions, very carefully arrived at, but concerned with points on which the circumstances of individual offices varied widely. In regard to the question of general expenses, for instance, the assumptions were based on the experience of life offices only and took no cognizance of the great and important "mixed offices," which might have to regard the matter in quite a different way.

The Paper was practically a commentary upon one feature of recent life assurance business—the advent of lower non-profit premiums than had prevailed in the past. The question was whether this feature was to be welcomed. He thought the general tone of the discussion that night had been adverse to it. Reference had been made to two old actuarial principles upon which all of them had been more or less brought up. The first of these laid it down that non-profit business sprang from what were called "financial transactions," and was therefore subject to a comparatively high mortality. They had been reminded in that connection of the facts relating to "self-selection", and the superiority from that point of view of endowment assurance to whole life experience. Mr. Rietschel had said that it was rather difficult to believe in self-selection, but that there was no questioning the facts which proved its existence. Personally he (the speaker) had little difficulty in believing in the principle in the case of endowment assurance, where the entire contract was different and the premiums substantially higher. He was not inclined to attach so much weight to it when applied to such a comparatively small thing as the difference between profit and non-profit whole life rates. A period of twenty years had elapsed since the tables on which their calculations were based had been completed. He had made a rough estimate from the Board of Trade Returns as to the relative increase of whole life assurances, with-profit and non-profit, during the interval. With-profit life assurance policies had increased in number by about 12 per-cent and in amount by 2 per-cent. Non-profit policies had increased in number by 94 per-cent and in amount by $17\frac{1}{2}$ per-cent. The average amount assured by with-profit policies had decreased from £589 to £445, which was a comparatively small proportion. In the non-profit business there was a decrease from £652 to £395. It seemed to him that these figures were interesting and significant. They showed that non-profit assurance had been growing in popularity, and among another class than that usually asociated with "financial" business. There seemed to be a popular demand for non-profit assurance. That business had become more democratic and less specialized in character. In those circumstances it was difficult to see why, if equal selective care were exercised in each case, and intending assurants, profit or non-profit, subjected to the same preliminary ordeal of medical examination and general enquiry, a higher mortality need be assumed for the latter than the former.

The other old actuarial principle with which they had renewed acquaintance that night was that of selection against the company

in the matter of surrender values. To this he thought a great deal too much importance was attached. It was obvious that the man known to be at the point of death would not be allowed to surrender his policy, but short of that one's experience was that so many and such complex causes led to the step being taken that on the whole the selection against the company in the matter of mortality was not of serious account. One of the author's tables seemed to show that a minimum surrender value of one-third of the premiums during the early years of the duration of the contract was a high one. It was necessary, however, to look at those things more or less from the practical standpoint of "the man in the street." If he desired to surrender his policy after three or five years he would not be easily induced to think that one-third of the money he had paid was an extravagant allowance; and if the defence was set up that the office had incurred very large expenditure, and therefore could not afford to give him so much, that defence would scarcely commend itself to him. A very large proportion of surrenders took place in the early years, and surrenders which took place after ten years were very few in number compared with those which occurred earlier. If they wished to make the contract attractive to the public a fixed surrender value, especially during the early years of the contract, was of very great importance, and they must so arrange their expenditure as to make such an allowance possible without loss. He was inclined on the whole to regard the growing popularity of non-profit assurance, as shown by the figures quoted, as not an undesirable thing. In the case, for instance, of an office which had experienced difficulties in keeping up its bonuses, the problem of competition was very severe, and one could hardly see how it was to procure a substantial amount of with-profit business except at an extravagant price. Was it not a better thing that such an office should approach the public with a non-profit table, carefully considered, and brought down to the lowest point consistent with safety, at a moderate expenditure, and offer that as a fair, straightforward contract; restricting the commission to such a reasonable return as the intermediary might justly expect? Was it not possible that a business of that sort, energetically worked, might do something to cleanse their calling from some of the abuses due to the payment of high commissions? Such a table would bring the assured and the office into more direct contact. The terms would be low, and designedly so: they would be low because the intention was that the policyholder should reap from his contract the fullest possible benefit and should have that benefit fixed and certain. He thought that non-profit business, while it would never assume the dimensions of the with-profit, was likely yet largely to increase; nor did he think that such a growth would be a thing to be regretted or deprecated in the highest interests of their business generally.

The PRESIDENT said it was his pleasing duty to move a vote of thanks to Mr. Cameron for his admirable and interesting Paper. They had had an excellent discussion and he did not propose to add

to it. The Paper was an eminently practical one, and dealt with many old controversial questions. He would ask Mr. Cameron to reply to any criticisms.

The vote was carried with acclamation.

Mr. F. J. CAMERON, in reply, said he appreciated very much the kind way in which his Paper had been received, and he specially wished to thank those who had taken part in the discussion for the leniency of their criticisms. In dealing with his subject he had found it somewhat difficult to preserve the equipoise between theory and practice; in some places he had inclined to one side and in some places to the other. For instance, in the case of the formula which he had derived for premium rates, it did not necessarily follow that he would recommend it in practice for any particular office. There were many considerations to take into account, and in fact every office was of necessity more or less a "law unto itself" in such matters. At the same time he felt that an interchange of opinions on the bases of office premiums was of considerable practical value.

He was glad to have Mr. Rietschel's support—and a more or less modified support from some other members—in his contention that the $O^{[NM]}$ table was probably more suitable than the $O^{[M]}$ in the case of whole life non-profit premiums. Most of the speakers, however, appeared to be opposed to the adoption of a table so stringent as the $O^{[NM]}$, but with all deference to their arguments he felt that it was largely a question of statistics—a question which could no doubt be better answered after the next investigation into the mortality of the participating and non-participating whole life classes respectively.

In regard to the matter of actual expenses as the basis of premium loadings he agreed with Mr. Warner that an analysis of the expenses of other offices was not altogether reliable, and he had therefore not attempted to go very minutely into the different kinds of expenditure, or to differentiate between the expenses due to the various classes of assurance. At the same time he felt that some such analysis was better than none at all. He was much interested in Mr. Marr's comments on the ratios of the $O^{[NM]}$ to the $H^{[M]}$ net premiums, and he intended to look further into the matter.

The factor of competition had been referred to by most of the speakers, and undoubtedly it played a large part—too large in his opinion—in the question of premium rates. Closely connected therewith was the subject of rebating. Mr. Coutts did not seem to think that his (the author's) remarks on this subject went far enough, and did not see why we should not have prohibitive legislation on this matter similar to that in force in the United States or in Canada. Personally, however, he was strongly adverse to prohibitive legislation on such a matter; firstly, because it was somewhat degrading, and, secondly, because he was doubtful (from what he had heard of one of the countries mentioned) if it really cured the evil. It was to the mutual interest of the offices and the public to discourage rebating, and he hoped that the opinions expressed by

members of the Institute would have some influence in repressing the practice.

The existence of adverse selection on surrender had been disputed by a number of those who had taken part in the discussion. On this question he had nothing to add to the opinions expressed in the Paper, but he wished to emphasize what he had stated as to the effect on adverse selection of communicating with a policyholder who wished to surrender, or arranging an interview with him —with a view to inducing him to maintain his policy in force.

LEGAL NOTES.

By Arthur Rhys Barrand, F.I.A., *Barrister-at-Law.*

Policies of life assurance comprised in property settled by will. Payment of premiums.

The case of *In re Sherry, Sherry* v. *Sherry* [1913]. 2 Ch. 508, is concerned, *inter alia*, with the position of trustees in regard to policies of life assurance comprised in the settled property, in the following circumstances. A testator, one Henry Sacheverel Sherry, by his will devised and bequeathed his residuary real and personal estate to trustees in trust, at their absolute discretion, to sell it and invest the proceeds in trustee securities, and pay the income to his wife for life or during widowhood, with remainder, on her death or re-marriage, to his children in equal shares. By a codicil he provided that the trustees might permit the whole or any part of his estate to remain in the state of investment in which it was at the time of his death.

The testator died on 3 March 1913, and at his death part of his estate consisted of an equal fourth share of the net income during the life of one B. A. T. Harwood, of the funds forming part of the residuary estate of Colonel Edward Harwood, deceased. This life interest was originally mortgaged by B. A. T. Harwood to the testator together with a policy on Harwood's life for £1,000. In 1898 the testator purchased the equity of redemption and took an assignment of the life interest and policy. At a later date the testator effected a further policy for £750 on the same life. The annual premiums on these policies amounted to £60. 17*s*. 6*d*. The surrender-value of the two policies was £380. The annual income of the life estate was £244. 10*s*. 6*d*. The *cestui que vie* was about sixty-one years. He had disappeared some years before, and his whereabouts was unknown, but he was believed to be in America.

In these circumstances the trustees took out a summons for the determination, *inter alia*, of the following questions:—(1) whether the widow was entitled during her widowhood to the whole of the income of the Harwood property, or whether any, and if so, what part of that income ought to be treated as capital; (2) how provision ought to be made for the payment of the annual premiums on the policies of assurance, and how the incidence of such payment should be borne by the parties interested; and (3) whether the trustees had power to retain the life interest and the policies unconverted, or whether they ought to be sold and surrendered, or how otherwise they ought to be dealt with.

The case came before Warrington, J., in July last, and in the course of his judgment he said:—" The main question in " this case is whether the trustees have a discretion and may " properly exercise it by retaining unsold a certain estate *pur* " *autre vie*, and certain policies of assurance effected on the life " of the *cestui que vie*. . . . The trustees are in this position. " The *cestui que vie* disappeared some years ago and is not in " touch with the Harwood trustees. . . . The Harwood trustees " continue to pay the income to the trustees of the testator's " will, who pay it to the testator's widow. Can the trustees of " the will properly postpone the sale of the life interest . . . ? " If they have power to do so on the true construction of the will " and codicils, and if they can in the circumstances properly " exercise that power, they are prepared to exercise it by re- " taining the life interest unconverted. The testator has given " the trustees the fullest power to postpone the sale, and they " feel a difficulty about converting. They think that to sell the " life estate would be ruinous, as they are not at present in a " position to satisfy a purchaser that the life has not dropped. " The *cestui que vie* is supposed to be sixty-one years of age. " The policies on his death will produce £1,750, which will form " part of the testator's estate. In these circumstances I think " the trustees have power to postpone the realisation both of " the life interest and the policies, and I will so declare.

" The widow being entitled . . . to the income actually " produced by the testator's estate is entitled to the whole of the " income of this life interest, which is quite securely invested " without the policies. The life policies are reversionary interests " and must be treated quite separately from the life interest of " the *cestui que vie*. The premiums on the policies are paid for

"the preservation of an item of the testator's property, the "benefit of which goes to capital. I think, therefore, that in "this case, as in *In re Bennett* [1896], 1 Ch. 778, 787, the premiums "may properly be paid by the trustees as part of their costs, "charges and expenses in the administration of the estate, and "are properly payable out of capital.

"I therefore answer question (3) of the summons by declaring "that the trustees have power to retain the Harwood life "interest and policies unconverted, and question (1) by declaring "that the widow is entitled to the whole of the income derived "from the life interest, and question (2) by declaring that the "premiums ought to be borne by the capital of the testator's "estate."

Payment of first premium after commencement of fatal illness. Position of assurance company.

The recent case of *Harrington* v. *Pearl Life Assurance Company (Limited)*, 1913, 30 T.L.R. 24, is concerned with the position of an assurance company which has accepted the first premium on an assurance after the commencement of the fatal illness of the life proposed and in ignorance of the existence of that illness. The material facts are as follows:—

In May 1912, a Mr. Bentley signed a proposal for an assurance of £300 with the defendant company, the transaction being submitted by an agent named Goodswin. The proposer was examined by the company's medical referee and passed by him as a first class life, and the proposal was thereupon accepted. A further proposal for an assurance of £200 was afterwards made by Bentley; but eventually both these proposals came to nothing owing to the fact that the first premiums were not paid within the 30 days allowed by the company for such payment.

On 1 October 1912, fresh proposals for £300 and £200 were made by Bentley on precisely similar terms, the policies to run from 18 October. The company did not require a further medical examination in the circumstances, but the proposer was required to make a declaration that there had been no material change in his health since the previous examination. This was done, and the proposals were thereupon accepted by the defendants, who stated that the policies would be forwarded on receipt of the premiums provided these were received within 30 days.

In the early part of November the plaintiff, who had lent various sums of money to Bentley, was approached by him for a further loan, and on 4 November, an assignment was drawn

up by which Bentley assigned all his interest in the policy for £300 to the plaintiff, it being arranged that the premium should be paid by him. On 8 November, the agent Goodswin called on the plaintiff for the amount of the premium, and it was paid on that day. At that time Bentley was seriously ill, and had been so for the preceding 48 hours, and he died shortly afterwards. The policy proposed to be issued was in the ordinary form issued by the company, and contained a condition that there should be no insurance until the first premium was paid.

It was contended on behalf of the plaintiff that the documents forming the contract were the proposal form and the acceptance. The proposal was dated 1 October, and made it clear that the contract was to begin on 18 October, provided that the premium was paid within 30 days from that date. The premium having been paid within that period, the plaintiff was entitled to recover the money due under the policy. Against this, it was contended on behalf of the defendants, *inter alia*, that there was no completed contract, and that they were therefore not liable under the policy. This latter view was upheld by A. T. Lawrence, J., who, in giving judgment for the defendants, held that there was no contract between the deceased and the defendants, although there were negotiations which might have ripened into a contract if the circumstances had been favourable.

Right of agent to retain undisclosed commission.

The case of *E. Green & Son* (*Limited*) v. *G. Tughan & Co.* 1913, 30 T.L.R. 64, although concerned with marine insurance, deals with a matter relating to commission on premiums which may be of sufficient interest to life assurance officials to warrant a brief note on the matter.

In this case the plaintiffs employed the defendants, who were insurance brokers, to effect certain marine insurances, and the defendants rendered to the plaintiffs an account which stated that the premiums payable by the plaintiffs were at a net rate, whereas the amounts paid by the defendants to the underwriters were subject to a commission of 5 per-cent and to a further discount of 10 per-cent; and in the case of two large insurances the defendants got a special rebate of 25 per-cent from the insurance companies.

On these facts coming to the knowledge of the plaintiffs, they brought an action against the defendants to recover the 5 per-cent commission, the 10 per-cent brokerage and the 25 per-cent

special rebate ; and contended that when premiums were stated to be net, this was a well-known term among insurance brokers, and implied that they were subject only to a deduction of 5 per-cent.

The case came before Pickford, J., and he held that although in an ordinary case the defendants would be entitled to retain the commission of 5 per-cent, nevertheless if a broker represented to his principal that the premiums charged were subject only to a deduction of 5 per-cent commission, the broker was not entitled to retain an additional 10 per-cent as discount, and that the same rule applied to the 25 per-cent special rebate ; and that as an agent could not retain a commission which he had obtained by acting dishonestly towards his principal the plaintiffs were entitled also to recover the 5 per-cent commission, although in ordinary circumstances the defendants would be entitled to retain that commission. He gave judgment for the plaintiffs accordingly, for the full amount claimed.

ACTUARIAL NOTE.

On the Valuation of Whole-Life Policies by Select Tables. By H. HOSKING TAYLER, A.I.A.

THIS subject has already been dealt with by Messrs. King, Diver, Ackland, and Brown, and the methods proposed by them apply equally well, whether annual or quinquennial valuations are required. The object of the present note is to offer a solution of the problem in the rather more limited aspect which it assumes to an office requiring strict accuracy at quinquennial valuations only.

The reserve under a whole-life policy when the net premium is derived from the select table and the reversion and annuity factors from the ultimate table, exceeds the true select reserve by the quantity

$$(P_{[x]}+d)(\mathbf{a}_{[x]+n}-\mathbf{a}_{x+n})$$

The methods of Messrs. Ackland and Diver are directed to obtaining an approximation to the total of this correction in respect of each duration, when the sums assured (or sums assured and net premiums) are grouped by durations, irrespective of age.

The method now proposed is that the quantity be recorded on the valuation cards, at the inception of the contract, in

respect of the two durations corresponding to the two quinquennial valuations which fall within the select period.

Suppose an office valuing quinquennially on 31 December 1915, 1920, &c. Let the age at entry of a policy entering in 1913 be x, and let this also be the valuation age on 31 December 1913, and let the sum assured be S. Then the correction to be made on

31 December $1915 = S(P_{[x]}+d)(\mathbf{a}_{[x]+2}-\mathbf{a}_{x+2})$
and on 31 December $1920 = S(P_{[x]}+d)(\mathbf{a}_{[x]+7}-\mathbf{a}_{x+7})$

These amounts would be recorded on the valuation cards for each policy and entered in the classification books in two columns headed say ΔV 1915 and ΔV 1920. The totals of these columns give the exact adjustment for a select valuation on the dates indicated. After the 1915 valuation the column ΔV 1915 would be replaced by ΔV 1925, which would receive the entries in respect of the 1916 new business, and so on.

Prepared tables of $(P_{[x]}+d)(\mathbf{a}_{[x]+t}-\mathbf{a}_{x+t})$ in which the values for quinquennial differences of t were placed in proximity, would render the process of recording the individual corrections as simple as the calculation of the net premium. If the method of obtaining entry and valuation ages does not secure that the correction at the end of the year of entry is $(P_{[x]}+d)(\mathbf{a}_{[x]}-\mathbf{a}_{x})$ it would be better to use a table of $(P_{[x]}+d)$ in connection with the values of $(\mathbf{a}_{[x]+t}-\mathbf{a}_{x+t})$.

The following points call for mention:

(1) The correction is negative and, as with Mr. Diver's method, its analysis into sum assured and premium corrections (if necessary) presents a difficulty. The deduction of one-half the total correction from the value of the sums assured and the addition of one-half to the value of the premiums will give a close approximation. This is indicated in the following examples extracted from Messrs. Diver's and Ackland's papers. The figures give the percentages of the total difference in the liability:

Office and Basis	Δ Value of Sums Assured	Δ Value of Net Premiums
Model Office No. 2. $O^{[M]}$ $2\frac{1}{2}\%$ . . .	49·8	50·2
Actual Example. $O^{[M]}$ 3% . . .	48·9	51·1
Model Office No. 2. $O^{[M]}$ $2\frac{1}{2}\%$ . . . (Adjusted sums assured.)	50·1	49·9
Model Office No. 2. $O^{[M]}$ 3% . . .	55·9	44·1

(2) Reversionary bonuses may be valued by the ultimate table with only slight overstatement of the reserve. If, however, strict accuracy were desired, the correction for the second valuation could be adjusted on each card by $Bd(\mathbf{a}_{[x]+t}-\mathbf{a}_{x+t})$ where B is the reversionary bonus allotted at the first valuation.

(3) Similarly for reductions of premium, the adjustment in this case being $R(\mathbf{a}_{[x]+t}-\mathbf{a}_{x+t})$.

(4) The difference of the annuities may be written $(a_{[x]+t}-a_{x+t})$ without change of value, so that the same correction applies if the premiums are valued by $\frac{1}{2}+a$, or by $\mathbf{a}$ and a for first half year and second half year cases respectively.

(5) If the influx of business were uniform in amount and age incidence, the total correction would remain the same for successive valuations. If the new business were increasing the correction would also increase. Therefore, if intervaluation estimates are required, the use of the total correction as at the previous valuation would produce a slight overstatement of liability.

To set out the advantages of the method :

(1) A single classification by attained ages is preserved.

(2) An exact valuation of the policies is secured by the use of two simple constants, without supplementary processes.

(3) An exact adjustment for reversionary bonuses and reductions of premium, may readily be made if desired.

It may be mentioned that but for the difficulty and labour of calculating tables of the differences between the ultimate reserves with select net premiums, and the select reserves, the method is equally applicable to other classes of assurance.

REVIEW.

NATIONAL INSURANCE ACT, 1911.

New Tables of Sickness and Issue Rates.

In the reprint—*J.I.A.*, xlvii, p. 548—of the Life Tables and the Memorandum on their Construction appended to the Report for 1912–13 on the Administration of the National Insurance Act

(Part I), reference was also made to the other appendices to that Report dealing with actuarial questions. The memoranda on Rates of Sickness and Disablement and Rates of Issue, which were amongst the reports referred to, present features of interest quite apart from their direct bearing on the National Insurance Scheme, and should be of considerable service in the solution of certain problems that come occasionally before the consulting actuary.

In the Memorandum on Rates of Sickness and Disablement, the Manchester Unity (Whole Society) rates are analyzed into the rates applicable to each week for periods up to 26 weeks, and to 4-weekly periods for longer durations. It will be remembered that in the Manchester Unity Tables the sickness experience was split up into the amounts applicable to the first three months of illness, second three months, second six months, and second year and subsequent periods, and the new tables supply a decided want where it is necessary to take into account in calculations the sickness rates for other periods, as well as to discriminate between the rates for various entry ages. Besides, the function tabulated is a development of the rate of invalidity in use in pension fund valuations, since the differences between the rates of sickness for the consecutive weekly periods give the proportion of members at each age invalided by illness of different durations. The table will, therefore, have a further use in investigations into the incidence of sickness claims in different societies. In order to ascertain the proportion of the first three months' sickness that falls within the first six weeks recourse was had to the more detailed information in the Friendly Societies experience (1876-80). Series of rectangles were then plotted out for each year of age with areas proportional to the amount of sickness applicable to successive periods, and the appropriate rates were obtained by replacing these rectangles by an equal area bounded by a continuous curve—the ordinate at any point showing the proportion of sickness of the duration represented by the abscissa of the point. The initial ordinate was fixed by the proportion of new cases of sickness as distinguished from the total sickness in any year of age (which, of course, includes cases of sickness continued from the preceding year) and this was obtained from the published information of the Manchester Unity Experience by an ingenious use of the central rate of sickness—a method suggested by Sir G. F. Hardy, and described by Mr. A. W. Watson in *J.I.A.*, xlvii, p. 100. The redistribution of the areas was then made graphically, slight adjustments being made so that the differences formed regular series. The points through which the curve should pass for durations of three months, six months, &c., were checked by determining y_t, the proportion of cases of sickness of duration t, from the relation

$$y_t = \mathrm{L}_{h=0}\frac{s_t - s_{t+h}}{h} = -\frac{ds_t}{dt} = -s_t\frac{d\log_e s_t}{dt}$$

$$= -\mathrm{M}^{-1}s_t\{\Delta - \tfrac{1}{2}\Delta^2 + \tfrac{1}{3}\Delta^3 - \&c.\}\log_{10} s_t$$

where $M = \log_{10}e$ and s_t, expressed in terms of the unit of time, denotes the amount of sickness of duration t and over. In analyzing the sickness experience into the amounts applicable to the 3rd, 5th, 6th, and 7th quarter years, a clever system of direct interpolation was adopted, the function $\log s_t$ being operated upon. The differences tended to a constant for each age group, and this fact was utilized in splitting up the amount of sickness of over two years' duration. The actual experience of a representative group of Lodges was available for checking this part of the work, and a close agreement was found to exist between the theoretical and the actual distribution of the claims.

The analyses of the "after 2 years" sickness figures will prove of use in ascertaining the reserves for disablement benefits during the first few years after entry into insurance as well as for calculating theoretical premiums for late age entrants. The function s_t includes illnesses contracted prior to the age at entry, and as in such calculations these illnesses should, strictly speaking, be ignored, the new tables have a further statistical value.

Tables are given showing for quinquennial ages the Rate of Sickness per annum for periods up to the first 12 months and the Percentage of the "after two years" sickness applicable to the successive years of sickness.

The following figures are extracts from these Tables.

Rate of Sickness per annum for durations of illness up to the number of weeks specified.

	Manchester Unity.		*Whole Society.*	
No. of Weeks	Age 20	Age 35	Age 50	Age 65
1	·200	·188	·218	·276
2	·345	·328	·398	·520
5	·599	·572	·745	1·064
10	·732	·715	·983	1·555
20	·814	·843	1·240	2·246
30	·847	·906	1·393	2·754
39	·863	·941	1·479	3·106
47	·874	·963	1·538	3·378

Percentage of the "after 2 years" sickness applicable to the successive years of sickness.

Age Group	3rd year	4th year	5th year	6th year	7th year	8th year and after
20–24	53	24	13	6	3	1
35–39	23	17	13	10	8	29
50–54	19	15	12	10	8	36
65–69	19	16	12	10	8	35
Weighted mean	20·5	15·7	12·8	10	8	33

The outstanding feature of the Memorandum on Issue Rates is the formation of a series of Select Tables, showing for six selected ages at marriage of females the central issue rates at each age attained besides Aggregate Tables for husbands and wives irrespective of the age at marriage. The statistics used were the recent census figures of the Metropolitan Borough of Camberwell. These showed at the date of the census in quinquennial age groups :

(1) The number of husbands present with wives ;

(2) The number of husbands present with one or more children under one year of age ;

(3) The number of wives present with husbands ;

(4) The number of wives present with one or more children under one year of age.

The information in (3) and (4) was also available according to the duration of marriage in quinquennial age groups. Dealing in the first place with the Aggregate Tables the statistics were redistributed according to years of age, and adjusted to allow for deaths of children prior to the date of the census, &c. The births were assumed to have taken place on the average six months before the census, and in consequence the ratio of the adjusted number at any age x in class (2) to the corresponding number at age $x-\frac{1}{2}$ in class (1), gave the central issue rate for husbands between ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. Similarly the central issue rates for wives were obtained from classes (3) and (4).

Each of the four classes of data was graduated by the curve

$$\log \frac{y}{\mathrm{N}-y} = \mathrm{K} + \frac{m}{a+x} + \frac{n}{b+x}$$

where N = total number of cases, y = the number below age x and $a\ b\ k\ m$ and n are constants : a and b being respectively the starting and terminating points of the curve. The use of this curve in graduation is described on p. 51 of Sir G. F. Hardy's Lectures. The issue rates were tested by applying them to the married population of the United Kingdom and comparing the estimated with the actual births.

In calculating the select issue rates, the adjusted numbers of wives with children, and of wives present with husbands, were set out in the quinquennial groups according to age and duration of marriage, and in order to determine the exact age and duration to which the central issue rate referred—obtained from the ratio of the two sets of numbers in each class—use was made of the statistics relating to marriages in London in 1909. The calculated issue rates were then assumed to be those applicable to the ages and duration of marriage so ascertained, and the curves for each of the six selected ages at marriage were determined by graphic interpolation from these values.

Examples of the rates as deduced are shown in the following table. All the curves of select rates display similar features. Each

curve descends rapidly at first, cuts through those for marriages at earlier ages, and lying below these curves runs into the ultimate curve about age 40. It will be noticed that the aggregate rate for women at ages under 22 is greater than the select rate at corresponding ages. In the former there is a full year's experience at each age, whereas the select rate for the first year represents virtually only six months' births. The tables are evidently practically useless for calculating the probability of a marriage proving fruitless, owing to the inclusion of children other than first-born, but they would seem to facilitate research into questions of fertility and the probable size of families as influenced by the age at marriage. They point to the fact that the rate of issue is dependent on the duration of marriage for the first 5 or 6 years of marriage, but for longer durations is more influenced by the age at marriage, being greater the earlier the age at which the marriage was contracted.

In this respect the results appear to differ from those of the late Dr. Matthews Duncan, who probably may be regarded as the pioneer of select issue tables in this country. In his work "Fecundity, Fertility, and Sterility", Edinburgh, 1866, he employed statistics relating to wives in Edinburgh and Glasgow in 1855. Tables are given which show for quinquennial ages at marriage the probability of issue according to attained ages in quinquennial groups. These tables were analyzed by the late Professor Tait, who concluded that (1) the probability of issue within a year was proportional to the number of years a woman's age is under fifty; (2) the total probability of issue was proportional to the square of the same number, and (3) the advent of sterility was hastened by early marriages. There are several Aggregate Tables in existence, the most recent and important being probably that giving the experience of the Hearts of Oak Benefit Society in respect of Lying-in Claims (Mr. Watson's lectures, p. 117), and those given in Messrs. Hardy and Wyatt's Report on the National Insurance Scheme (*J.I.A.*, xlv, p. 406). The new rates are higher than those given by Mr. Watson, but a comparison with those of Messrs. Hardy and Wyatt is scarcely

Central Issue Rates.

Age	Married Men	Married Women irrespective of age at marriage	Married Women whose Age at Marriage was					
			18	21	25	30	35	40
18	·660	·690	·654	—	—	—	—	—
21	·625	·534	·455	·531	—	—	—	—
25	·396	·359	·330	·335	·458	—	—	—
30	·292	·261	·277	·259	·247	·374	—	—
35	·218	·194	·220	·199	·175	·183	·290	—
40	·135	·106	·121	·111	·097	·089	·093	·191
45	·068	·022	·022	·022	·022	·022	·022	·021
50	·029	·000	·000	·000	·000	·000	·000	·000

practical as they show the probability of issue within a year, and the usual relation between m_x and q_x is not applicable owing to the number of marriages, or the number exposed to risk, rapidly increasing at the ages at which the rates are highest. It will be remembered that Messrs. Hardy and Wyatt in calculating their tables had to resort to New Zealand Statistics as to orphanhood, and adjusted the probabilities of issue obtained from that source to harmonize with the relative birth rates in the United Kingdom.

The investigations under review may be regarded as typical of problems of interest to the actuarial profession which may be expected to arise out of the National Insurance Scheme.

V. M.

NOTICES OF RECENT PUBLICATIONS.

The Theory of Money. By D. A. BARKER.

(Cambridge University Press.)

THE animated discussion evoked by the paper on Currency and Credit, read before the Institute by Mr. W. T. May, in December 1912, showed that the Members not only take a considerable interest in the problems of economics, but also hold widely divergent views.

Mr. Barker is a strong supporter of the modernised Quantity Theory, which has received even more attention in the United States than in England. Consequently many of the arguments and results are based on the recent work of American writers. The book will be useful to those who are anxious to follow the latest developments of the Quantity Theory, but are unable, from lack of leisure or opportunity, to study the voluminous literature which has grown up around the subject. A student who had not already some familiarity with the technical terms, and a general idea of economics would probably have difficulty in following the arguments, and he would need to refer to other works for a more detailed exposition of rival theories.

The Evolution of German Banking. By LEOPOLD JOSEPH.

(C. & E. Layton, London.)

THIS book is a reprint of a course of lectures delivered at the London School of Economics and Political Science, and gives, in compact form, a very interesting account of German banks and banking methods, and also, as a necessary introduction, a review of the economic development of Germany.

The social, political and economic upheavals of the nineteenth century left their mark on the history of banking institutions. We read that in 1851 the Prussian Bank found it necessary to call upon depositors to withdraw credit balances, because it had more money than it could employ. In 1870–1872, over £50,000,000 capital was raised for banks, but between 1872 and 1876 the general depression forced seventy-three banks, representing a capital of

£21,500,000, into liquidation. The recent tendency in Germany, as elsewhere, appears to be towards absorption or amalgamation of the smaller banks.

The description of the constitution and objects of the various classes of banks shows that the scope of the transactions is very wide. The association with trade seems to be more intimate and direct, the banks undertaking not only the financing of important industrial enterprises, but also the promotion and management, in many cases.

One large and old established class, the Mortgage and Land Banks, make advances, directly or by issue of marketable obligations, in the nature of debentures, on real estate. The total amount of their obligations in circulation at the end of 1910 was not far short of £800,000,000. This must include a considerable amount of business which would, in this country, be undertaken by Insurance Companies by way of mortgage.

The figures and facts have evidently been selected and arranged with great care, so as to give a well proportioned summary of a very large subject.

Valuation Tables. By RICHARD PARRY, F.S.I.
(Estates Gazette, Ltd., London.)

THESE Tables are intended for the use of Valuers, Surveyors, and others, and appear, so far as the functions involving interest only are concerned, to be well adapted to their requirements. Simplicity is secured by recording the values at yearly intervals (half-yearly in the case of $a_{\overline{n}|}$) and at rates of interest convertible yearly. The results are recorded to three places of decimals (v^n and $\frac{1}{s_{\overline{n}|}}$ to seven places). In his calculations the Author seems to have erred on the side of caution. Thus, an extract from the working figures shows that the values of v^n were calculated to 13 places of decimals, and it is stated elsewhere that, in most of the tables worked out, fifteen places were retained, the results being then cut down to three.

The usual interest functions are given, for the most part, at rates of interest increasing by $\frac{1}{4}$ per-cent from 2 per-cent to 4 per-cent, by $\frac{1}{2}$ per-cent from 4 per-cent to 8 per-cent, and thereafter by 1 per-cent to 15 per-cent. In addition, the values of $a^{i\,\&\,i'}_{\overline{n}|}$ at reproductive rates varying from $2\frac{1}{2}$ per-cent to 4 per-cent, are shown in parallel columns to the value of $a_{\overline{n}|}$ at the remunerative rate—a very convenient arrangement.

Two tables, required in connection with the Finance (1909–10) Act 1910 show (*a*) the 4 per-cent values of $\frac{a_{\overline{n}|}}{a_x}$ or $ia_{\overline{n}|}$ and its complement, for calculating the "proper proportion" of duty payable in respect of the interest in possession or the interest in reversion respectively on transfer of less than the freehold estate, and (*b*) the 4 per-cent values of $a_{\overline{e_x}|}$ based on the Northampton Table, since,

according to the Rules of the Commissioners of Inland Revenue, $\mathring{e}_x$ represents the term of an interest dependent on a life, and $a_{\overline{\mathring{e}_x}|}$ the present value of an annuity during that term.

Examples are given of the methods of applying the tables, in which connection it is worth noting that for valuing a deferred term annuity with different reproductive and remunerative rates, the rule indicates the use of the reproductive rate for discounting during the period of deferment.

From the actuarial point of view, the most interesting part of the book is that relating to the valuation of interests based on lives. It is the practice of valuers to advise on the value of such interests, too often without sufficient knowledge of the problems involved, and it is worth while to examine some of the views put forward, as these are probably representative of the opinions current among valuers of the best class.

Mr. Parry is of opinion that the mortality tables based on the experience of assured lives cannot be considered as giving average results, and "are misleading to a valuer of property. The best Mortality Table for a valuer to use as his own guide for average cases would be that known as the English Life Table." He discards "the" English Life Table on the ground that the necessary functions are not available, recommending the Carlisle as giving results sufficiently near. It will be readily agreed that for valuing life interests the Carlisle will give quite as good an approximation as is desirable to any of the English Life Tables. The Tables actually published are based on the Carlisle and Northampton, including the 3 per-cent to 6 per-cent values of a_x and A_x, and of a_{xy} and $a_{\overline{xy}}$ for quinquennial intervals of the younger age, combined with quinquennial intervals of the older age for the Carlisle and decennial for the Northampton.

In the hands of a valuer, other factors would probably have more effect on the result than the choice of a mortality table. For example, instructions are given for finding the value of $a_{\overline{xyz}}$, by means of Simpson's Rule, but no help is afforded in interpolating for the values of a_{xy}, although a chapter is added showing how numbers containing decimals must be treated if it is desired to perform the operations of addition, subtraction, multiplication and division. Having arrived at a result (and only then), the valuer "will modify it to the best of his ability, according to the state of health of the various persons whose life interests are being valued." Possibly at this stage he will also begin to consider the sex of the lives.

On the whole it may be said that the purely interest functions will prove useful, owing to the wide range of the rates, particularly the annuities at different remunerative and reproductive rates, and also the Finance Act tables, but the combined interest and mortality tables are likely to do more harm than good in the hands of those who have not been specially trained to appreciate the difficulties of valuing interests depending upon human life.

S. G. D.

CORRESPONDENCE.

To the Editors of the Journal of the Institute of Actuaries.

Whole Life Non-Profit Assurances.

Dear Sirs,—In the discussion that followed the reading of Mr. Cameron's paper on 22 December, I gave short particulars of the mortality experienced in this class for the past eight years by an office transacting a considerable non-profit business, and it has been suggested that I should give the Institute a somewhat fuller account of the experience.

I was led to make the investigation by the fact that in the Office in question this class, in which there is a high average sum assured, generally shows a profit from mortality, and I expected to find some indication of a positive correlation between sum assured and longevity. Accordingly, the mortality was investigated on the basis of sums assured as well as policies. Nearest entry ages and nearest durations were employed, each policy in force in 1905 being followed from its renewal date in that year to its renewal date in 1913 or earlier withdrawal, the new business being treated similarly. Sums re-assured were deducted, so that in no case was there a larger net sum assured than £15,000 on any life. Policies on invalid lives (which constitute about 16 per-cent of the experience) were taken at their rated-up ages, but on the other hand policies at climatic or occupation extras were included at their true ages. The combined errors from these two sources, acting as they probably do in opposite directions, cannot, I think, vitiate the results as a whole to any serious extent.

The grand total of the Exposed to Risk gave 4,845 years of life for sums assured of £7,483,015, showing an average policy of £1,544 net.

Tables A, B, C, and D give the results of the investigation. They were worked out for each individual age and duration up to 10 years in force and thereafter for each attained age, but I have grouped them for conciseness.

The figures in heavy type refer to policies, those in ordinary type to sums assured.

TABLE A.

Exposed to Risk.

Years elapsed since date of Assurance	Ages at Entry: Under 30	30 to 39	40 to 49	50 to 59	60 and over	All ages combined
0	**78** 154,680	**121** 217,192	**107** 201,292	**86** 171,245	**28** 39,833	**420** 784,242
1	**61** 96,380	**115** 176,592	**99** 196,717	**78** 137,385	**27** 35,233	**380** 642,307
2	**58** 77,430	**104** 139,992	**96** 183,273	**76** 116,210	**24** 32,633	**358** 549,538
3	**56** 73,330	**101** 131,181	**97** 188,223	**63** 77,125	**17** 16,175	**334** 486,034
4	**42** 49,000	**91** 110,441	**88** 151,911	**44** 51,225	**8** 8,905	**273** 371,482
5	**38** 45,500	**80** 91,842	**77** 128,811	**43** 45,945	**7** 16,250	**245** 328,348
6	**33** 38,300	**72** 87,892	**71** 111,100	**38** 32,015	**6** 12,750	**220** 282,057
7	**36** 38,434	**69** 89,480	**68** 116,300	**37** 23,915	**6** 12,750	**216** 280,879
8	**27** 24,334	**56** 71,105	**60** 101,930	**33** 26,365	**5** 12,250	**181** 235,984
9	**24** 21,450	**49** 58,405	**54** 90,003	**28** 21,365	**5** 9,750	**160** 200,973
Totals	**453** 618,838	**858** 1,174,122	**817** 1,469,560	**526** 702,795	**133** 196,529	**2,787** 4,161,844

TABLE B

Expected Deaths and Claims under the $O^{[M]}$ *Table.*

Years elapsed since date of Assurance	Ages at Entry: Under 30	30 to 39	40 to 49	50 to 59	60 and over	All ages combined
0	**·21** 447	**·44** 809	**·59** 1,057	**·87** 1,715	**·56** 810	**2·67** 4,838
1	**·27** 454	**·66** 1,041	**·79** 1,508	**1·07** 1,837	**·75** 936	**3·54** 5,776
2	**·31** 420	**·69** 943	**·92** 1,687	**1·22** 1,877	**·76** 1,048	**3·90** 5,975
3	**·34** 430	**·74** 970	**1·03** 1,962	**1·16** 1,408	**·60** 544	**3·87** 5,314
4	**·28** 309	**·70** 894	**1·03** 1,736	**·92** 1,118	**·29** 311	**3·22** 4,368
5	**·27** 305	**·67** 796	**1·01** 1,609	**1·01** 1,137	**·29** 645	**3·25** 4,492
6	**·24** 275	**·66** 826	**1·00** 1,583	**·97** 882	**·40** 622	**3·27** 4,188
7	**·27** 293	**·68** 896	**1·07** 1,800	**1·02** 677	**·43** 688	**3·47** 4,354
8	**·24** 193	**·59** 769	**1·04** 1,735	**1·01** 805	**·28** 667	**3·16** 4,169
9	**·21** 182	**·54** 640	**1·02** 1,718	**·93** 720	**·47** 734	**3·17** 3,994
Totals	**2·64** 3,308	**6·37** 8,584	**9·50** 16,395	**10·18** 12,176	**4·83** 7,005	**33·52** 47,468

TABLE C.

Actual Deaths and Claims.

Years elapsed since date of Assurance	Ages at Entry					
	Under 30	30 to 39	40 to 49	50 to 59	60 and over	All ages combined
0	...	...	1 2,500	...	...	1 2,500
1	...	2 8,500	...	1 1,000	...	3 9,500
2	...	...	...	1 15,000	2 2,408	3 17,408
3	...	...	...	...	...	...
4	...	...	2 3,500	...	..	2 3,500
5	1 200	...	...	1 100	1 3,000	3 3,300
6	...	...	...	1 5,000	...	1 5,000
7	...	...	...	...	1 500	1 500
8	...	...	...	1 1,500	...	1 1,500
9	1 3,534	...	...	...	...	1 3,534
Totals	2 3,734	2 8,500	3 6,000	5 22,600	4 5,908	16 46,742

TABLE D.

Experience after the first 10 years' duration.

—	Ages attained							
	Under 40	40 to 49	50 to 59	60 to 69	70 to 79	80 to 89	90 & over	All ages combined
osed to Risk . ..	78 68,319	311 500,408	630 1,196,549	557 805,403	380 540,892	90 199,400	12 10,200	2,058 3,321,171
ected Deaths & Claims	·6 610	3·8 6,191	13·2 24,418	22·1 32,558	32·3 45,219	17·1 37,089	4·0 3,164	93·1 149,579
ual Deaths & Claims	...	4 6,100	6 11,830	16 20,700	28 29,483	10 16,300	...	64 84,713

The results are summarised in the following Table:

—	Duration					
	Less than 10 years		10 years & upwards		All Durations combined	
	Policies	Sums Assured	Policies	Sums Assured	Policies	Sums Assured
Exposed to Risk ...	2,787	4,161,844	2,058	3,321,171	4,845	7,483,015
Expected Deaths ...	33·5	47,468	93·1	149,579	126·6	197,047
Actual Deaths	16	46,742	64	84,713	80	131,455
Ratio of Actual to Expected	·477	·985	·687	·566	·632	·667

Perhaps the most interesting feature is the lightness of the mortality experienced after the selection has worn off. This is specially noticeable among the largest assurances, for the deaths after 10 years amount to 68·7 per-cent of the expected and the claims to only 56·6 per-cent. For the first 10 years' duration the mortality among the larger policies appears to have been above that of the smaller ones, but I might add that here the figures were seriously affected by one claim for £15,000 (under an Estate Duty policy) which only escaped exclusion from the experience by a margin of three days.

It might be contended that the experience is rendered valueless by the inclusion of invalid lives at their rated-up ages. These lives might all have been included at their true ages,and the expected claims would have been about $7\frac{1}{2}$ per-cent less than those shown above. It would have been unsatisfactory to reduce the dimensions of the experience still further by excluding invalid lives altogether, and, allowing that $7\frac{1}{2}$ per-cent is perhaps rather too much to add for what is left of the initial invalidity, I think there is sufficient margin left over to establish a *prima facie* case that "financial lives" are not necessarily bad lives, and that an office basing its non-profit rates on the $O^{[M]}$ Table may still expect a substantial profit from mortality.

Yours faithfully,

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On the extension of existing Valuation Methods of grouping policies by the employment of a system of Weights. By Albert E. King, F.I.A., *of the Equitable Life Assurance Society.*

[Read before the Institute, 26 January 1914.]

I.—Introduction.

Improvement in valuation methods.

THE advances which have been made in valuation methods were brought home very forcibly to my mind when perusing the records of a valuation made many years ago.

2. During the past 50 or 60 years the valuation of complicated benefits has been, generally speaking, rendered a comparatively easy task, thanks to the energies of members of this and of other Actuarial Institutes—the Law of Makeham, to take one instance, has been the means of simplifying the solutions of a great number of difficult problems—and the numerous approximations to various intricate expressions, which have appeared from time to time in the pages of the *Journal*, have further lightened the labour of valuations.

3. The outstanding feature of modern valuation methods is, however, the system of grouping policies wherever it is possible to do so. The valuation of Whole Life Assurances merely consists of the evaluation of about 90 sets of particulars, one set for each office year of birth. In the case of Endowment Assurances the policies are usually grouped for valuation purposes according to the year of maturity, and only about 50 sets of particulars have to be evaluated. The use of methods of grouping also naturally paved the way to the

adoption of continuous valuation classification books, which make a yearly valuation a matter of quite easy performance.

Outline of Paper.

4. As the title of this Paper shows, it is not my present purpose to consider the valuation of complicated benefits, but to suggest that the existing methods of grouping might be still further developed to save both time and labour without sacrificing very much of the accuracy which any method must preserve to be of practical service, and it may be well first of all to give an outline of the ground which the Paper will cover.

5. The existing methods of grouping combine policies with a common Office year of birth, year of maturity, or year of issue, according to the class of policy; I endeavour to show that it is possible to combine the particulars in respect of a number of consecutive Office years of birth, years of maturity, or years of issue, as the case may be, without introducing any considerable error in the final result. The new method, which for convenience may be termed the weighted year method, forms, therefore, a useful check on the results brought out by existing methods, but, what is even more important, it is to be considered whether the existing methods might not be discarded in favour of more extended groupings.

II. Basis of the Method.

Two methods of approximate valuation commonly employed.

6. There are two approximate valuation methods commonly employed at the present day for checking the valuation results. The first may be termed the central age method, for it consists of collecting the sums assured, office premiums, net premiums, &c., in groups in respect of a number of consecutive ages, say t ages, and valuing the total of the figures for each group by the factors corresponding to the central age of the group. Thus, for the value of the Sums Assured, we have the equation $\Sigma S_x A_x = A_{\bar{x}} \times \Sigma S_x$ approximately, where $\bar{x}$ represents the central age of the group and ΣS_x represents the total Sums Assured in the group. The second method is that suggested by Woolhouse, *J.I.A.*, vol. xxvii, p. 433, for Whole Life Assurances, Endowment Assurances and Annuities.

7. It is well known that both of the methods last mentioned produce results which are often not free from considerable error, and probably neither will ever be applied to supersede the more detailed methods at present in use for the purpose of supplying the necessary particulars for the Board of Trade Returns.

8. The method which I propose to discuss depends on an extremely simple system of weighting, and the valuation of the Sums Assured of Whole Life Assurances will serve as an illustration of the principles on which it is based.

Weighted year method.

9. Assume that we have the figures for valuation arranged according to present age, in other words, arranged according to the Office year of birth. First of all the particulars should be marked off in groups of a fixed number of consecutive ages, say 10 ages, and the sum of the figures for each group should be recorded. It will be appreciated that instead of using the central age of a group much greater accuracy would be secured if a weighted age were determined, giving effect to the weight of the Sums Assured against each attained age. This is therefore our *first* step; find ω, where $\omega = \frac{\Sigma x S_x}{\Sigma S_x}$, and then evaluate $(\Sigma S_x) A_\omega$.

10. The example (O^M 3%) on the following page shews the method of procedure when ten ages are grouped together.

11. It will be seen on referring to the example that no advantage was secured by working on the Sums Assured correct in the units figure to find the weighted age, in fact identically the same age was obtained to two places of decimals when the Sums Assured cut down to the nearest £100 were employed, and there would have been no appreciable loss of accuracy if we had set down the Sums Assured to the nearest £1,000 for the purpose of weighting. The number of figures which we should retain in any particular case must obviously depend on the size of the business we are valuing.

12. Reverting to the central age method, it is obvious from an inspection of the equation already given, § 6, that this method might only be expected to give reliable results if the Sums Assured were fairly evenly distributed age by age throughout the group. Even if the Sums Assured were of equal amount at each age, however, an error would be introduced if $A_{\bar{x}}$ were employed to evaluate the total Sums Assured; in this particular case a closer approximation to $\frac{\Sigma A_x}{t}$ (which would be the correct value of A_x to employ) is $A_{\bar{x}} + \frac{t^2-1}{24}\delta^2 A_{\bar{x}}$, *i.e.*, the correction to $A_{\bar{x}}$ is $\frac{t^2-1}{24}\delta^2 A_{\bar{x}}$ where t is the range of, or the number of consecutive ages included in, the group (cf. G. F. Hardy, *J.I.A.*, vol. xxxii, p. 371).

Age	S_x	$S_x \times A_x$	Distance (K) from age 52*	$S_x \times K$	$\frac{S}{100}$	$\frac{S_x}{100} \times K$
	£	£				
57	192,702	122,551	5	963,510	1,927	9,635
56	180,889	113,085	4	723,556	1,809	7,236
55	206,423	126,826	3	619,269	2,064	6,192
54	129,708	78,302	2	259,416	1,297	2,594
53	147,563	87,508	1	147,563	1,476	1,476
52	126,722	73,808	0	Σ + 2,713,314	1,267	Σ + 27,133
51	134,623	76,996	− 1	134,623	1,346	1,346
50	114,779	64,453	− 2	229,558	1,148	2,296
49	115,348	63,583	− 3	346,044	1,153	3,459
48	165,686	89,643	− 4	662,744	1,657	6,628
				Σ − 1,372,969		Σ − 13,729
Totals	1,514,443	896,755	...	+ 1,340,345	15,144	+ 13,404

Approximate Valuation, weighting with S_x—

$$\frac{1,340,345}{1,514,443} = \cdot 89$$

$$\therefore \quad \omega = 52 + \cdot 89 = 52\cdot 89$$

Approximate Valuation, weighting with $\frac{S_x}{100}$—

$$\frac{13,404}{15,144} = \cdot 89$$

$$\therefore \quad \omega = 52 + \cdot 89 = 52\cdot 89$$

$$£1,514,443 \times A_{52\cdot 89} = £1,514,443 \times \cdot 59186 = £896,338$$

$$\text{Error} = £896,755 - £896,338 = £417$$

* The distance (K) can of course be calculated from any other age of the group.

13. The consideration of a correction to $A_{\overline{x}}$ in the last paragraph leads us to our second step, which is to find a similar correction to A_ω. This correction is at once obtained by assuming (but *only* for the purpose of obtaining this correction) that the Sums Assured are of equal amount at each age of the group. If, therefore, $\overline{\omega}$ represent the corrected weighted age, we have $A_{\overline{\omega}} = A_\omega + \frac{t^2-1}{24}\delta^2 A_\omega$, and our valuation rule becomes

Find $\qquad \omega$ where $\omega = \frac{\Sigma x S_x}{\Sigma S_x}$

and then evaluate $\Sigma S_x\left(A_\omega + \frac{t^2-1}{24}\delta^2 A_\omega\right)$*

14. If we fix definitely upon a suitable range of ages for grouping, tables of $f_{\bar{\omega}}$ can be prepared and tabulated to correspond to each value of ω (such tables for groups of ten consecutive ages appear at the end of the Paper). The only work entailed by the weighted year method is therefore (1) to determine ω on the lines of the example given in § 10, (2) to enter a table of $f_{\bar{\omega}}$ with ω, (3) to multiply the quantity to be valued by $f_{\bar{\omega}}$. Thus in the above example we have $\omega = 52{\cdot}89$. The table on page 134 gives $A'_{52\cdot 89} = {\cdot}59215$, and £1,514,443 × ·59215 produces the value £896,777, the error being only £22.

15. The valuation of the Sums Assured of Whole Life policies has been taken as an illustration of the working of the method; the valuation of Reversionary Bonuses and Premiums could be dealt with in identically the same way, and if the maximum accuracy obtainable by the weighted year method be desired, separate weighted ages should be obtained for the Sums Assured, Reversionary Bonuses, and Premiums, but in practice it is generally possible, without introducing any great error into the final results, to value the Reversionary Bonuses and Premiums by making small adjustments to the weighted age for the Sums Assured.

16. If the Reversionary Bonuses and Premiums were a fixed percentage of the Sums Assured, age by age, throughout a group, the weighted age for the Sums Assured, Reversionary Bonuses and Premiums would clearly be identical. As however, the average Reversionary Bonuses and Premiums tend to increase with the valuation age, the weighted age for these quantities will generally be somewhat higher than the weighted ages for the Sums Assured.

17. The exact adjustments which have to be made to the weighted age for the Sums Assured in order to produce the

* A more accurate correction to employ for the weighted year method would have been

$$\left(\frac{\Sigma x^2 S_x}{\Sigma S_x} - \omega^2\right)\frac{\delta^2}{2}A_\omega$$

This fact is brought out by Mr. Lidstone in his Note, *J. I. A.*, vol. xlv, p. 483, on the Determination of the Weighted Mean Value of a Variable; the expression as it stands is, however, impracticable for the solution of our problem, because of the labour involved in weighting with both x and x^2.

† Corrected values of $f\omega$ (*i.e.* $f\bar{\omega}$) are represented in the Tables for convenience by accented symbols, thus: A'_ω.

weighted ages for the Reversionary Bonuses and Premiums respectively, depend clearly on the average rate of increase and the distribution of the Bonuses or Premiums as the case may be throughout the group. The problem of determining the required adjustments approximately is an interesting one, and for convenience the adjustment in the case of the Reversionary Bonuses is alone considered, the argument applying equally to the Premiums.

18. If we divide the total Bonuses of a group by the total Sums Assured we obtain the average Bonus for the group, and by comparing the average Bonuses for different groups we can obtain an approximation to the rate of increase age by age. Thus if B_1, B_2, B_3, B_4, &c., and S_1, S_2, S_3, S_4, &c., are the total Bonuses and total Sums Assured for groups each consisting of say t ages, the average Bonuses are $\frac{B_1}{S_1}$, $\frac{B_2}{S_2}$, $\frac{B_3}{S_3}$, $\frac{B_4}{S_4}$, &c. If we now assume these average Bonuses to increase in geometrical progression, age by age, and if

$(1+i_1)$ be the rate of increase over the group with Bonus of B_1

$(1+i_2)$,, ,, ,, ,, B_2

⋮

$(1+i_m)$,, ,, ,, ,, B_m

⋮

we have, approximately,

$$(1+i_m)^{2t} = \frac{\dfrac{B_{m-1}}{S_{m-1}}}{\dfrac{B_{m+1}}{S_{m+1}}} = \text{K say}.$$

19. Having now determined the rate of increase, the adjustment to be made to the weighted age for the Sums Assured, in order to arrive at the corresponding weighted age for the Bonuses (assuming the Reversionary Bonuses to be of equal amount throughout the group for the purpose of finding this adjustment only) is equal to

$$\frac{0(1+i)^0 + 1(1+i)^1 + \ldots + (t-1)(1+i)^{t-1}}{(1+i)^0 + (1+i)^1 + \ldots + (1+i)^{t-1}} - \frac{t-1}{2}$$

$$= \frac{1}{i}\left(\frac{t}{a_{\overline{t}|}} - 1\right) - \frac{t+1}{2}$$

where $$i = K^{\frac{1}{2t}} - 1.$$

For groups of 10 years the adjustment is therefore

$$\frac{1}{i}\left(\frac{10}{a_{\overline{10}|}}-1\right)-5\cdot5$$

where $$i=K^{\frac{1}{20}}-1.$$

20. The following table shows the adjustments corresponding to various values of K and $(1+i)$.

For groups of 10 *years.*

Increase in Age	K	$1+i$
·1	1·27	1·012
·2	1·62	1·025
·3	2·07	1·037
·4	2·65	1·050
·5	3·39	1·063
·6	4·34	1·076
·7	5·58	1·090
·8	7·19	1·104
·9	9·29	1·118
1 0	12·03	1·132

The adjustments may obviously be determined approximately in several ways, but the above method has been found to give very satisfactory results. In practice it will generally be found sufficient to assume the same rate of increase for each group and to adopt the same adjustment throughout.

III. Application of the Method.

21. Attained ages in the case of Whole Life Assurances usually range from about age 15 to age 100. It has been found by experiment and experience that groups of 10 ages prove very convenient; f_{ϖ} in this case takes the form $f_{\omega}+4\frac{1}{8}\delta^2 f_{\omega}$.

22. It sometimes happens that there are no policies existing at some of the younger and older ages, and therefore the "end" groups are sometimes incomplete. In such cases it is better (although there is not generally a great deal to be gained) to use f_{ω} uncorrected, instead of f_{ϖ}.

23. Endowment Assurances are generally grouped for Valuation purposes according to years of maturity, and the Z method is in general use for determining the average maturity age for each year of maturity. It will be well,

therefore, to explain the procedure that would be adopted if the Z method were employed in conjunction with the weighted year method.

24. The particulars to be valued should be marked off in groups of, say t consecutive years of maturity. The Sums Assured, Bonuses, Z's, and Premiums, should then be summed for each group. When this is done the average maturity age for each group of t years of maturity may be found by entering a Table of $Z_{\overline{M}}$ with the result of dividing the total Z's for the group by the corresponding total Sums Assured.*

25. The average year of maturity is found by weighting each year of maturity in the same manner as we weighted the Office year of birth (or attained age) in the case of Whole Life Assurances.

If M be the average maturity age at the end of the calendar year of maturity (Method I, *J.I.A.*, vol. xxxviii, p. 24) and $1900+q$ be the weighted year of maturity for a group, $1900+p$ be the year of valuation, and n the curtate unexpired term, we have

$$n = (1900+q)-(1900+p+1).$$

26. Both M and n are now known, and if we enter a Table of $\bar{a}'_{\mathrm{M}-\overline{n+1}:\overline{n}|}$ which has been obtained by the addition of $\left(\frac{t^2-1}{24}\right)\delta^2 f_\omega$ to $\bar{a}_{\mathrm{M}-\overline{n+1}:n}$ (*vide* Table) we have at once $\bar{a}'$, the factor for the valuation of premiums. A$'$ corresponding to $\bar{a}'$ is obtained as usual by entering the Conversion Tables with $\bar{a}'-\frac{1}{2}$.

27. In the case of Endowment Assurances the maximum unexpired term will be about 50 years, and, if groups of 10 years are employed, 5 groups will therefore suffice.

28. The remarks as to incomplete groups (§22 *supra*) will also apply in the case of Endowment Assurances. In the case of an incomplete group containing those policies nearest maturity, it is generally advisable to allow for the smaller range, and to employ $\bar{a}+\frac{t^2-1}{24}\delta^2\bar{a}$ for the valuation factor, where t is the number of consecutive years in the incomplete group. This factor can be easily obtained by multiplying the

* Mr. Elderton has shown (*J.I.A.*, vol. xlviii, p. 1) that it is possible to fix a maturity age for the whole of the Endowment Assurance business from the outset. If this simplified method be in use the above operation $\Sigma Z \div \Sigma S$ is dispensed with, and, having the age of maturity fixed, we may proceed at once to find our weighted year of maturity.

difference between $\bar{a}$ and $\bar{a}'$ by $\frac{t^2-1}{99}$ and deducting the result from $\bar{a}$.

29. Those classes of policies which are usually grouped according to year of issue, such as Deferred Assurances, before the deferred age is attained, Capital Redemption Assurances, &c., are readily valued by the weighted year method, but space prevents their detailed treatment here. The general idea of the working principles having been given, it is a simple matter to extend them to other classes of contracts.

IV. Special Classification Books.

30. In order to obtain the particulars for a valuation by the methods at present in vogue, it is necessary to divide the class books into about 90 separate sections for Whole Life Assurances, and about 50 sections for Endowment Assurances. If the weighted year method be adopted the number of sections is very considerably reduced; consequently the work involved in keeping the classification books is much simplified, and it is possible to obtain a yearly valuation with a minimum of labour.

31. This system of Class Books may be explained by dealing, first of all, with the Sums Assured under Whole Life policies. Let the groups be chosen for 10 years of age in respect of policies with Office years of birth 1810-19, 1820-29, 1830-39, and so on. Now in arriving at the weights for the class books it is advisable to modify the method followed in § 10, as in order to save columns in the class books it is necessary to keep the same sign for all the weights. We accordingly proceed as follows:

Multiply the Sum Assured

by 0	if the Office year of birth	be	1810, 1820, 1830, &c.
,, 1	,,	,,	1811, 1821, 1831, &c.
,, 2	,,	,,	1812, 1822, 1832, &c

and so on, and we have a series of weights (which should be divided by 100 to avoid large figures); these weights may be recorded on the valuation cards, and in the class books.

32. The division of the total weights in any given group by the total Sums Assured of the group, will of course give the units figure of the weighted Office year of birth in respect of the Sums Assured, from which we arrive at the weighted age ω; and with ω we enter the Table of $f_{\bar{\omega}}$

(calculated for groups of 10) and obtain at once the proper valuation factor.

33. Similar methods could be pursued in tabulating the weights in respect of the Premiums, Reversionary Bonuses, &c., but as already indicated it is thought that it will be generally sufficient to arrive at the weighted age for these quantities from the weighted age for the Sums Assured.

34. Endowment Assurances are dealt with in a similar manner, the only variation being that the multiplier for weighting purposes is the units figure of the year of maturity instead of the units figure of the Office year of birth.

35. This system of class books has now been actually adopted by one of the largest Offices in this country, and I am confident that the method, while saving a great deal of time and labour, gives results which are accurate enough to enable an Actuary to obtain a close estimate of the liabilities of his Office.

36. At this point I submit summary of particulars together with valuations made by the weighted year method.

[When the values by the weighted year method are in excess of the detailed valuation figures the deviations are marked (+), when in defect (−)].

Summary of Particulars. Whole-Life Assurance.

Years of Birth	Sums Assured		Reversionary Bonuses		Net Premiums	
	Weighted Ages	Totals	Weighted Ages	Totals	Weighted Ages	Totals
1821–29	83·52	76,146	83·31	23,807	83·55	2,953
1830–39	75·09	390,773	75·33	106,207	75·36	11,298
1840–49	65·41	1,234,675	65·52	284,742	65·61	30,420
1850–59	55·72	1,399,823	56·29	221,412	55·73	29,991
1860–69	46·61	863,531	47·02	76,955	46·67	16,846
1870–79	37·10	321,033	38·17	11,224	37·25	5 560
1880–89	28·21	117,250	29·04	1,122	28·32	1,639
1890–95	16·98	9,720	...	...	17·05	114
		4,113,251	..	725,469	..	98,821

Summary of Particulars. Endowment Assurances.

Years of Maturity	Average Maturity Ages	Sums Assured		Reversionary Bonuses		Net Premiums	
		Weighted Durations n	Totals	Weighted Durations n	Totals	Weighted Durations n	Totals
1911–19	58·1	4·36	506,497	4·41	63,120	4·20	19,846
1920–29	58·7	13·54	704,548	12·97	22,272	13·39	23,397
1930–39	59·8	22·50	347,700	22·13	6,421	22·23	9,267
1940–49	60·3	31·54	60,435	30·75	399	31·51	1,237
1950–54	62·7	40·68	1,550	...	...	40·58	25
...	...	...	1,620,730	...	92,212	...	53,772

Valuation of Whole-Life Assurances. O^{M} 3 %.

Weighted Year Method.

Years of Birth	Sums Assured		Reversionary Bonuses		Net Premiums		Percentage Deviation in Value of			
	Value*	Deviation	Value†	Deviation	Value‡	Deviation	S.A.	R.B.	N.P.	Net L.
1821–29	67,423	−36	20,965	−65	10,470	+62	·05	·31	·60	·21
1830–39	319,852	− 4	87,149	+ 4	63,857	−30	...	...	·05	·01
1840–49	895,941	+31	206,948	+18	269,154	−60	...	·01	·02	·01
1850–59	871,124	−29	139,142	+ 4	373,783	+30	...	...	·01	·01
1860–69	455,608	+ 4	40,918	+ 1	264,453	−43	...	...	·01	·02
1870–79	140,994	+ 2	5,034	+ 2	104,003	− 5	...	·04	...	·02
1880–89	43,087	+14	419	...	34,723	− 8	·03	...	·02	·25
1890–95	2,832	...	...	...	2,720	+ 1	...	...	...	...
Totals	2,796,861	−15	500,575	−36	1,123,163	−53	...	·01	...	...

* Using weighted age found for Sums Assured.
† Using weighted age found for Reversionary Bonuses.
‡ Using weighted age found for Net Premiums.

Total net liability...2,174,273

Deviation + 2

Valuation of Whole-Life Assurances. O^M 3 %.

Weighted Year Method.

Years of Birth	Sums Assured		Reversionary Bonuses		Net Premiums		Percentage Deviation in Value of			
	Value	Deviation	Value†	Deviation	Value†	Deviation	S.A.	R.B.	N.P.	Net L.
1821–29	67,423	−36	21,021	− 9	10,361	− 47	·05	·04	·45	...
1830–39	319,852	− 4	87,078	− 67	64,095	+ 208	...	·08	·33	·08
1840–49	895,941	+34	207,330	+400	270,006	+ 792	...	·19	·29	·04
1850–59	871,124	−29	138,737	−401	372,583	−1170	...	·29	·31	·11
1860–69	455,608	+ 4	41,087	+170	264,116	− 380	...	·42	·14	·24
1870–79	140,994	+ 2	5,027	− 5	104,003	− 5		·10	...	...
1880–89	43,087	+14	412	− 7	34,702	− 29	·03	1·67	·08	·41
1890–95	2,832	.	.	...	2,721	+ 2	.	...	...	...
Totals ..	2,796,861	−15	500,692	+ 81	1,122,587	− 629	...	·02	·06	·03

* Using weighted age found for Sums Assured.
† Using weighted age found for Sums Assured with adjustments as described §§16–20.

Total net liability... 2,174,966

Deviation +695

Valuation of Endowment Assurances. O^M 3 %.

Weighted Year Method.

Years of Maturity	Sums Assured		Reversionary Bonuses		Net Premiums		Percentage Deviation in Value of			
	Value*	Deviation	Value†	Deviation	Value‡	Deviation	S.A.	R.B.	N.P.	Net L.
1911–19	443,210	+263	55,156	+77	72,597	−280	06	·14	·38	·15
1920–29	485,651	−143	15,572	+10	235 795	+ 21	·03	·07	·01	·06
1930–39	193,544	+ 2	3,606	− 3	135,289	− 28	..	·08	·02	·04
1940–49	27,291	+ 28	184	+ 1	22,661	+ 27	·10	...	·12	·04
1950–54	577	+ 1	...	...	532		...	...	...	...
Totals ..	1,150.273	+151	74,518	+85	466,874	−260	·01	·11	·06	·07

* Using weighted age found for Sums Assured.
† Using weighted age found for Reversionary Bonus.
‡ Using weighted age found for Net Premiums.

Total net liability... 757,917

Deviation +496

Valuation of Endowment Assurances. O^{M} 3½ %.

Weighted Year Method.

Years of Maturity	Sums Assured		Reversionary Bonuses		Net Premiums		Percentage Deviation in Value of			
	Value*	Deviation	Value†	Deviation	Value†	Deviation	S.A.	R.B.	N.P.	Net L.
1911–19	443,210	+262	55,235	+156	72,597	−280	·06	·28	·38	·16
1920–29	485,651	−143	15,627	+ 65	235,655	−119	·03	·42	·05	·02
1930–39	193,544	+ 2	3,628	+ 19	135,585	+268	...	·53	·20	·40
1940–49	27,291	+ 28	180	− 3	22,584	− 50	·10	1·61	·16	1·56
1950–54	577	+ 1	—	..	531	− 1	...	...	...	...
Totals	1,150,273	+151	74,670	+237	466,952	−182	·01	·32	·01	·08

* Using weighted age found for Sums Assured.

† Using weighted age found for Sums Assured with adjustments as described §§16–20.

Total net liability... 757,991

Deviation +570

37. It will be seen that the results by the new method deviate but little from those obtained by the application of more detailed methods. In the case of most Offices, and especially those Offices which do business in many countries abroad, the saving of time and labour would be very great, and the saving would take place at the time when it is most valuable—when the work attending a valuation reaches its climax.

V. Concluding Remarks.

38. It may be that in the future the whole of the policies of one class may be combined into one group for valuation by means of a frequency curve, indeed this was naturally suggested when the weighting with x, x^2, &c., was being considered in §§ 9–13, but the possibility cannot be further considered in this paper.

39. In conclusion, I wish to record my indebtedness to Messrs. S. J. Rowland, A.I.A., and J. I. Gopp, for their kind assistance in checking the Tables.

For groups of **10** *years.*

O^M — **3 per-cent.**

x	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	x
	A'_x											
10	·25296	346	396	447	497	547	598	649	700	751	51	10
1	·25802	853	905	956	**008**	**060**	**113**	**165**	**218**	**270**	52	1
2	·26323	376	429	482	535	588	642	696	750	804	54	2
3	·26858	912	967	**021**	**076**	**131**	**186**	**241**	**297**	**352**	55	3
4	·27408	464	520	576	632	688	745	802	859	916	56	4
15	·27973	**030**	**088**	**146**	**203**	**261**	**319**	**377**	**435**	**493**	58	15
6	·28552	611	670	729	788	847	906	966	**026**	**086**	59	6
7	·29146	206	267	327	387	448	509	570	631	692	61	7
8	·29754	816	878	939	**001**	**064**	**126**	**188**	**251**	**314**	62	8
9	·30377	440	503	566	630	694	758	822	886	950	64	9
20	·31015	080	144	209	274	339	404	470	535	600	65	20
1	·31666	732	798	864	930	996	**062**	**129**	**196**	**263**	66	1
2	·32330	397	465	532	600	668	736	804	873	941	68	2
3	·33010	079	147	216	285	355	424	493	562	631	69	3
4	·33701	771	841	911	981	**051**	**122**	**192**	**262**	**333**	70	4
25	·34494	475	546	617	688	760	832	904	976	**048**	72	25
6	·35120	192	264	336	409	482	555	628	701	774	73	6
7	·35847	920	994	**068**	**142**	**216**	**290**	**365**	**440**	**515**	74	7
8	·36590	665	740	815	890	966	**042**	**118**	**194**	**270**	76	8
9	·37346	422	498	575	651	728	805	882	959	**036**	77	9
30	·38114	192	269	347	425	503	582	660	738	816	78	30
1	·38895	974	**053**	**132**	**211**	**290**	**369**	**448**	**527**	**607**	79	1
2	·39687	767	847	927	**007**	**087**	**167**	**248**	**329**	**410**	80	2
3	·40491	572	653	734	815	897	979	**061**	**143**	**225**	82	3
4	·41307	389	472	555	638	721	804	887	970	**053**	83	4
35	·42137	221	305	389	473	557	641	725	809	893	84	35
6	·42978	**063**	**148**	**233**	**318**	**404**	**489**	**575**	**661**	**747**	86	6
7	·43833	919	**006**	**093**	**180**	**267**	**353**	**440**	**527**	**614**	87	7
8	·44702	790	878	966	**054**	**142**	**230**	**319**	**408**	**497**	88	8
9	·45586	675	764	853	943	**033**	**123**	**213**	**303**	**393**	90	9
40	·46483	573	664	755	846	937	**028**	**120**	**211**	**302**	91	40
1	·47394	486	578	670	762	854	946	**039**	**132**	**225**	92	1
2	·48318	411	504	598	692	786	880	974	**068**	**162**	94	2
3	·49257	352	447	542	637	732	827	922	**018**	**114**	95	3
4	·50210	306	402	498	595	692	789	886	983	**080**	97	4
45	·51177	274	372	470	568	666	764	862	960	**058**	98	45
6	·52157	256	355	454	553	652	751	851	951	**051**	99	6
7	·53151	251	351	451	551	651	752	853	954	**055**	100	7
8	·54156	257	358	459	560	662	764	866	968	**070**	102	8
9	·55172	274	376	478	581	684	787	890	993	**096**	103	9
50	·56199	302	405	508	611	714	818	922	**026**	**130**	104	50
1	·57234	338	442	546	651	755	859	964	**069**	**174**	104	1
2	·58279	384	489	594	699	804	909	**014**	**120**	**226**	105	2

For groups of **10** *years.*

O

O^M **3 per-cent.**

x	N'_x ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff	x
53	·59332	438	544	650	756	862	968	074	180	286	106	53
4	·60393	499	606	712	819	926	033	140	247	354	107	4
55	·61461	568	675	782	889	997	104	211	318	425	107	55
6	·62532	639	747	854	962	069	176	284	391	499	107	6
7	·63606	714	821	929	037	145	252	360	468	575	108	7
8	·64683	791	898	006	113	221	329	436	544	651	108	8
9	·65759	867	974	082	190	293	405	513	621	728	108	9
60	·66836	943	051	158	266	373	480	587	694	801	107	60
1	67908	015	122	229	336	443	550	657	764	871	107	1
2	·68978	085	191	297	403	510	616	722	828	933	106	2
3	·70039	145	250	355	461	567	672	778	883	989	106	3
4	·71094	199	304	409	514	619	723	827	931	035	104	4
65	·72139	243	347	451	555	658	761	864	967	070	103	65
6	·73173	276	379	481	583	685	787	889	991	093	102	6
7	·74195	297	398	499	600	701	802	903	004	105	101	7
8	·75206	306	406	506	605	704	803	902	001	100	99	8
9	·76199	297	395	493	591	689	787	885	982	079	98	9
70	·77176	273	370	467	563	659	755	851	947	042	96	70
1	·78137	232	327	422	516	610	704	798	892	985	94	1
2	·79078	171	264	356	448	540	632	723	814	905	92	2
3	·79997	088	179	269	359	449	539	629	719	808	90	3
4	·80897	986	074	162	250	338	426	514	601	688	88	4
75	·81774	860	946	032	117	202	287	372	457	542	85	75
6	82626	710	793	876	959	042	125	207	289	371	83	6
7	·83452	533	614	695	776	857	937	017	097	176	80	7
8	·84255	334	412	490	568	646	723	800	877	954	78	8
9	·85030	106	182	258	333	408	483	558	632	706	75	9
80	·85780	854	927	000	073	145	217	289	361	432	72	80
1	·86503	574	644	714	784	854	923	992	061	129	69	1
2	·87197	265	332	399	466	533	599	665	731	796	66	2
3	·87861	926	991	056	120	184	248	311	374	437	64	3
4	·88500	563	625	687	749	810	871	932	993	054	61	4
85	·89114	174	233	292	351	410	468	526	583	640	58	85
6	·89697	754	811	867	923	979	034	089	144	198	56	6
7	·90252	306	360	414	467	520	573	625	677	729	53	7
8	·90781	832	883	934	984	034	083	132	181	229	50	8
9	·91278	327	375	423	471	519	567	614	661	708	48	9
90	·91755	801	846	891	936	989	024	068	111	154	44	90
1	·92197	241	284	327	370	413	455	497	539	581	43	1
2	·92623	664	704	743	782	821	860	899	938	977	40	2
3	·93016	056	095	135	174	214	253	292	331	370	39	3
4	·93409	447	485	522	559	596	632	668	704	739	36	4
95	·93774	808	842	876	909	942	975	008	040	072	33	95

For groups of **10** *years.*

O^M **3 per-cent.**

x	a'_x ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff	x
10	24·648	·631	·614	·597	·579	·562	·545	·527	·510	·492	17	10
1	·475	·457	·439	·422	·404	·386	·368	·350	·332	·314	18	1
2	·296	·278	·259	·241	·223	·205	·186	·168	·149	·131	18	2
3	·112	·093	·075	·056	·037	·018	·999	·980	·961	·942	19	3
4	23·923	·904	·885	·866	·846	·827	·808	·788	·768	·749	19	4
15	·729	·710	·690	·670	·650	·630	·610	·591	·571	·551	20	15
6	·530	·510	·490	·470	·449	·429	·409	·388	·368	·347	20	6
7	·327	·306	·285	·264	·244	·223	·202	·181	·160	·139	21	7
8	·118	·097	·075	·054	·033	·011	·990	·969	·947	·926	21	8
9	22·904	·882	·861	·839	·817	·795	·773	·751	·729	·707	22	9
20	·685	·662	·640	·618	·596	·574	·551	·529	·506	·484	22	20
1	·461	·439	·416	·393	·371	·348	·325	·302	·279	·256	23	1
2	·233	·210	·187	·164	·141	·117	·094	·071	·047	·023	23	2
3	·000	·976	·953	·929	·905	·881	·858	·834	·810	·787	24	3
4	21·763	·739	·715	·691	·667	·642	·618	·594	·570	·546	24	4
25	·521	·497	·472	·448	·424	·399	·374	·350	·325	·300	24	25
6	·275	·251	·226	·201	·176	·151	·126	·101	·076	·051	25	6
7	·026	·001	·976	·950	·924	·899	·874	·848	·822	·797	25	7
8	20·771	·745	·719	·693	·668	·642	·616	·590	·563	·537	26	8
9	·511	·485	·459	·433	·407	·380	·354	·327	·301	·274	26	9
30	·247	·221	·194	·168	·141	·114	·087	·060	·033	·006	27	30
1	19·979	·952	·925	·898	·871	·844	·817	·790	·762	·735	27	1
2	·707	·680	·652	·625	·598	·570	·543	·515	·487	·459	28	2
3	·431	·404	·376	·348	·320	·292	·264	·236	·208	·179	28	3
4	·151	·123	·094	·066	·038	·009	·981	·952	·924	·895	29	4
35	18·866	·838	·809	·780	·751	·722	·693	·664	·635	·606	29	35
6	·577	·548	·519	·490	·461	·431	·402	·373	·343	·313	29	6
7	·284	·254	·224	·194	·165	·135	·105	·076	·046	·016	30	7
8	17·985	·955	·925	·895	·865	·834	·804	·774	·743	·713	30	8
9	·682	·652	·621	·590	·560	·529	·498	·467	·436	·405	31	9
40	·374	·343	·312	·281	·250	·218	·187	·155	·124	·093	31	40
1	·061	·030	·998	·967	·935	·903	·872	·840	·808	·776	32	1
2	16·744	·712	·680	·648	·616	·583	·551	·519	·487	·454	32	2
3	·422	·389	·357	·324	·291	·259	·226	·193	·160	·127	33	3
4	·094	·062	·029	·996	·962	·929	·896	·863	·830	·796	33	4
45	15·762	·729	·696	·662	·628	·595	·561	·527	·494	·460	34	45
6	·426	·392	·358	·324	·290	·256	·222	·188	·153	·119	34	6
7	·085	·050	·016	·982	·948	·913	·879	·844	·809	·774	35	7
8	14·740	·705	·670	·636	·601	·566	·531	·496	·461	·426	35	8
9	·391	·356	·321	·286	·251	·215	·180	·144	·109	·074	35	9
50	·038	·003	·968	·932	·897	·861	·826	·790	·754	·719	36	50
1	13·683	·647	·612	·576	·540	·504	·468	·432	·396	·360	36	1
2	·324	·288	·252	·216	·180	·144	·108	·072	·035	·999	36	2

For groups of **10** *years.*

OM

3 per-cent.

x	a'_x ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	x
53	**12**·962	·926	·890	·853	·817	·781	·744	·708	·671	·635	36	53
4	·598	·562	·525	·489	·452	·415	·378	·342	·306	·269	37	4
55	·232	·195	·158	·122	·085	·048	·011	**·974**	**·938**	**·901**	37	55
6	**11**·864	·827	·790	·753	·717	·680	·643	·606	·569	·532	37	6
7	·495	·458	·421	·384	·347	·310	·273	·236	·199	·163	37	7
8	·125	·089	·052	·015	**·978**	**·941**	**·904**	**·867**	**·830**	**·793**	37	8
9	**10**·755	·719	·682	·645	·608	·571	·534	·497	·460	·423	37	9
60	·386	·350	·313	·276	·239	·202	·165	·129	·092	·055	37	60
1	·018	**·981**	**·945**	**·908**	**·871**	**·834**	**·798**	**·761**	**·724**	**·688**	37	1
2	**9**·651	·614	·578	·541	·505	·468	·432	·395	·359	·323	37	2
3	·287	·250	·214	·178	·142	·105	·069	·033	**·997**	**·960**	36	3
4	**8**·924	·888	·852	·816	·780	·744	·709	·673	·637	·601	36	4
65	·566	·530	·494	·459	·423	·387	·352	·317	·281	·246	36	65
6	·211	·175	·140	·105	·070	·035	·000	**·965**	**·930**	**·895**	35	6
7	**7**·860	·825	·790	·755	·721	·686	·651	·617	·582	·547	35	7
8	·513	·478	·444	·410	·376	·342	·308	·274	·240	·206	34	8
9	·172	·138	·104	·071	·037	·003	**·970**	**·936**	**·903**	**·870**	34	9
70	**6**·836	·803	·770	·736	·703	·670	·637	·604	·571	·539	33	70
1	·506	·474	·441	·409	·376	·344	·312	·279	·247	·215	32	1
2	·183	·151	·119	·088	·056	·024	**·993**	**·962**	**·931**	**·899**	32	2
3	**5**·868	·837	·806	·775	·744	·713	·682	·651	·620	·589	31	3
4	·559	·528	·498	·468	·438	·407	·377	·347	·317	·287	30	4
75	·258	·228	·199	·169	·140	·111	·081	·052	·023	**·994**	29	75
6	**4**·965	·936	·908	·879	·851	·822	·794	·766	·738	·709	28	6
7	·681	·654	·626	·598	·570	·542	·515	·488	·460	·433	28	7
8	·406	·379	·352	·325	·298	·271	·245	·219	·192	·166	27	8
9	·140	·114	·088	·062	·036	·010	**·984**	**·958**	**·933**	**·908**	26	9
80	**3**·882	·857	·832	·807	·782	·757	·732	·708	·683	·658	25	80
1	·634	·610	·585	·561	·537	·513	·490	·466	·442	·419	24	1
2	·396	·372	·349	·326	·303	·280	·258	·235	·212	·190	23	2
3	·167	·145	·123	·101	·079	·057	·035	·013	**·992**	**·970**	22	3
4	**2**·948	·927	·905	·884	·863	·842	·821	·800	·779	·758	21	4
85	·738	·717	·697	·677	·656	·636	·616	·596	·577	·557	20	85
6	·537	·518	·498	·479	·460	·441	·422	·403	·384	·365	19	6
7	·346	·328	·310	·291	·273	·255	·237	·219	·201	·183	18	7
8	·165	·147	·130	·113	·096	·078	·061	·044	·028	·011	17	8
9	**1**·994	·978	·961	·945	·928	·912	·896	·879	·863	·847	16	9
90	·832	·816	·800	·784	·769	·753	·738	·723	·709	·694	15	90
1	·679	·664	·649	·634	·620	·605	·590	·576	·562	·547	14	1
2	·533	·519	·505	·492	·478	·465	·451	·438	·424	·411	14	2
3	·398	·384	·371	·357	·343	·330	·317	·303	·290	·276	13	3
4	·263	·250	·237	·224	·211	·199	·186	·174	·162	·150	12	4
95	·138	·126	·114	·102	·091	·080	·069	·057	·046	·035	11	95

For groups of **10** *years.*

O^M — 3 per-cent.

x	$10^5 D_x^{-1}$* ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	x
20	1·883	·889	·896	·902	·909	·915	·921	·928	·934	·941	6	20
1	1·947	·954	·960	·967	·974	·981	·987	·994	**·001**	**·007**	7	1
2	2·014	·021	·028	·035	·042	·049	·055	·062	·069	·076	7	2
3	2·083	·090	·097	·105	·112	·119	·126	·133	·141	·148	7	3
4	2·155	·162	·170	·178	·185	·193	·200	·208	·215	·223	8	4
25	2·230	·238	·246	·254	·262	·269	·277	·285	·293	·301	8	25
6	2·309	·317	·325	·334	·342	·350	·358	·366	·375	·383	8	6
7	2·391	·399	·408	·417	·425	·434	·442	·451	·459	·468	8	7
8	2·476	·485	·494	·502	·511	·520	·529	·538	·546	·555	9	8
9	2·564	·573	·583	·592	·601	·611	·620	·629	·638	·648	9	9
30	2·657	·667	·676	·685	·695	·705	·714	·723	·733	·743	10	30
1	2·752	·762	·772	·781	·791	·802	·812	·822	·833	·843	10	1
2	2·853	·863	·874	·885	·895	·905	·916	·927	·937	·947	11	2
3	2·958	·969	·980	·991	**·002**	**·014**	**·025**	**·036**	**·047**	**·058**	11	3
4	3·069	·081	·092	·103	·115	·127	·138	·149	·161	·173	12	4
35	3·184	·196	·208	·219	·231	·243	·256	·268	·281	·293	12	35
6	3·305	·318	·330	·343	·355	·368	·381	·393	·406	·418	13	6
7	3·431	·444	·457	·469	·482	·496	·509	·522	·536	·549	13	7
8	3·562	·576	·589	·603	·617	·630	·644	·658	·672	·687	14	8
9	3·701	·715	·729	·743	·758	·773	·787	·802	·817	·832	15	9
40	3·847	·862	·877	·892	·907	·922	·938	·953	·969	·984	15	40
1	4·000	·016	·031	·047	·063	·079	·095	·111	·127	·144	16	1
2	4·160	·177	·193	·210	·227	·243	·260	·277	·294	·312	17	2
3	4·329	·347	·364	·382	·400	·418	·435	·453	·471	·490	18	3
4	4·508	·527	·545	·564	·582	·601	·620	·639	·658	·677	19	4
45	4·696	·716	·735	·755	·775	·795	·814	·834	·854	·875	20	45
6	4·895	·915	·936	·957	·978	**·000**	**·021**	**·042**	**·063**	**·085**	21	6
7	5·106	·128	·150	·172	·194	·216	·238	·261	·284	·307	22	7
8	5·330	·353	·376	·400	·423	·447	·471	·495	·519	·543	24	8
9	5·568	·593	·618	·642	·667	·692	·718	·743	·769	·795	25	9
50	5·821	·847	·873	·900	·927	·954	·981	**·008**	**·035**	**·063**	27	50
1	6·091	·119	·147	·176	·205	·233	·262	·291	·320	·350	29	1
2	6·380	·410	·440	·471	·501	·532	·563	·595	·626	·658	31	2
3	6·690	·722	·754	·787	·820	·853	·886	·920	·954	·988	33	3
4	7·022	·057	·092	·127	·163	·198	·234	·270	·307	·344	36	4
55	7·381	·419	·457	·495	·533	·571	·610	·650	·690	·730	39	55
6	7·770	·810	·851	·893	·935	·977	**·019**	**·062**	**·105**	**·148**	42	6
7	8·192	·236	·281	·326	·371	·417	·463	·509	·556	·603	46	7
8	8·651	·699	·748	·797	·846	·896	·947	·998	**·049**	**·100**	50	8
9	9·152	·205	·258	·312	·366	·421	·476	·532	·588	·644	55	9
60	9·701	·759	·818	·877	·937	·997	**·058**	**·120**	**·182**	**·245**	61	60
1	10·308	·372	·437	·503	·569	·636	·704	·772	·841	·911	67	1
2	10·981	**·052**	**·124**	**·197**	**·271**	**·345**	**·420**	**·496**	**·573**	**·650**	75	2
3	11·728	·807	·887	·969	**·052**	**·135**	**·219**	**·304**	**·390**	**·476**	84	3
4	12·564	·653	·743	·834	·927	**·020**	**·115**	**·210**	**·306**	**·403**	94	4

* Tabulated for use in connection with Altenburger's method.

For groups of **10** *years.*

O^M *Maturity Age* **45.** **3 per-cent.**

n	$\bar{a}'_{45-n+1:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{45-n+1:n} - \bar{a}'_{46-n+1:n}$	n
1	·824	·921	·018	·114	·210	·306	·401	·495	·589	·683	95	2	1
2	1·777	·870	·963	·056	·148	·240	·331	·422	·512	·602	92	2	2
3	2·692	·782	·871	·960	·048	·136	·224	·312	·399	·486	88	3	3
4	3·572	·658	·744	·830	·916	·001	·086	·170	·254	·338	85	4	4
5	4·421	·504	·587	·670	·752	·834	·916	·997	·078	·159	82	5	5
6	5·240	·320	·400	·480	·559	·638	·717	·796	·874	·952	79	7	6
7	6·030	·107	·184	·261	·338	·415	·491	·567	·643	·718	76	8	7
8	6·793	·868	·943	·017	·091	·165	·238	·311	·384	·457	74	10	8
9	7·530	·602	·674	·746	·818	·890	·961	·032	·103	·173	71	11	9
10	8·243	·313	·383	·453	·522	·591	·660	·728	·796	·864	69	14	10
11	8·932	·000	·067	·134	·201	·268	·334	·400	·466	·532	67	17	11
12	9·598	·664	·729	·794	·859	·924	·988	·052	·116	·180	65	18	12
13	10·244	·308	·371	·434	·497	·560	·622	·684	·746	·808	63	20	13
14	10·870	·932	·993	·054	·115	·176	·236	·296	·356	·416	61	22	14
15	11·476	·536	·595	·654	·713	·772	·831	·890	·948	·006	59	24	15
16	12·064	·122	·180	·238	·295	·352	·409	·466	·523	·579	57	26	16
17	12·635	·691	·747	·803	·859	·914	·969	·024	·079	·134	55	28	17
18	13·189	·244	·298	·352	·406	·460	·514	·567	·620	·673	54	30	18
19	13·726	·779	·832	·885	·937	·989	·041	·093	·145	·197	52	32	19
20	14·248	·299	·350	·401	·452	·503	·554	·604	·654	·704	51	33	20
21	14·754	·804	·854	·903	·952	·001	·050	·099	·148	·197	49	34	21
22	15·246	·294	·342	·390	·438	·486	·534	·582	·630	·677	48	35	22
23	15·724	·771	·818	·865	·911	·957	·003	·049	·095	·141	46	36	23
24	16·187	·233	·278	·323	·368	·413	·458	·503	·548	·593	45	37	24
25	16·637	·681	·725	·769	·813	·857	·901	·944	·987	·030	44	38	25
26	17·073	·116	·159	·202	·245	·287	·329	·371	·413	·455	42	38	26
27	17·497	·539	·581	·622	·663	·704	·745	·786	·827	·868	41	38	27
28	17·909	·949	·989	·029	·069	·109	·149	·189	·229	·269	40	39	28
29	18·308	·347	·386	·425	·464	·503	·542	·580	·618	·656	39	39	29
30	...	...	...	...	...	...	...	...	...	...	...	...	30
31	...	...	...	...	...	...	...	...	...	...	...	...	31
32	...	...	...	...	...	...	...	...	...	...	...	...	32
33	...	...	...	...	...	...	...	...	...	...	...	...	33
34	...	...	...	...	...	...	...	...	...	...	...	...	34
35	...	...	...	...	...	...	...	...	...	...	...	...	35
36	...	...	...	...	...	...	...	...	...	...	...	...	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **46.** **3 per-cent.**

n	$\bar{a}'_{46-n+1:\overline{n}\rvert}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{46-n+1:\overline{n}\rvert} - \bar{a}'_{47-n+1:\overline{n}\rvert}$	n
1	·822	·919	·016	·112	·208	·304	·399	·493	·587	·681	95	2	1
2	1·775	·868	·961	·053	·145	·237	·328	·419	·509	·599	91	2	2
3	2·689	·779	·868	·957	·045	·133	·221	·308	·395	·482	88	3	3
4	3·568	·654	·740	·826	·911	·996	·081	·165	·249	·333	85	4	4
5	4·416	·499	·582	·665	·747	·829	·910	·991	·072	·153	82	5	5
6	5·233	·313	·393	·473	·552	·631	·710	·788	·866	·944	79	7	6
7	6·022	·099	·176	·253	·330	·406	·482	·558	·633	·708	76	9	7
8	6·783	·858	·932	·006	·080	·154	·228	·301	·374	·447	74	11	8
9	7·519	·591	·663	·735	·806	·877	·948	·019	·089	·159	71	13	9
10	8·229	·299	·369	·438	·507	·575	·643	·711	·779	·847	69	15	10
11	8·915	·983	·050	·117	·184	·251	·317	·383	·449	·515	66	17	11
12	9·580	·645	·710	·775	·840	·905	·969	·033	·097	·161	64	19	12
13	10·224	·287	·350	·413	·476	·539	·601	·663	·725	·787	62	21	13
14	10·848	·909	·970	·031	·092	·153	·213	·273	·333	·393	60	24	14
15	11·452	·511	·570	·629	·688	·747	·806	·864	·922	·980	59	25	15
16	12·038	·096	·154	·211	·268	·325	·382	·439	·495	·551	57	26	16
17	12·607	·663	·719	·775	·830	·885	·940	·995	·050	·105	55	28	17
18	13·159	·213	·267	·321	·375	·429	·482	·535	·588	·641	54	30	18
19	13·694	·747	·800	·853	·906	·958	·010	·062	·113	·164	52	32	19
20	14·215	·266	·317	·368	·419	·470	·520	·570	·620	·670	50	34	20
21	14·720	·770	·820	·869	·918	·967	·016	·065	·114	·163	49	35	21
22	15·211	·259	·307	·355	·403	·451	·499	·547	·594	·641	48	37	22
23	15·688	·735	·782	·828	·874	·920	·966	·012	·058	·104	46	38	23
24	16·150	·196	·241	·286	·331	·376	·421	·466	·511	·555	45	39	24
25	16·599	·643	·687	·731	·775	·819	·863	·906	·949	·992	44	39	25
26	17·035	·078	·121	·164	·207	·249	·291	·333	·375	·417	42	40	26
27	17·459	·501	·543	·584	·625	·666	·707	·748	·789	·830	41	40	27
28	17·870	·910	·950	·990	·030	·070	·110	·150	·190	·230	40	40	28
29	18·269	·308	·347	·386	·425	·464	·503	·542	·580	·618	39	40	29
30	18·656	·694	·732	·770	·808	·846	·883	·920	·957	·994	38	40	30
31	...	...	...	...	...	...	...	...	...	...	...	...	31
32	...	...	...	...	...	...	...	...	...	...	...	...	32
33	...	...	...	...	...	...	...	...	...	...	...	...	33
34	...	...	...	...	...	...	...	...	...	...	...	...	34
35	...	...	...	...	...	...	...	...	...	...	...	...	35
36	...	...	...	...	...	...	...	...	...	...	...	...	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M — *Maturity Age* **47.** — **3 per-cent.**

n	$\bar{a}'_{47-n+1:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{47-n+1:n} - \bar{a}'_{48-n+1:n}$	n
1	·820	·917	·014	·110	·206	·302	·397	·491	·585	·679	95	3	1
2	1·773	·866	·959	·051	·143	·235	·326	·416	·506	·596	91	3	2
3	2·686	·776	·865	·954	·042	·130	·218	·305	·392	·478	88	3	3
4	3·564	·650	·736	·822	·907	·992	·076	·160	·244	·328	85	4	4
5	4·411	·494	·577	·659	·741	·823	·904	·985	·066	·146	82	6	5
6	5·226	·306	·386	·465	·544	·623	·701	·779	·857	·935	79	7	6
7	6·013	·090	·167	·244	·320	·396	·472	·547	·622	·697	76	9	7
8	6·772	·847	·921	·995	·069	·143	·216	·289	·362	·434	73	11	8
9	7·506	·578	·650	·721	·792	·863	·934	·004	·074	·144	71	14	9
10	8·214	·284	·353	·422	·491	·559	·627	·695	·763	·831	68	16	10
11	8·898	·965	·032	·099	·166	·232	·298	·364	·430	·496	66	18	11
12	9·561	·626	·691	·756	·821	·885	·949	·013	·077	·140	64	20	12
13	10·203	·266	·329	·392	·455	·517	·579	·641	·702	·763	62	22	13
14	10·824	·885	·946	·007	·068	·128	·188	·248	·308	·368	60	24	14
15	11·427	·486	·545	·604	·663	·722	·780	·838	·896	·954	58	26	15
16	12·012	·070	·128	·185	·242	·299	·356	·412	·468	·524	57	29	16
17	12·579	·635	·690	·745	·800	·855	·910	·965	·020	·075	55	31	17
18	13·129	·183	·237	·291	·344	·397	·450	·503	·556	·609	53	33	18
19	13·662	·715	·768	·820	·872	·924	·976	·028	·079	·130	52	34	19
20	14·181	·232	·283	·334	·385	·435	·485	·535	·585	·635	50	36	20
21	14·685	·735	·784	·833	·882	·931	·980	·029	·078	·126	49	37	21
22	15·174	·222	·270	·318	·366	·414	·462	·509	·556	·603	48	39	22
23	15·650	·697	·743	·789	·835	·881	·927	·973	·019	·065	46	40	23
24	16·111	·157	·202	·247	·292	·337	·382	·427	·472	·516	45	40	24
25	16·560	·604	·648	·692	·736	·780	·823	·866	·909	·952	44	41	25
26	16·995	·038	·081	·124	·167	·209	·251	·293	·335	·377	42	42	26
27	17·419	·461	·503	·544	·585	·626	·667	·708	·749	·790	41	44	27
28	17·830	·870	·910	·950	·990	·030	·070	·110	·150	·190	40	44	28
29	18·229	·268	·307	·346	·385	·424	·463	·502	·540	·578	39	43	29
30	18·616	·654	·692	·730	·768	·806	·844	·881	·918	·955	38	43	30
31	18·992	·029	·066	·103	·140	·177	·213	·249	·285	·321	37	43	31
32	...	...	...	...	...	...	...	...	...	...	...	...	32
33	...	...	...	...	...	...	...	...	...	...	...	...	33
34	...	...	...	...	...	...	...	...	...	...	...	...	34
35	...	...	...	...	...	...	...	...	...	...	...	...	35
36	...	...	...	...	...	...	...	...	...	...	...	...	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O[M] *Maturity Age* **48.** **3 per-cent.**

n	$\bar{a}'_{48-n+1:\overline{n}}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{48-n+1:\overline{n}} - \bar{a}'_{49-n+1:\overline{n}}$	n
1	·817	·914	·011	·107	·203	·299	·394	·488	·582	·676	95	2	1
2	1·770	·863	·956	·048	·140	·232	·323	·413	·503	·593	91	3	2
3	2·683	·772	·861	·950	·038	·126	·214	·301	·388	·474	88	4	3
4	3·560	·646	·732	·817	·902	·987	·071	·155	·239	·322	84	5	4
5	4·405	·488	·571	·653	·735	·817	·898	·979	·059	·139	81	7	5
6	5·219	·299	·379	·458	·537	·616	·694	·772	·850	·927	78	8	6
7	6·004	·081	·158	·235	·311	·387	·463	·538	·613	·687	76	10	7
8	6·761	·835	·909	·983	·056	·129	·202	·275	·348	·420	73	12	8
9	7·492	·563	·634	·705	·776	·847	·918	·988	·058	·128	71	14	9
10	8·198	·268	·337	·406	·475	·543	·611	·679	·746	·813	68	16	10
11	8·880	·947	·014	·081	·147	·213	·279	·345	·411	·476	66	18	11
12	9·541	·606	·671	·736	·800	·864	·928	·992	·055	·118	64	21	12
13	10·181	·244	·307	·370	·432	·494	·556	·617	·678	·739	62	23	13
14	10·800	·861	·922	·982	·042	·102	·162	·222	·282	·342	60	25	14
15	11·401	·460	·519	·578	·636	·694	·752	·810	·868	·926	58	27	15
16	11·983	·040	·097	·154	·211	·268	·324	·380	·436	·492	56	29	16
17	12·548	·604	·659	·714	·769	·824	·879	·934	·988	·042	55	32	17
18	13·096	·150	·204	·258	·312	·365	·418	·471	·524	·576	53	34	18
19	13·628	·680	·732	·784	·836	·888	·940	·992	·043	·094	52	35	19
20	14·145	·196	·247	·298	·349	·399	·449	·499	·549	·599	50	37	20
21	14·648	·697	·746	·795	·844	·893	·942	·991	·039	·087	49	39	21
22	15·135	·183	·231	·279	·327	·375	·422	·469	·516	·563	48	40	22
23	15·610	·657	·704	·750	·796	·842	·888	·934	·980	·026	46	42	23
24	16·071	·116	·161	·206	·251	·296	·341	·386	·431	·475	45	43	24
25	16·519	·563	·607	·651	·695	·738	·781	·824	·867	·910	43	44	25
26	16·953	·996	·039	·082	·124	·166	·208	·250	·292	·334	42	44	26
27	17·375	·417	·459	·500	·541	·582	·623	·664	·705	·746	41	45	27
28	17·786	·826	·866	·906	·946	·986	·026	·066	·106	·146	40	46	28
29	18·186	·225	·264	·303	·342	·381	·420	·459	·497	·535	39	47	29
30	18·573	·611	·649	·687	·725	·763	·801	·838	·875	·912	38	47	30
31	18·949	·986	·023	·060	·097	·134	·170	·206	·242	·278	36	47	31
32	19·314	·350	·386	·422	·458	·493	·528	·563	·598	·633	35	46	32
33	...	...	...	...	...	...	...	...	...	...	...	...	33
34	...	...	...	...	...	...	...	...	...	...	...	...	34
35	...	...	...	...	...	...	...	...	...	...	...	...	35
36	...	...	...	...	...	...	...	...	...	...	...	...	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **49.** **3 per-cent.**

n	$\bar{a}'_{49-n+1:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{49-n+1:n} - \bar{a}'_{50-n+1:n}$	n
1	·815	·911	·007	·103	·199	·294	·389	·484	·579	·673	95	2	1
2	1·767	·860	·953	·045	·136	·227	·318	·409	·499	·589	91	3	2
3	2·679	·769	·858	·946	·034	·122	·209	·296	·383	·469	88	4	3
4	3·555	·641	·727	·812	·897	·982	·066	·150	·233	·316	84	5	4
5	4·398	·480	·562	·644	·726	·807	·888	·969	·050	·131	81	7	5
6	5·211	·290	·369	·448	·527	·605	·683	·761	·839	·917	78	9	6
7	5·994	·071	·148	·224	·300	·376	·451	·526	·601	·675	76	11	7
8	6·749	·823	·897	·970	·043	·116	·189	·262	·334	·406	73	13	8
9	7·478	·549	·620	·691	·762	·832	·902	·972	·042	·112	70	15	9
10	8·182	·251	·320	·389	·457	·525	·593	·661	·728	·795	68	17	10
11	8·862	·929	·996	·063	·129	·195	·261	·326	·391	·456	66	19	11
12	9·520	·585	·650	·714	·778	·842	·906	·969	·032	·095	64	22	12
13	10·158	·221	·284	·346	·408	·470	·531	·592	·653	·714	62	25	13
14	10·775	·836	·897	·957	·017	·077	·137	·197	·256	·315	60	27	14
15	11·374	·433	·492	·551	·609	·667	·725	·783	·840	·897	58	29	15
16	11·954	·011	·068	·125	·182	·238	·294	·350	·406	·461	56	31	16
17	12·516	·571	·626	·681	·736	·791	·846	·900	·954	·008	55	33	17
18	13·062	·116	·170	·224	·277	·330	·383	·436	·489	·541	53	35	18
19	13·593	·645	·697	·749	·801	·853	·904	·955	·006	·057	52	37	19
20	14·108	·159	·210	·261	·311	·361	·411	·461	·511	·560	50	39	20
21	14·609	·658	·707	·756	·805	·854	·903	·951	·999	·047	48	41	21
22	15·095	·143	·191	·239	·286	·333	·380	·427	·474	·521	47	43	22
23	15·568	·615	·662	·709	·755	·801	·847	·893	·938	·983	46	44	23
24	16·028	·073	·118	·163	·208	·253	·298	·343	·387	·431	44	45	24
25	16·475	·519	·563	·607	·651	·695	·738	·781	·824	·867	43	47	25
26	16·909	·952	·995	·038	·080	·122	·164	·206	·248	·289	42	48	26
27	17·330	·371	·412	·453	·494	·535	·576	·617	·658	·699	41	48	27
28	17·740	·780	·820	·860	·900	·940	·980	·020	·060	·100	40	49	28
29	18·139	·178	·217	·256	·295	·334	·373	·412	·450	·488	39	49	29
30	18·526	·564	·602	·640	·678	·716	·754	·791	·828	·865	38	49	30
31	18·902	·939	·976	·013	·050	·087	·124	·160	·196	·232	37	49	31
32	19·268	·304	·340	·376	·412	·447	·482	·517	·552	·587	35	49	32
33	19·622	·657	·692	·727	·762	·797	·831	·865	·899	·933	34	48	33
34	...	...	...	...	...	...	...	...	...	...	...	...	34
35	...	...	...	...	...	...	...	...	...	...	...	...	35
36	...	...	...	...	...	...	...	...	...	...	...	...	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **50.** **3 per-cent.**

n	$\bar{a}'_{50-n+1:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{50-n+1:n} - \bar{a}'_{51-n+1:n}$	n
1	·813	·909	**·005**	**·101**	**·197**	**·292**	**·387**	**·482**	**·577**	**·671**	95	4	1
2	**1**·764	·857	·949	**·041**	**·132**	**·223**	**·314**	**·405**	**·495**	**·585**	91	4	2
3	**2**·675	·764	·853	·941	**·029**	**·117**	**·204**	**·291**	**·378**	**·464**	88	4	3
4	**3**·550	·636	·721	·806	·891	·975	**·059**	**·143**	**·226**	**·309**	84	5	4
5	**4**·391	·473	·555	·637	·718	·799	·880	·961	**·042**	**·122**	81	6	5
6	**5**·202	·281	·360	·439	·517	·595	·673	·751	·829	·906	78	9	6
7	**5**·983	**·060**	**·136**	**·212**	**·288**	**·363**	**·438**	**·513**	**·588**	**·662**	75	11	7
8	**6**·736	·810	·883	·956	**·029**	**·102**	**·175**	**·247**	**·319**	**·391**	73	14	8
9	**7**·463	·534	·605	·676	·746	·816	·886	·956	**·026**	**·096**	70	16	9
10	**8**·165	·234	·303	·371	·439	·507	·575	·642	·709	·776	68	19	10
11	**8**·843	·910	·976	**·042**	**·108**	**·174**	**·239**	**·304**	**·369**	**·434**	66	21	11
12	**9**·498	·563	·627	·691	·755	·819	·882	·945	**·008**	**·071**	64	23	12
13	**10**·133	·196	·258	·320	·382	·444	·505	·566	·627	·688	62	26	13
14	**10**·748	·809	·869	·929	·989	**·049**	**·109**	**·168**	**·227**	**·286**	60	28	14
15	**11**·345	·404	·462	·520	·578	·636	·694	·752	·809	·866	58	31	15
16	**11**·923	·980	**·037**	**·093**	**·149**	**·205**	**·261**	**·317**	**·373**	**·428**	56	33	16
17	**12**·483	·538	·593	·648	·703	·757	·811	·865	·919	·973	54	34	17
18	**13**·027	·081	·135	·188	·241	·294	·347	·400	·452	·504	53	37	18
19	**13**·556	·608	·660	·712	·763	·814	·865	·916	·967	**·018**	51	40	19
20	**14**·069	·120	·171	·221	·271	·321	·371	·421	·470	·519	50	41	20
21	**14**·568	·617	·666	·715	·764	·812	·860	·908	·956	**·004**	48	43	21
22	**15**·052	·100	·148	·195	·242	·289	·336	·383	·430	·477	47	45	22
23	**15**·524	·571	·618	·664	·710	·756	·802	·848	·893	·938	46	47	23
24	**15**·983	**·028**	**·073**	**·118**	**·163**	**·208**	**·252**	**·296**	**·340**	**·384**	44	48	24
25	**16**·428	·472	·516	·560	·604	·647	·690	·733	·776	·819	43	49	25
26	**16**·861	·904	·947	·990	**·032**	**·074**	**·116**	**·158**	**·200**	**·241**	42	51	26
27	**17**·282	·323	·364	·405	·446	·487	·528	·569	·610	·651	41	52	27
28	**17**·691	·731	·771	·811	·851	·891	·931	·971	**·011**	**·051**	40	52	28
29	**18**·090	·129	·168	·207	·246	·285	·324	·363	·401	·439	39	52	29
30	**18**·477	·515	·553	·591	·629	·667	·705	·742	·779	·816	38	52	30
31	**18**·853	·890	·927	·964	**·001**	**·038**	**·075**	**·111**	**·147**	**·183**	37	52	31
32	**19**·219	·255	·291	·327	·363	·399	·434	·469	·504	·539	36	52	32
33	**19**·574	·609	·644	·679	·714	·749	·783	·817	·851	·885	35	52	33
34	**19**·919	·953	·987	**·021**	**·055**	**·089**	**·122**	**·155**	**·188**	**·221**	34	52	34
35	...	...	...	...	...	...	...	...	...	...	...	...	35
36	...	...	...	...	...	...	...	...	...	...	...	...	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

0[M] *Maturity Age* **51.** **3 per-cent.**

n	$\bar{a}'_{51-n+1:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{51-n+1:n} - \bar{a}'_{52-n+1:n}$	n
1	·809	·905	·001	·097	·193	·288	·383	·478	·573	·667	95	4	1
2	1·760	·853	·945	·037	·128	·219	·310	·401	·491	·581	91	4	2
3	2·671	·760	·849	·937	·025	·113	·200	·287	·373	·459	87	5	3
4	3·545	·631	·716	·801	·886	·970	·054	·137	·220	·303	84	6	4
5	4·385	·467	·549	·630	·711	·792	·873	·953	·033	·113	81	8	5
6	5·193	·272	·351	·429	·507	·585	·663	·741	·818	·895	78	10	6
7	5·972	·048	·124	·200	·275	·350	·425	·500	·574	·648	75	13	7
8	6·722	·795	·868	·941	·014	·087	·159	·231	·303	·375	72	15	8
9	7·447	·518	·589	·659	·729	·799	·869	·939	·008	·077	70	18	9
10	8·146	·215	·283	·351	·419	·487	·554	·621	·688	·755	68	20	10
11	9·822	·888	·954	·020	·085	·150	·215	·280	·345	·410	65	23	11
12	9·475	·539	·603	·667	·731	·794	·857	·920	·983	·045	63	25	12
13	10·107	·169	·231	·293	·355	·416	·477	·538	·599	·660	61	27	13
14	10·720	·780	·840	·900	·960	·019	·078	·137	·196	·255	59	30	14
15	11·314	·372	·430	·488	·546	·604	·662	·719	·776	·833	58	32	15
16	11·890	·947	·003	·059	·115	·171	·227	·283	·339	·394	56	35	16
17	12·449	·504	·559	·613	·667	·721	·775	·829	·883	·937	54	37	17
18	12·990	·043	·096	·149	·202	·255	·308	·360	·412	·464	53	39	18
19	13·516	·568	·620	·671	·722	·773	·824	·875	·926	·977	51	41	19
20	14·028	·078	·128	·178	·228	·278	·328	·378	·427	·476	50	44	20
21	14·525	·574	·623	·671	·719	·767	·815	·863	·911	·959	48	46	21
22	15·007	·055	·102	·149	·196	·243	·290	·337	·384	·431	47	47	22
23	15·477	·524	·570	·616	·662	·708	·754	·800	·845	·890	46	49	23
24	15·935	·980	·025	·070	·115	·159	·203	·247	·291	·335	44	51	24
25	16·379	·423	·467	·510	·553	·596	·639	·682	·725	·768	43	52	25
26	16·810	·853	·896	·938	·980	·022	·064	·106	·148	·189	42	53	26
27	17·230	·271	·312	·353	·394	·435	·476	·517	·558	·599	41	54	27
28	17·639	·679	·719	·759	·799	·839	·879	·919	·959	·999	40	55	28
29	18·038	·077	·116	·155	·194	·233	·272	·311	·349	·387	39	56	29
30	18·425	·463	·501	·539	·577	·615	·653	·690	·727	·764	38	56	30
31	18·801	·838	·875	·912	·949	·986	·023	·059	·095	·131	37	56	31
32	19·167	·203	·239	·275	·311	·347	·382	·417	·452	·487	36	56	32
33	19·522	·557	·592	·627	·662	·697	·731	·765	·799	·833	35	56	33
34	19·867	·901	·935	·969	·003	·037	·070	·103	·136	·169	34	55	34
35	20·202	·235	·268	·301	·334	·367	·399	·431	·463	·495	33	55	35
36	...	...	...	...	...	...	...	...	...	...	...	...	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

OM — *Maturity Age* **52.** — **3 per-cent.**

n	$\bar{a}'_{52-n+1:\overline{n}\vert}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff	$\bar{a}'_{52-n+1:\overline{n}\vert} - \bar{a}'_{53-n+1:\overline{n}\vert}$	n
1	·805	·901	·997	·093	·189	·284	·379	·474	·569	·663	95	4	1
2	1·756	·849	·941	·033	·124	·215	·306	·396	·486	·576	91	4	2
3	2·666	·755	·844	·932	·020	·108	·195	·282	·368	·454	87	5	3
4	3·539	·624	·709	·794	·878	·962	·046	·129	·212	·295	84	7	4
5	4·377	·459	·540	·621	·702	·783	·863	·943	·023	·103	81	9	5
6	5·183	·262	·340	·418	·496	·574	·651	·728	·805	·882	78	10	6
7	5·959	·035	·111	·186	·261	·336	·411	·485	·559	·633	75	12	7
8	6·707	·780	·853	·926	·998	·070	·142	·214	·286	·358	72	15	8
9	7·429	·500	·570	·640	·710	·780	·850	·919	·988	·057	70	18	9
10	8·126	·194	·262	·330	·398	·465	·532	·599	·666	·733	67	21	10
11	8·799	·865	·931	·996	·061	·126	·191	·256	·321	·386	65	23	11
12	9·450	·514	·577	·640	·703	·766	·829	·892	·955	·018	63	26	12
13	10·080	·142	·204	·266	·327	·388	·449	·510	·570	·630	61	29	13
14	10·690	·750	·810	·869	·928	·987	·046	·105	·164	·223	59	31	14
15	11·282	·340	·398	·456	·513	·570	·627	·684	·741	·798	57	33	15
16	11·855	·912	·968	·024	·080	·136	·192	·247	·302	·357	56	35	16
17	12·412	·467	·522	·576	·630	·684	·738	·792	·845	·898	54	39	17
18	12·951	·004	·057	·110	·163	·215	·267	·319	·371	·423	52	41	18
19	13·475	·527	·578	·629	·680	·731	·782	·833	·884	·934	51	43	19
20	13·984	·034	·084	·134	·184	·234	·283	·332	·381	·430	50	46	20
21	14·479	·528	·576	·624	·672	·720	·768	·816	·864	·912	48	48	21
22	14·960	·007	·054	·101	·148	·195	·242	·289	·336	·382	47	50	22
23	15·428	·474	·520	·566	·612	·658	·704	·749	·794	·839	46	51	23
24	15·884	·929	·974	·019	·063	·107	·151	·195	·239	·283	44	53	24
25	16·327	·371	·414	·457	·500	·543	·586	·629	·672	·715	43	55	25
26	16·757	·800	·842	·884	·926	·968	·010	·052	·094	·135	42	56	26
27	17·176	·217	·258	·299	·340	·381	·422	·463	·504	·544	41	57	27
28	17·584	·624	·664	·704	·744	·784	·824	·864	·904	·943	40	58	28
29	17·982	·021	·060	·099	·138	·177	·216	·255	·293	·331	39	58	29
30	18·369	·407	·445	·483	·521	·559	·597	·634	·671	·708	38	59	30
31	18·745	·782	·819	·856	·893	·930	·967	·003	·039	·075	37	60	31
32	19·111	·147	·183	·219	·255	·291	·326	·361	·396	·431	36	60	32
33	19·466	·501	·536	·571	·606	·641	·676	·710	·744	·778	35	60	33
34	19·812	·846	·880	·914	·948	·982	·015	·048	·081	·114	34	60	34
35	20·147	·180	·213	·246	·279	·312	·345	·378	·410	·442	33	60	35
36	20·474	·506	·538	·570	·602	·634	·666	·698	·730	·762	32	60	36
37	...	...	...	...	...	...	...	...	...	...	...	...	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

0ᴹ *Maturity Age* **53.** **3 per-cent.**

n	$\bar{a}'_{53-n+1:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{53-n+1:n} - \bar{a}'_{54-n+1:n}$	n
1	·801	·897	·993	**·089**	**·185**	**·280**	**·375**	**·470**	**·565**	**·659**	95	3	1
2	**1**·752	·845	·937	**·029**	**·120**	**·211**	**·302**	**·392**	**·482**	**·572**	91	4	2
3	**2**·661	·750	·839	·927	**·014**	**·101**	**·188**	**·275**	**·361**	**·447**	87	6	3
4	**3**·532	·617	·702	·787	·871	·955	**·038**	**·121**	**·204**	**·286**	84	7	4
5	**4**·368	·450	·532	·613	·694	·775	·855	·935	**·015**	**·094**	80	9	5
6	**5**·173	·252	·331	·409	·487	·565	·642	·719	·795	·871	77	12	6
7	**5**·9[illegible]7	**·023**	**·099**	**·175**	**·250**	**·325**	**·399**	**·473**	**·546**	**·619**	74	15	7
8	**6**.692	·765	·838	·911	·983	**·055**	**·127**	**·198**	**·269**	**·340**	72	17	8
9	**7**·411	·481	·551	·621	·691	·761	·830	·899	·968	**·037**	69	20	9
10	**8**·105	·173	·241	·309	·376	·443	·510	·577	·644	·710	67	23	10
11	**8**·776	·842	·908	·973	**·038**	**·103**	**·168**	**·232**	**·296**	**·360**	65	25	11
12	**9**·424	·488	·552	·615	·678	·741	·804	·866	·928	·990	63	28	12
13	**10**·051	·113	·174	·235	·296	·357	·418	·479	·539	·599	61	30	13
14	**10**·659	·719	·779	·839	·898	·957	**·016**	**·075**	**·133**	**·191**	59	32	14
15	**11**·249	·307	·365	·423	·480	·537	·594	·651	·708	·764	57	36	15
16	**11**·820	·876	·932	·988	**·044**	**·099**	**·154**	**·209**	**·264**	**·319**	55	39	16
17	**12**·373	·427	·481	·535	·589	·643	·697	·751	·804	·857	54	41	17
18	**12**·910	·963	**·016**	**·069**	**·121**	**·173**	**·225**	**·277**	**·329**	**·381**	52	44	18
19	**13**·432	·483	·534	·585	·636	·687	·738	·788	·838	·888	51	46	19
20	**13**·938	·988	**·038**	**·088**	**·138**	**·187**	**·236**	**·285**	**·334**	**·383**	49	48	20
21	**14**·431	·480	·528	·576	·624	·672	·720	·768	·816	·863	48	50	21
22	**14**·910	·957	**·004**	**·051**	**·098**	**·145**	**·192**	**·239**	**·285**	**·331**	47	51	22
23	**15**·377	·423	·469	·515	·561	·606	·651	·696	·741	·786	45	53	23
24	**15**·831	·876	·920	·964	**·008**	**·052**	**·096**	**·140**	**·184**	**·228**	44	55	24
25	**16**·272	·315	·358	·401	·444	·487	·530	·573	·616	·659	43	57	25
26	**16**·701	·743	·785	·827	·869	·911	·953	·995	**·037**	**·078**	42	58	26
27	**17**·119	·160	·201	·242	·283	·324	·365	·406	·446	·486	41	59	27
28	**17**·526	·566	·606	·646	·686	·726	·766	·806	·846	·885	40	60	28
29	**17**·924	·963	**·002**	**·041**	**·080**	**·119**	**·158**	**·196**	**·234**	**·272**	39	62	29
30	**18**·310	·348	·386	·424	·462	·500	·537	·574	·611	·648	38	63	30
31	**18**·685	·722	·759	·796	·833	·870	·907	·943	·979	**·015**	37	62	31
32	**19**·051	·087	·123	·159	·195	·231	·266	·301	·336	·371	36	63	32
33	**19**·406	·441	·476	·511	·546	·581	·616	·650	·684	·718	35	63	33
34	**19**·752	·786	·820	·854	·888	·922	·955	·988	**·021**	**·054**	34	63	34
35	**20**·087	·120	·153	·186	·219	·252	·285	·318	·350	·382	33	62	35
36	**20**·414	·446	·478	·510	·542	·574	·606	·638	·670	·701	32	62	36
37	**20**·732	·763	·794	·825	·856	·887	·918	·949	·980	**·011**	31	63	37
38	...	...	...	...	...	...	...	...	...	...	...	...	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **54.** **3 per-cent.**

n	$\bar{a}'_{54-n+1:\overline{n}}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{54-n+1:\overline{n}} - \bar{a}'_{55-n+1:\overline{n}}$	n
1	·798	·895	·991	·087	·183	·278	·373	·467	·561	·655	95	4	1
2	1·748	·841	·933	·025	·116	·207	·297	·387	·477	·566	91	5	2
3	2·655	·744	·832	·920	·007	·094	·181	·268	·354	·440	87	6	3
4	3·525	·610	·695	·779	·863	·946	·029	·112	·195	·277	83	8	4
5	4·359	·441	·522	·603	·684	·764	·844	·924	·003	·082	80	10	5
6	5·161	·239	·317	·395	·473	·551	·628	·704	·780	·856	77	13	6
7	5·932	·008	·083	·158	·233	·308	·382	·456	·529	·602	74	15	7
8	6·675	·748	·820	·892	·964	·036	·108	·179	·250	·321	72	19	8
9	7·391	·461	·531	·601	·671	·740	·809	·878	·946	·014	69	21	9
10	8·082	·150	·218	·285	·352	·419	·486	·553	·619	·685	67	23	10
11	8·751	·817	·882	·947	·012	·076	·140	·204	·268	·332	64	27	11
12	9·396	·460	·523	·586	·649	·712	·774	·836	·898	·960	62	30	12
13	10·021	·083	·144	·205	·266	·327	·387	·447	·507	·567	61	33	13
14	10·627	·687	·746	·805	·864	·923	·981	·039	·097	·155	59	36	14
15	11·213	·271	·328	·385	·442	·499	·556	·613	·669	·725	57	38	15
16	11·781	·837	·893	·949	·004	·059	·114	·169	·224	·278	55	41	16
17	12·332	·386	·440	·494	·548	·602	·655	·708	·761	·814	53	44	17
18	12·866	·919	·972	·024	·076	·128	·180	·232	·284	·335	52	46	18
19	13·386	·437	·488	·539	·590	·640	·690	·740	·790	·840	50	48	19
20	13·890	·940	·990	·040	·089	·138	·187	·236	·285	·333	49	49	20
21	14·381	·430	·478	·526	·574	·622	·670	·718	·765	·812	48	52	21
22	14·859	·906	·953	·000	·047	·094	·140	·186	·232	·278	46	54	22
23	15·324	·370	·416	·462	·507	·552	·597	·642	·687	·732	45	57	23
24	15·776	·820	·864	·908	·952	·996	·040	·084	·128	·172	44	59	24
25	16·215	·258	·301	·344	·387	·430	·473	·516	·559	·601	43	60	25
26	16·643	·685	·727	·769	·811	·853	·895	·937	·978	·019	42	62	26
27	17·060	·101	·142	·183	·224	·265	·306	·346	·386	·426	41	63	27
28	17·466	·506	·546	·586	·626	·666	·706	·745	·784	·823	40	64	28
29	17·862	·901	·940	·979	·018	·057	·095	·133	·171	·209	39	66	29
30	18·247	·285	·323	·361	·399	·437	·475	·512	·549	·586	38	66	30
31	18·623	·660	·697	·734	·771	·808	·844	·880	·916	·952	37	68	31
32	18·988	·024	·060	·096	·132	·168	·203	·238	·273	·308	36	68	32
33	19·343	·378	·413	·448	·483	·518	·553	·587	·621	·655	35	68	33
34	19·689	·723	·757	·791	·825	·859	·893	·926	·959	·992	34	69	34
35	20·025	·058	·091	·124	·157	·190	·223	·256	·288	·320	33	68	35
36	20·352	·384	·416	·448	·480	·512	·544	·576	·607	·638	32	67	36
37	20·669	·700	·731	·762	·793	·824	·855	·886	·917	·948	31	66	37
38	20·978	·008	·038	·068	·098	·128	·158	·188	·218	·248	30	66	38
39	...	...	...	...	...	...	...	...	...	...	...	...	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

0^{M}

Maturity Age **55.**

3 per-cent.

n	$\bar{a}'_{55-n+1:\overline{n}}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{55-n+1:\overline{n}} - \bar{a}'_{56-n+1:\overline{n}}$	n
1	·794	·890	·986	·082	·178	·273	·368	·462	·556	·650	95	5	1
2	1·743	·835	·927	·019	·110	·201	·291	·381	·471	·560	91	5	2
3	2·649	·737	·825	·913	·000	·087	·174	·260	·346	·432	87	6	3
4	3·517	·602	·686	·770	·854	·937	·020	·103	·185	·267	83	8	4
5	4·349	·430	·511	·592	·672	·752	·832	·911	·990	·069	80	10	5
6	5·148	·226	·304	·382	·459	·536	·613	·689	·765	·841	77	13	6
7	5·917	·992	·067	·142	·216	·290	·364	·437	·510	·583	74	17	7
8	6·656	·728	·800	·872	·944	·016	·087	·158	·229	·300	71	20	8
9	7·370	·440	·510	·580	·649	·718	·787	·855	·923	·991	69	23	9
10	8·059	·126	·193	·260	·327	·394	·460	·526	·592	·658	66	26	10
11	8·724	·789	·854	·919	·984	·048	·112	·176	·240	·303	64	29	11
12	9·366	·429	·492	·555	·618	·680	·742	·804	·866	·927	62	32	12
13	9·988	·049	·110	·171	·232	·292	·352	·412	·472	·532	60	35	13
14	10·591	·650	·709	·768	·827	·885	·943	·001	·059	·117	58	38	14
15	11·175	·232	·289	·346	·403	·460	·516	·572	·628	·684	56	41	15
16	11·740	·795	·850	·905	·960	·015	·070	·125	·180	·234	55	43	16
17	12·288	·342	·396	·449	·502	·555	·608	·661	·714	·767	53	46	17
18	12·820	·872	·924	·976	·028	·080	·132	·184	·236	·287	52	48	18
19	13·338	·389	·440	·491	·541	·591	·641	·691	·741	·791	50	51	19
20	13·841	·890	·939	·988	·037	·086	·135	·184	·233	·281	49	54	20
21	14·329	·377	·425	·473	·521	·569	·617	·664	·711	·758	48	56	21
22	14·805	·852	·899	·945	·991	·037	·083	·129	·175	·221	46	58	22
23	15·267	·312	·357	·402	·447	·492	·537	·582	·627	·672	45	60	23
24	15·717	·761	·805	·849	·893	·937	·981	·025	·069	·112	44	62	24
25	16·155	·198	·241	·284	·327	·370	·413	·455	·497	·539	43	64	25
26	16·581	·623	·665	·707	·749	·791	·833	·874	·915	·956	42	65	26
27	16·997	·038	·079	·120	·161	·202	·242	·282	·322	·362	40	67	27
28	17·402	·442	·482	·522	·562	·601	·640	·679	·718	·757	39	69	28
29	17·796	·835	·874	·913	·952	·991	·029	·067	·105	·143	38	69	29
30	18·181	·219	·257	·295	·333	·370	·407	·444	·481	·518	37	70	30
31	18·555	·592	·629	·666	·703	·740	·776	·812	·848	·884	36	70	31
32	18·920	·956	·992	·028	·064	·100	·135	·170	·205	·240	35	70	32
33	19·275	·310	·345	·380	·415	·450	·484	·518	·552	·586	34	71	33
34	19·620	·654	·688	·722	·756	·790	·824	·858	·891	·924	34	70	34
35	19·957	·990	·023	·056	·089	·122	·155	·188	·221	·253	33	71	35
36	20·285	·317	·349	·381	·413	·445	·477	·509	·541	·572	32	71	36
37	20·603	·634	·665	·696	·727	·758	·789	·820	·851	·882	31	70	37
38	20·912	·943	·974	·004	·034	·064	·094	·124	·154	·184	30	69	38
39	21·214	·244	·274	·304	·334	·363	·392	·421	·450	·479	29	69	39
40	...	...	...	...	...	...	...	...	...	...	...	...	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **56.** **3 per-cent.**

$\bar{a}'_{56-\overline{n+1}:n}$

n	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{56-\overline{n+1}:n} - \bar{a}'_{57-\overline{n+1}:n}$	n
1	·789	·885	·981	·077	·173	·268	·363	·457	·551	·645	95	5	1
2	1·738	·830	·922	·014	·105	·196	·286	·376	·465	·554	91	6	2
3	2·643	·731	·819	·907	·994	·081	·167	·253	·339	·424	87	7	3
4	3·509	·593	·677	·761	·844	·927	·010	·093	·175	·257	83	9	4
5	4·339	·420	·501	·582	·662	·742	·821	·900	·979	·057	80	12	5
6	5·135	·213	·291	·368	·445	·522	·598	·674	·750	·825	77	15	6
7	5·900	·975	·050	·124	·198	·272	·345	·418	·491	·564	74	18	7
8	6·636	·708	·780	·852	·923	·994	·065	·136	·207	·277	71	21	8
9	7·347	·417	·487	·556	·625	·694	·762	·830	·898	·966	69	25	9
10	8·033	·100	·167	·234	·301	·367	·433	·499	·565	·630	66	29	10
11	8·695	·760	·825	·889	·953	·017	·081	·145	·208	·271	64	32	11
12	9·334	·397	·460	·522	·584	·646	·708	·770	·831	·892	62	35	12
13	9·953	·014	·075	·135	·195	·255	·315	·375	·435	·494	60	38	13
14	10·553	·612	·671	·729	·787	·845	·903	·961	·019	·077	58	42	14
15	11·134	·191	·248	·305	·362	·418	·474	·530	·586	·642	56	45	15
16	11·697	·752	·807	·862	·917	·972	·026	·080	·134	·188	54	48	16
17	12·242	·296	·349	·402	·455	·508	·561	·614	·667	·720	53	50	17
18	12·772	·824	·876	·928	·980	·032	·083	·134	·185	·236	52	52	18
19	13·287	·338	·388	·438	·488	·538	·588	·638	·688	·738	50	54	19
20	13·787	·836	·885	·934	·983	·032	·081	·129	·177	·225	49	56	20
21	14·273	·321	·369	·417	·465	·512	·559	·606	·653	·700	47	59	21
22	14·747	·793	·839	·885	·931	·977	·023	·069	·115	·161	46	62	22
23	15·207	·252	·297	·342	·387	·432	·477	·522	·567	·611	45	63	23
24	15·655	·699	·743	·787	·831	·875	·919	·962	·005	·048	44	65	24
25	16·091	·134	·177	·220	·263	·306	·348	·390	·432	·474	42	67	25
26	16·516	·558	·600	·642	·684	·725	·766	·807	·848	·889	41	68	26
27	16·930	·971	·012	·053	·093	·133	·173	·213	·253	·293	40	70	27
28	17·333	·373	·413	·453	·493	·532	·571	·610	·649	·688	39	71	28
29	17·727	·766	·805	·844	·883	·921	·959	·997	·035	·073	38	73	29
30	18·111	·149	·187	·225	·263	·300	·337	·374	·411	·448	37	74	30
31	18·485	·522	·559	·596	·633	·670	·706	·742	·778	·814	36	74	31
32	18·850	·886	·922	·958	·994	·029	·064	·099	·134	·169	35	76	32
33	19·204	·239	·274	·309	·344	·379	·414	·448	·482	·516	35	76	33
34	19·550	·584	·618	·652	·686	·720	·754	·787	·820	·853	34	76	34
35	19·886	·919	·952	·985	·018	·051	·084	·117	·150	·182	33	76	35
36	20·214	·246	·278	·310	·342	·374	·406	·438	·470	·502	32	76	36
37	20·533	·564	·595	·626	·657	·688	·719	·750	·781	·812	31	75	37
38	20·843	·874	·905	·935	·965	·995	·025	·055	·085	·115	30	74	38
39	21·145	·175	·205	·235	·265	·294	·323	·352	·381	·410	29	73	39
40	21·439	·468	·497	·526	·555	·584	·613	·641	·669	·697	29	72	40
41	...	...	...	...	...	...	...	...	...	...	...	...	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **57.** **3 per-cent.**

n	$\bar{a}'_{57-\overline{n+1}:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{57-\overline{n+1}:n} - \bar{a}'_{58-\overline{n+1}:n}$	n
1	·784	·881	·978	·074	·169	·264	·359	·453	·546	·639	95	5	1
2	1·732	·824	·916	·008	·099	·190	·280	·370	·459	·548	90	5	2
3	2·636	·724	·812	·900	·987	·074	·160	·246	·331	·416	86	7	3
4	3·500	·584	·668	·752	·835	·918	·001	·083	·165	·246	83	10	4
5	4·327	·408	·488	·568	·648	·727	·806	·885	·964	·042	79	13	5
6	5·120	·198	·275	·352	·429	·505	·581	·657	·732	·807	76	16	6
7	5·882	·957	·031	·105	·179	·253	·326	·399	·471	·543	73	20	7
8	6·615	·687	·758	·829	·900	·971	·042	·112	·182	·252	71	24	8
9	7·322	·392	·461	·530	·599	·667	·735	·803	·870	·937	68	27	9
10	8·004	·071	·138	·204	·270	·336	·402	·468	·533	·598	66	30	10
11	8·663	·727	·791	·855	·919	·983	·047	·110	·173	·236	64	35	11
12	9·299	·362	·424	·486	·548	·610	·671	·732	·793	·854	62	38	12
13	9·915	·975	·035	·095	·155	·215	·275	·334	·393	·452	60	41	13
14	10·511	·569	·627	·685	·743	·801	·859	·917	·975	·032	58	44	14
15	11·089	·146	·202	·258	·314	·370	·426	·482	·538	·594	56	47	15
16	11·649	·704	·759	·814	·868	·922	·976	·030	·084	·138	54	50	16
17	12·192	·245	·298	·351	·404	·457	·510	·563	·616	·668	53	52	17
18	12·720	·772	·824	·876	·927	·978	·029	·080	·131	·182	51	55	18
19	13·233	·283	·333	·383	·433	·483	·533	·583	·633	·682	50	58	19
20	13·731	·780	·829	·878	·926	·974	·022	·070	·118	·166	48	60	20
21	14·214	·262	·309	·356	·403	·450	·497	·544	·591	·638	47	62	21
22	14·685	·731	·777	·823	·869	·915	·961	·007	·053	·099	46	65	22
23	15·144	·189	·234	·279	·324	·369	·414	·458	·502	·546	45	68	23
24	15·590	·634	·678	·722	·766	·809	·852	·895	·938	·981	43	70	24
25	16·024	·067	·110	·153	·196	·238	·280	·322	·364	·406	42	71	25
26	16·448	·490	·532	·573	·614	·655	·696	·737	·778	·819	41	73	26
27	16·860	·901	·942	·982	·022	·062	·102	·142	·182	·222	40	74	27
28	17·262	·302	·342	·381	·420	·459	·498	·537	·576	·615	39	75	28
29	17·654	·693	·732	·771	·809	·847	·885	·923	·961	·999	38	76	29
30	18·037	·075	·113	·151	·189	·226	·263	·300	·337	·374	37	77	30
31	18·411	·448	·485	·522	·558	·594	·630	·666	·702	·738	36	78	31
32	18·774	·810	·846	·882	·918	·953	·988	·023	·058	·093	35	79	32
33	19·128	·163	·198	·233	·268	·303	·338	·372	·406	·440	35	79	33
34	19·474	·508	·542	·576	·610	·644	·678	·711	·744	·777	34	79	34
35	19·810	·843	·876	·909	·942	·975	·008	·041	·074	·106	33	78	35
36	20·138	·170	·202	·234	·266	·298	·330	·362	·394	·426	32	78	36
37	20·458	·490	·521	·552	·583	·614	·645	·676	·707	·738	31	79	37
38	20·769	·800	·831	·862	·892	·922	·952	·982	·012	·042	30	79	38
39	21·072	·102	·132	·162	·192	·222	·251	·280	·309	·338	30	78	39
40	21·367	·396	·425	·454	·483	·512	·540	·568	·596	·624	29	77	40
41	21·652	·680	·708	·736	·764	·792	·820	·847	·874	·901	28	75	41
42	...	...	...	...	...	...	...	...	...	...	...	...	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **58.** **3 per-cent.**

n	$\bar{a}'_{58-\overline{n+1}:\overline{n}\rvert}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{58-\overline{n+1}:\overline{n}\rvert} - \bar{a}'_{59-\overline{n+1}:\overline{n}\rvert}$	n
1	·779	·876	·973	**·069**	**·164**	**·259**	**·354**	**·448**	**·541**	**·634**	95	6	1
2	1·727	·819	·911	**·003**	**·094**	**·184**	**·274**	**·364**	**·453**	**·541**	90	6	2
3	2·629	·717	·804	·891	·978	**·064**	**·150**	**·235**	**·320**	**·405**	86	8	3
4	3·490	·574	·658	·741	·824	·907	·989	**·071**	**·152**	**·233**	82	11	4
5	4·314	·394	·474	·554	·633	·712	·791	·870	·948	**·026**	79	14	5
6	5·104	·181	·258	·335	·411	·487	·563	·638	·713	·788	76	18	6
7	5·862	·936	**·010**	**·084**	**·158**	**·231**	**·303**	**·375**	**·447**	**·519**	73	21	7
8	6·591	·663	·734	·805	·876	·946	**·016**	**·086**	**·156**	**·226**	70	24	8
9	7·295	·364	·433	·502	·570	·638	·706	·773	·840	·907	68	29	9
10	7·974	**·041**	**·107**	**·173**	**·239**	**·304**	**·369**	**·434**	**·499**	**·564**	65	34	10
11	8·628	·692	·756	·820	·884	·947	**·010**	**·073**	**·136**	**·199**	63	37	11
12	9·261	·323	·385	·447	·509	·570	·631	·692	·753	·814	61	40	12
13	9·874	·934	·994	**·054**	**·114**	**·173**	**·232**	**·291**	**·350**	**·409**	59	43	13
14	10·467	·525	·583	·641	·699	·757	·814	·871	·928	·985	58	46	14
15	11·042	·099	·155	·211	·267	·323	·379	·434	·489	·544	56	50	15
16	11·599	·654	·709	·764	·818	·872	·926	·980	**·034**	**·087**	54	53	16
17	12·140	·193	·246	·299	·352	·405	·457	·509	·561	·613	52	56	17
18	12·665	·717	·769	·820	·871	·922	·973	**·024**	**·075**	**·125**	51	59	18
19	13·175	·225	·275	·325	·375	·425	·475	·524	·573	·622	50	62	19
20	13·671	·720	·769	·818	·866	·914	·962	**·010**	**·058**	**·105**	48	64	20
21	14·152	·199	·246	·293	·340	·387	·434	·481	·528	·574	47	66	21
22	14·620	·666	·712	·758	·804	·850	·896	·941	·986	**·031**	46	68	22
23	15·076	·121	·166	·211	·256	·300	·344	·388	·432	·476	44	70	23
24	15·520	·564	·608	·652	·695	·738	·781	·824	·867	·910	43	72	24
25	15·953	·996	**·039**	**·081**	**·123**	**·165**	**·207**	**·249**	**·291**	**·333**	42	74	25
26	16·375	·417	·458	·499	·540	·581	·622	·663	·704	·745	41	76	26
27	16·786	·827	·867	·907	·947	·987	**·027**	**·067**	**·107**	**·147**	40	77	27
28	17·187	·227	·266	·305	·344	·383	·422	·461	·500	·539	39	79	28
29	17·578	·617	·656	·694	·732	·770	·808	·846	·884	·922	38	80	29
30	17·960	·998	**·036**	**·074**	**·111**	**·148**	**·185**	**·222**	**·259**	**·296**	37	82	30
31	18·333	·370	·407	·443	·479	·515	·551	·587	·623	·659	36	83	31
32	18·695	·731	·767	·803	·839	·874	·909	·944	·979	**·014**	35	83	32
33	19·049	·084	·119	·154	·189	·224	·259	·293	·327	·361	35	84	33
34	19·395	·429	·463	·497	·531	·565	·599	·633	·666	·699	34	84	34
35	19·732	·765	·798	·831	·864	·897	·930	·963	·996	**·028**	33	85	35
36	20·060	·092	·124	·156	·188	·220	·252	·284	·316	·348	32	85	36
37	20·380	·411	·442	·473	·504	·535	·566	·597	·628	·659	31	84	37
38	20·690	·721	·752	·783	·814	·844	·874	·904	·934	·964	30	83	38
39	20·994	**·024**	**·054**	**·084**	**·114**	**·144**	**·174**	**·203**	**·232**	**·261**	30	83	39
40	21·290	·319	·348	·377	·406	·435	·464	·493	·521	·549	29	82	40
41	21·577	·605	·633	·661	·689	·717	·745	·773	·801	·829	28	81	41
42	21·857	·885	·913	·940	·967	·994	**·021**	**·048**	**·075**	**·102**	27	81	42
43	...	...	...	...	...	...	...	...	...	...	...	...	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O^M *Maturity Age* **59.** **3 per-cent.**

n	$\bar{a}'_{59-n+1:\overline{n}}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{59-n+1:\overline{n}} - \bar{a}'_{60-n+1:\overline{n}}$	n
1	·773	·870	·967	**·063**	**·158**	**·253**	**·348**	**·442**	**·535**	**·628**	95	6	1
2	1·721	·813	·905	·996	**·087**	**·177**	**·267**	**·356**	**·445**	**·533**	90	7	2
3	2·621	·708	·795	·882	·969	**·055**	**·141**	**·226**	**·311**	**·395**	86	9	3
4	3·479	·562	·645	·728	·811	·893	·975	**·057**	**·138**	**·219**	82	11	4
5	4·300	·380	·460	·540	·619	·698	·776	·854	·932	**·009**	79	15	5
6	5·086	·162	·238	·314	·390	·466	·541	·616	·691	·766	76	19	6
7	5·841	·915	·988	**·061**	**·134**	**·207**	**·280**	**·352**	**·424**	**·496**	73	24	7
8	6·567	·638	·709	·779	·849	·919	·989	**·059**	**·128**	**·197**	70	28	8
9	7·266	·335	·403	·471	·539	·607	·674	·741	·808	·874	67	31	9
10	7·940	**·006**	**·072**	**·137**	**·202**	**·267**	**·332**	**·397**	**·462**	**·527**	65	36	10
11	8·591	·655	·718	·781	·844	·907	·970	**·033**	**·096**	**·159**	63	41	11
12	9·221	·282	·343	·404	·465	·526	·587	·648	·709	·770	61	45	12
13	9·831	·891	·950	**·009**	**·068**	**·127**	**·186**	**·245**	**·304**	**·363**	59	49	13
14	10·421	·479	·537	·594	·651	·708	·765	·822	·879	·936	57	53	14
15	10·992	**·048**	**·104**	**·160**	**·216**	**·271**	**·326**	**·381**	**·436**	**·491**	55	56	15
16	11·546	·601	·655	·709	·763	·817	·871	·925	·978	**·031**	54	59	16
17	12·084	·137	·190	·242	·294	·346	·398	·450	·502	·554	52	62	17
18	12·606	·657	·708	·759	·810	·861	·912	·963	**·013**	**·063**	51	65	18
19	13·113	·163	·213	·263	·313	·362	·411	·460	·509	·558	49	67	19
20	13·607	·656	·704	·752	·800	·848	·896	·944	·992	**·039**	48	69	20
21	14·086	·133	·180	·227	·274	·321	·368	·414	·460	·506	47	71	21
22	14·552	·598	·644	·690	·736	·781	·826	·871	·916	·961	45	74	22
23	15·006	·051	·096	·140	·184	·228	·272	·316	·360	·404	44	76	23
24	15·448	·492	·535	·578	·621	·664	·707	·750	·793	·836	43	77	24
25	15·879	·921	·963	**·005**	**·047**	**·089**	**·131**	**·173**	**·215**	**·257**	42	79	25
26	16·299	·340	·381	·422	·463	·504	·545	·586	·627	·668	41	81	26
27	16·709	·749	·789	·829	·869	·909	·949	·989	**·029**	**·069**	40	83	27
28	17·108	·147	·186	·225	·264	·303	·342	·381	·420	·459	39	84	28
29	17·498	·536	·574	·612	·650	·688	·726	·764	·802	·840	38	85	29
30	17·878	·916	·954	·991	**·028**	**·065**	**·102**	**·139**	**·176**	**·213**	37	86	30
31	18·250	·287	·324	·360	·396	·432	·468	·504	·540	·576	36	88	31
32	18·612	·648	·684	·720	·755	·790	·825	·860	·895	·930	35	89	32
33	18·965	**·000**	**·035**	**·070**	**·105**	**·140**	**·175**	**·209**	**·243**	**·277**	35	89	33
34	19·311	·345	·379	·413	·447	·481	·515	·548	·581	·614	34	90	34
35	19·647	·680	·713	·746	·779	·812	·845	·878	·911	·943	33	90	35
36	19·975	**·007**	**·039**	**·071**	**·103**	**·135**	**·167**	**·199**	**·231**	**·263**	32	89	36
37	20·295	·327	·359	·390	·421	·452	·483	·514	·545	·576	31	89	37
38	20·607	·638	·669	·700	·731	·761	·791	·821	·851	·881	30	89	38
39	20·911	·941	·971	**·001**	**·031**	**·061**	**·091**	**·121**	**·150**	**·179**	30	88	39
40	21·208	·237	·266	·295	·324	·353	·382	·411	·440	·468	29	88	40
41	21·496	·524	·552	·580	·608	·636	·664	·692	·720	·748	28	87	41
42	21·776	·804	·832	·860	·887	·914	·941	·968	·995	**·022**	27	86	42
43	22·049	·076	·103	·130	·157	·184	·211	·237	·263	·289	27	85	43
44	...	...	...	...	...	...	...	...	...	...	...	...	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

O[M] *Maturity Age* **60.** **3 per-cent.**

n	$\bar{a}'_{60-\overline{n+1}:\overline{n}\rvert}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{60-\overline{n+1}:\overline{n}\rvert} - \bar{a}'_{61-\overline{n+1}:\overline{n}\rvert}$	n
1	·767	·864	·960	**·056**	**·151**	**·246**	**·341**	**·435**	**·528**	**·621**	95	6	1
2	1·714	·806	·897	·988	**·078**	**·168**	**·258**	**·347**	**·436**	**·524**	90	7	2
3	2·612	·699	·786	·873	·959	**·045**	**·130**	**·215**	**·300**	**·384**	86	9	3
4	3·468	·551	·634	·717	·799	·881	·963	**·044**	**·125**	**·205**	82	13	4
5	4·285	·365	·444	·523	·602	·680	·758	·836	·913	·990	78	18	5
6	5·067	·143	·219	·295	·370	·445	·520	·595	·669	·743	75	22	6
7	5·817	·890	·963	**·036**	**·109**	**·181**	**·253**	**·325**	**·397**	**·468**	72	26	7
8	6·539	·610	·680	·750	·820	·890	·960	**·029**	**·098**	**·167**	70	30	8
9	7·235	·303	·371	·438	·505	·572	·639	·706	·772	·838	67	35	9
10	7·904	·970	**·035**	**·100**	**·165**	**·230**	**·294**	**·358**	**·422**	**·486**	65	38	10
11	8·550	·613	·676	·739	·802	·865	·928	·990	**·052**	**·114**	63	42	11
12	9·176	·237	·298	·359	·420	·481	·542	·602	·662	·722	61	47	12
13	9·782	·841	·900	·959	**·018**	**·077**	**·136**	**·194**	**·252**	**·310**	59	52	13
14	10·368	·426	·483	·540	·597	·654	·711	·768	·824	·880	57	56	14
15	10·936	·992	**·048**	**·103**	**·158**	**·213**	**·268**	**·323**	**·378**	**·433**	55	59	15
16	11·487	·541	·595	·649	·703	·757	·810	·863	·916	·969	54	62	16
17	12·022	·075	·127	·179	·231	·283	·335	·387	·439	·490	52	65	17
18	12·541	·592	·643	·694	·745	·796	·846	·896	·946	·996	50	68	18
19	13·046	·096	·146	·195	·244	·293	·342	·391	·440	·489	49	71	19
20	13·538	·586	·634	·682	·730	·778	·826	·874	·921	·968	48	74	20
21	14·015	·062	·109	·156	·202	·248	·294	·340	·386	·432	46	76	21
22	14·478	·524	·570	·615	·660	·705	·750	·795	·840	·885	45	77	22
23	14·930	·975	**·019**	**·063**	**·107**	**·151**	**·195**	**·239**	**·283**	**·327**	44	79	23
24	15·371	·414	·457	·500	·543	·586	·629	·672	·715	·758	43	82	24
25	15·800	·842	·884	·926	·968	**·010**	**·052**	**·094**	**·136**	**·177**	42	84	25
26	16·218	·259	·300	·341	·382	·423	·464	·505	·546	·586	41	85	26
27	16·626	·666	·706	·746	·786	·826	·866	·906	·946	·985	40	86	27
28	17·024	·063	·102	·141	·180	·219	·258	·297	·336	·375	39	88	28
29	17·413	·451	·489	·527	·565	·603	·641	·679	·717	·755	38	90	29
30	17·792	·829	·866	·903	·940	·977	**·014**	**·051**	**·088**	**·125**	37	91	30
31	18·162	·199	·235	·271	·307	·343	·379	·415	·451	·487	36	92	31
32	18·523	·559	·595	·631	·666	·701	·736	·771	·806	·841	35	93	32
33	18·876	·911	·946	·981	**·016**	**·051**	**·085**	**·119**	**·153**	**·187**	34	93	33
34	19·221	·255	·289	·323	·357	·391	·425	·458	·491	·524	34	94	34
35	19·557	·590	·623	·656	·689	·722	·755	·788	·821	·854	33	94	35
36	19·886	·918	·950	·982	**·014**	**·046**	**·078**	**·110**	**·142**	**·174**	32	94	36
37	20·206	·238	·270	·301	·332	·363	·394	·425	·456	·487	31	94	37
38	20·518	·549	·580	·611	·642	·673	·703	·733	·763	·793	30	94	38
39	20·823	·853	·883	·913	·943	·973	**·003**	**·033**	**·062**	**·091**	30	94	39
40	21·120	·149	·178	·207	·236	·265	·294	·323	·352	·381	29	93	40
41	21·409	·438	·466	·494	·522	·550	·578	·606	·634	·662	28	92	41
42	21·690	·718	·746	·774	·802	·829	·856	·883	·910	·937	27	90	42
43	21·964	·991	**·018**	**·045**	**·072**	**·099**	**·126**	**·153**	**·179**	**·205**	27	89	43
44	22·231	·257	·283	·309	·335	·361	·387	·413	·439	·465	26	88	44
45	...	...	...	...	...	...	...	...	...	...	...	...	45

For groups of **10** *years.*

0^M *Maturity Age* **61.** **3 per-cent.**

n	$\bar{a}'_{61-\overline{n+1}:n}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{61-\overline{n+1}:n} - \bar{a}'_{62-\overline{n+1}:n}$	n
1	·761	·858	·954	·050	·146	·241	·335	·429	·522	·615	95	7	1
2	1·707	·799	·890	·981	·071	·161	·250	·339	·428	·516	90	9	2
3	2·603	·690	·777	·863	·949	·034	·119	·204	·288	·372	85	11	3
4	3·455	·538	·621	·703	·785	·866	·947	·028	·108	·188	81	14	4
5	4·267	·346	·425	·504	·582	·660	·738	·815	·892	·969	78	17	5
6	5·045	·121	·197	·272	·347	·422	·496	·570	·644	·718	75	22	6
7	5·791	·864	·937	·009	·081	·153	·225	·296	·367	·438	72	27	7
8	6·509	·579	·649	·719	·789	·858	·927	·996	·064	·132	69	33	8
9	7·200	·268	·335	·402	·469	·536	·603	·669	·735	·801	67	38	9
10	7·866	·931	·996	·061	·126	·190	·254	·318	·382	·445	64	42	10
11	8·508	·571	·634	·697	·759	·821	·883	·945	·007	·068	62	46	11
12	9·129	·190	·251	·312	·372	·432	·492	·552	·612	·671	60	51	12
13	9·730	·789	·848	·907	·965	·023	·081	·139	·197	·255	58	56	13
14	10·312	·369	·426	·483	·540	·597	·653	·709	·765	·821	56	60	14
15	10·877	·933	·988	·043	·098	·153	·208	·263	·317	·371	55	63	15
16	11·425	·479	·533	·587	·640	·693	·746	·799	·852	·905	53	67	16
17	11·957	·009	·061	·113	·165	·217	·269	·320	·371	·422	52	70	17
18	12·473	·524	·575	·626	·676	·726	·776	·826	·876	·926	50	73	18
19	12·975	·025	·075	·124	·173	·222	·271	·320	·368	·416	49	76	19
20	13·464	·512	·560	·608	·656	·704	·751	·798	·845	·892	48	79	20
21	13·939	·986	·033	·079	·125	·171	·217	·263	·309	·355	46	81	21
22	14·401	·446	·491	·536	·581	·626	·671	·716	·761	·806	45	83	22
23	14·851	·895	·939	·983	·027	·071	·115	·159	·203	·246	44	85	23
24	15·289	·332	·375	·418	·461	·504	·547	·590	·632	·674	43	87	24
25	15·716	·758	·800	·842	·884	·926	·968	·010	·051	·092	42	89	25
26	16·133	·174	·215	·256	·297	·338	·379	·420	·460	·500	41	90	26
27	16·540	·580	·620	·660	·700	·740	·780	·819	·858	·897	40	92	27
28	16·936	·975	·014	·053	·092	·131	·170	·209	·247	·285	39	94	28
29	17·323	·361	·399	·437	·475	·513	·551	·589	·627	·664	38	96	29
30	17·701	·738	·775	·812	·849	·886	·923	·960	·997	·034	37	97	30
31	18·070	·106	·142	·178	·214	·250	·286	·322	·358	·394	36	98	31
32	18·430	·466	·502	·538	·573	·608	·643	·678	·713	·748	35	98	32
33	18·783	·818	·853	·888	·923	·957	·991	·025	·059	·093	34	99	33
34	19·127	·161	·195	·229	·263	·297	·331	·364	·397	·430	34	100	34
35	19·463	·496	·529	·562	·595	·628	·661	·694	·727	·760	33	100	35
36	19·792	·824	·856	·888	·920	·952	·984	·016	·048	·080	32	100	36
37	20·112	·144	·176	·207	·238	·269	·300	·331	·362	·393	31	100	37
38	20·424	·455	·486	·517	·548	·579	·609	·639	·669	·699	30	99	38
39	20·729	·759	·789	·819	·849	·879	·909	·939	·969	·998	30	99	39
40	21·027	·056	·085	·114	·143	·172	·201	·230	·259	·288	29	98	40
41	21·317	·346	·375	·404	·432	·460	·488	·516	·544	·572	28	97	41
42	21·600	·628	·656	·684	·712	·740	·767	·794	·821	·848	28	96	42
43	21·875	·902	·929	·956	·983	·010	·037	·064	·091	·117	27	95	43
44	22·143	·169	·195	·221	·247	·273	·299	·325	·351	·377	26	94	44
45	22·403	·429	·455	·481	·506	·531	·556	·581	·606	·631	25	92	45

For groups of **10** *years.*

O^M *Maturity Age* **62.** **3 per-cent.**

n	$\bar{a}'_{\overline{62-n+1:n}}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{\overline{62-n+1:n}} - \bar{a}'_{\overline{63-n+1:n}}$	n
1	·754	·851	·947	·043	·138	·233	·327	·421	·514	·606	94	8	1
2	1·698	·789	·880	·971	·061	·151	·240	·329	·417	·505	89	9	2
3	2·592	·679	·765	·851	·936	·021	·106	·190	·274	·358	85	12	3
4	3·441	·524	·606	·688	·769	·850	·931	·011	·091	·171	81	15	4
5	4·250	·329	·408	·486	·564	·641	·718	·795	·871	·947	77	19	5
6	5·023	·098	·173	·248	·323	·397	·471	·545	·618	·691	74	24	6
7	5·764	·836	·908	·980	·052	·123	·194	·265	·336	·406	71	30	7
8	6·476	·546	·616	·685	·754	·823	·891	·959	·027	·095	69	35	8
9	7·162	·229	·296	·363	·430	·496	·562	·628	·694	·759	66	41	9
10	7·824	·889	·954	·018	·082	·146	·210	·273	·336	·399	64	47	10
11	8·462	·524	·586	·648	·710	·772	·834	·895	·956	·017	62	52	11
12	9·078	·138	·198	·258	·318	·378	·438	·497	·556	·615	60	57	12
13	9·674	·733	·791	·849	·907	·965	·023	·081	·138	·195	58	61	13
14	10·252	·309	·366	·422	·478	·534	·590	·646	·702	·758	56	64	14
15	10·814	·869	·924	·979	·034	·088	·142	·196	·250	·304	54	69	15
16	11·358	·412	·465	·518	·571	·624	·677	·730	·783	·835	53	73	16
17	11·887	·939	·991	·043	·095	·146	·197	·248	·299	·350	51	77	17
18	12·400	·451	·501	·551	·601	·651	·701	·751	·801	·850	50	79	18
19	12·899	·948	·997	·046	·095	·144	·193	·241	·289	·337	49	81	19
20	13·385	·433	·481	·529	·576	·623	·670	·717	·764	·811	47	84	20
21	13·858	·905	·951	·997	·043	·089	·135	·181	·227	·273	46	87	21
22	14·318	·363	·408	·453	·498	·543	·588	·633	·678	·722	45	89	22
23	14·766	·810	·854	·898	·942	·986	·030	·073	·116	·159	44	91	23
24	15·202	·245	·288	·331	·374	·417	·459	·501	·543	·585	43	93	24
25	15·627	·669	·711	·753	·795	·837	·879	·920	·961	·002	42	95	25
26	16·043	·084	·125	·166	·207	·248	·288	·328	·368	·408	40	97	26
27	16·448	·488	·528	·568	·608	·647	·686	·725	·764	·803	39	98	27
28	16·842	·881	·920	·959	·998	·037	·075	·113	·151	·189	38	99	28
29	17·227	·265	·303	·341	·379	·417	·455	·493	·530	·567	38	100	29
30	17·604	·641	·678	·715	·752	·789	·826	·863	·900	·936	37	101	30
31	17·972	·008	·044	·080	·116	·152	·188	·224	·260	·296	36	102	31
32	18·332	·368	·404	·439	·474	·509	·544	·579	·614	·649	35	103	32
33	18·684	·719	·754	·789	·823	·857	·891	·925	·959	·993	34	104	33
34	19·027	·061	·095	·129	·163	·197	·231	·264	·297	·330	34	104	34
35	19·363	·396	·429	·462	·495	·528	·561	·594	·627	·660	33	105	35
36	19·692	·724	·756	·788	·820	·852	·884	·916	·948	·980	32	106	36
37	20·012	·044	·076	·108	·139	·170	·201	·232	·263	·294	31	105	37
38	20·325	·356	·387	·418	·449	·480	·510	·540	·570	·600	30	105	38
39	20·630	·660	·690	·720	·750	·780	·810	·840	·870	·900	30	105	39
40	20·929	·959	·988	·017	·046	·075	·104	·133	·162	·191	29	105	40
41	21·220	·249	·278	·307	·336	·364	·392	·420	·448	·476	28	104	41
42	21·504	·532	·560	·588	·616	·644	·672	·699	·726	·753	28	103	42
43	21·780	·807	·834	·861	·888	·915	·942	·969	·996	·023	27	101	43
44	22·049	·076	·103	·129	·155	·181	·207	·233	·259	·285	26	99	44
45	22·311	·337	·363	·389	·415	·441	·466	·491	·516	·541	26	98	45

For groups of **10** *years.*

O^M^ *Maturity Age* **63.** **3 per-cent.**

n	$\bar{a}'_{63-\overline{n+1}:\overline{n}}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{63-\overline{n+1}:\overline{n}} - \bar{a}'_{64-\overline{n+1}:\overline{n}}$	n
1	·746	·843	·939	**·035**	**·130**	**·225**	**·319**	**·412**	**·505**	**·597**	94	10	1
2	1·689	·781	·872	·963	**·053**	**·142**	**·230**	**·318**	**·406**	**·493**	89	10	2
3	2·580	·667	·753	·839	·924	**·009**	**·093**	**·177**	**·260**	**·343**	85	12	3
4	3·426	·509	·590	·671	·752	·833	·913	·993	**·073**	**·152**	81	16	4
5	4·231	·309	·387	·465	·542	·619	·696	·772	·848	·924	77	21	5
6	4·999	**·074**	**·149**	**·223**	**·297**	**·371**	**·444**	**·517**	**·590**	**·662**	74	26	6
7	5·734	·806	·877	·948	**·019**	**·090**	**·161**	**·231**	**·301**	**·371**	71	32	7
8	6·441	·510	·579	·648	·716	·784	·852	·920	·987	**·054**	68	39	8
9	7·121	·188	·254	·320	·386	·452	·517	·582	·647	·712	66	45	9
10	7·777	·841	·905	·969	**·033**	**·096**	**·159**	**·222**	**·285**	**·348**	63	50	10
11	8·410	·472	·534	·596	·657	·718	·779	·840	·901	·961	61	56	11
12	9·021	·081	·141	·201	·260	·319	·378	·437	·496	·555	59	61	12
13	9·613	·671	·729	·787	·845	·903	·960	**·017**	**·074**	**·131**	58	65	13
14	10·188	·245	·301	·357	·413	·469	·525	·580	·635	·690	56	70	14
15	10·745	·800	·855	·909	·963	**·017**	**·071**	**·125**	**·179**	**·232**	54	74	15
16	11·285	·338	·391	·444	·497	·550	·602	·654	·706	·758	52	77	16
17	11·810	·862	·913	·964	**·015**	**·066**	**·117**	**·168**	**·219**	**·270**	51	81	17
18	12·321	·371	·421	·471	·521	·571	·621	·671	·720	·769	50	85	18
19	12·818	·867	·916	·965	**·013**	**·061**	**·109**	**·157**	**·205**	**·253**	48	88	19
20	13·301	·349	·396	·443	·490	·537	·584	·631	·678	·725	47	91	20
21	13·771	·817	·863	·909	·955	**·001**	**·047**	**·093**	**·139**	**·184**	46	94	21
22	14·229	·274	·319	·364	·409	·454	·499	·543	·587	·631	45	97	22
23	14·675	·719	·763	·807	·851	·894	·937	·980	**·023**	**·066**	43	99	23
24	15·109	·152	·195	·238	·280	·322	·364	·406	·448	·490	42	100	24
25	15·532	·574	·616	·658	·700	·741	·782	·823	·864	·905	41	101	25
26	15·946	·987	**·028**	**·069**	**·110**	**·150**	**·190**	**·230**	**·270**	**·310**	40	103	26
27	16·350	·390	·430	·470	·509	·548	·587	·626	·665	·704	39	105	27
28	16·743	·782	·821	·860	·899	·937	·975	**·013**	**·051**	**·089**	38	106	28
29	17·127	·165	·203	·241	·279	·317	·355	·392	·429	·466	38	107	29
30	17·503	·540	·577	·614	·651	·688	·725	·762	·798	·834	37	108	30
31	17·870	·906	·942	·978	**·014**	**·050**	**·086**	**·122**	**·158**	**·194**	36	108	31
32	18·229	·265	·300	·335	·370	·405	·440	·475	·510	·545	35	109	32
33	18·580	·615	·650	·685	·719	·753	·787	·821	·855	·889	34	111	33
34	18·923	·957	·991	**·025**	**·059**	**·093**	**·126**	**·159**	**·192**	**·225**	34	112	34
35	19·258	·291	·324	·357	·390	·423	·456	·489	·522	·554	33	112	35
36	19·586	·619	·651	·683	·715	·747	·779	·811	·843	·875	32	112	36
37	19·907	·939	·971	**·003**	**·034**	**·065**	**·096**	**·127**	**·158**	**·189**	31	112	37
38	20·220	·251	·282	·313	·344	·375	·405	·435	·465	·495	30	112	38
39	20·525	·555	·585	·615	·645	·675	·705	·735	·765	·795	30	111	39
40	20·824	·854	·884	·913	·942	·971	**·000**	**·029**	**·058**	**·087**	29	110	40
41	21·116	·145	·174	·203	·232	·261	·289	·317	·345	·373	28	109	41
42	21·401	·429	·457	·485	·513	·541	·569	·597	·625	·652	28	109	42
43	21·679	·707	·734	·761	·788	·815	·842	·869	·896	·923	27	109	43
44	21·950	·977	**·004**	**·031**	**·057**	**·083**	**·109**	**·135**	**·161**	**·187**	26	107	44
45	22·213	·239	·265	·291	·317	·343	·369	·394	·419	·444	26	104	45

For groups of **10** *years.*

O^M *Maturity Age* **64.** **3 per-cent.**

n	$\bar{a}'_{64-\overline{n+1}:\overline{n}\rceil}$ ·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	1 diff.	$\bar{a}'_{64-\overline{n+1}:\overline{n}\rceil}-\bar{a}'_{65-\overline{n+1}:\overline{n}\rceil}$	n
1	·736	·833	·929	·025	·120	·215	·309	·402	·495	·587	94	10	1
2	1·679	·770	·861	·952	·042	·131	·219	·307	·395	·482	89	11	2
3	2·568	·654	·740	·826	·911	·995	·079	·162	·245	·328	84	15	3
4	3·410	·492	·573	·654	·735	·815	·895	·974	·053	·132	80	19	4
5	4·210	·288	·366	·443	·520	·596	·672	·748	·823	·898	76	23	5
6	4·973	·048	·122	·196	·269	·342	·415	·487	·559	·631	73	28	6
7	5·702	·773	·844	·915	·986	·056	·126	·195	·264	·333	70	34	7
8	6·402	·471	·539	·607	·675	·742	·809	·876	·943	·010	67	41	8
9	7·076	·142	·208	·274	·339	·404	·469	·534	·599	·663	65	47	9
10	7·727	·791	·855	·918	·981	·044	·106	·168	·230	·292	63	53	10
11	8·354	·416	·477	·538	·599	·660	·720	·780	·840	·900	61	58	11
12	8·960	·020	·079	·138	·197	·256	·315	·374	·432	·490	59	64	12
13	9·548	·606	·664	·721	·778	·835	·892	·949	·006	·062	57	70	13
14	10·118	·174	·230	·286	·342	·397	·452	·507	·562	·617	55	75	14
15	10·671	·725	·779	·833	·887	·941	·995	·049	·102	·155	54	80	15
16	11·208	·261	·314	·367	·419	·471	·523	·575	·627	·678	52	84	16
17	11·729	·780	·831	·882	·933	·984	·035	·086	·136	·186	51	87	17
18	12·236	·286	·336	·386	·436	·485	·534	·583	·632	·681	49	91	18
19	12·730	·779	·827	·875	·923	·971	·019	·067	·115	·163	48	95	19
20	13·210	·257	·304	·351	·398	·445	·492	·539	·585	·631	47	98	20
21	13·677	·723	·769	·815	·861	·907	·952	·997	·042	·087	46	100	21
22	14·132	·177	·222	·267	·312	·356	·400	·444	·488	·532	44	102	22
23	14·576	·620	·664	·708	·751	·794	·837	·880	·923	·966	43	104	23
24	15·009	·052	·095	·137	·179	·221	·263	·305	·347	·389	42	106	24
25	15·431	·473	·515	·556	·597	·638	·679	·720	·761	·802	41	108	25
26	15·843	·884	·925	·965	·005	·045	·085	·125	·165	·205	40	110	26
27	16·245	·285	·325	·364	·403	·442	·481	·520	·559	·598	39	111	27
28	16·637	·676	·715	·754	·792	·830	·868	·906	·944	·982	38	112	28
29	17·020	·058	·096	·134	·172	·210	·247	·284	·321	·358	38	113	29
30	17·395	·432	·469	·506	·543	·580	·617	·653	·689	·725	37	114	30
31	17·762	·798	·834	·870	·906	·942	·978	·014	·050	·085	36	115	31
32	18·120	·155	·190	·225	·260	·295	·330	·365	·400	·435	35	116	32
33	18·469	·504	·539	·573	·607	·641	·675	·709	·743	·777	34	116	33
34	18·811	·845	·879	·913	·947	·981	·014	·047	·080	·113	34	117	34
35	19·146	·179	·212	·245	·278	·311	·344	·377	·410	·442	33	117	35
36	19·474	·507	·539	·571	·603	·635	·667	·699	·731	·763	32	117	36
37	19·795	·827	·859	·891	·922	·953	·984	·015	·046	·077	31	118	37
38	20·108	·139	·170	·201	·232	·263	·294	·324	·354	·384	31	117	38
39	20·414	·444	·474	·504	·534	·564	·594	·624	·654	·684	30	117	39
40	20·714	·744	·774	·804	·833	·862	·891	·920	·949	·978	29	117	40
41	21·007	·036	·065	·094	·123	·152	·180	·208	·236	·264	28	116	41
42	21·292	·320	·348	·376	·404	·432	·460	·488	·516	·543	28	115	42
43	21·570	·598	·626	·654	·681	·708	·735	·762	·789	·816	27	113	43
44	21·843	·870	·897	·924	·951	·978	·005	·031	·057	·083	26	113	44
45	22·109	·135	·161	·187	·213	·239	·265	·291	·317	·343	26	111	45

For groups of **10** *years.*

O^M *Maturity Age* **65.** **3 per-cent.**

n	·0	·1	·2	·3	·4	·5	·6	·7	·8	·9	Diff.	$\bar{a}'_{65-n+1:\bar{n}} - \bar{a}'_{65-n+1:\bar{n}}$	n
	$\bar{a}'_{65-n+1:\bar{n}}$												
1	·726	·823	·919	·014	·109	·204	·298	·391	·484	·576	94	10	1
2	1·668	·759	·850	·940	·029	·118	·206	·294	·381	·467	89	12	2
3	2·553	·639	·724	·809	·893	·977	·061	·144	·227	·309	84	15	3
4	3·391	·472	·553	·634	·714	·794	·873	·952	·031	·109	80	19	4
5	4·187	·264	·341	·418	·494	·570	·646	·721	·796	·871	76	25	5
6	4·945	·019	·092	·165	·238	·310	·382	·454	·526	·597	72	31	6
7	5·668	·739	·809	·879	·949	·018	·087	·156	·225	·293	69	36	7
8	6·361	·429	·497	·564	·631	·698	·765	·831	·897	·963	67	43	8
9	7·029	·095	·160	·225	·290	·355	·419	·483	·547	·611	64	51	9
10	7·674	·737	·800	·863	·925	·987	·049	·111	·173	·235	62	57	10
11	8·296	·357	·418	·478	·538	·598	·658	·718	·778	·837	60	63	11
12	8·896	·955	·014	·073	·131	·189	·247	·305	·363	·421	58	70	12
13	9·478	·535	·592	·649	·706	·763	·819	·875	·931	·987	56	76	13
14	10·043	·099	·154	·209	·264	·319	·374	·429	·483	·537	55	81	14
15	10·591	·645	·699	·753	·806	·859	·912	·965	·018	·071	53	86	15
16	11·124	·177	·230	·282	·334	·386	·438	·489	·540	·591	52	91	16
17	11·642	·693	·744	·795	·845	·895	·945	·995	·045	·095	50	96	17
18	12·145	·195	·244	·293	·342	·391	·440	·489	·538	·587	49	99	18
19	12·635	·683	·731	·779	·827	·875	·923	·971	·018	·065	48	103	19
20	13·112	·159	·206	·253	·300	·347	·393	·439	·485	·531	46	105	20
21	13·577	·623	·669	·715	·760	·805	·850	·895	·940	·985	45	108	21
22	14·030	·075	·120	·164	·208	·252	·296	·340	·384	·428	44	110	22
23	14·472	·516	·559	·602	·645	·688	·731	·774	·817	·860	43	113	23
24	14·903	·945	·987	·029	·071	·113	·155	·197	·239	·281	42	115	24
25	15·323	·364	·405	·446	·487	·528	·569	·610	·651	·692	41	116	25
26	15·733	·774	·814	·854	·894	·934	·974	·014	·054	·094	40	117	26
27	16·134	·174	·213	·252	·291	·330	·369	·408	·447	·486	39	119	27
28	16·525	·564	·603	·641	·679	·717	·755	·793	·831	·869	38	119	28
29	16·907	·945	·983	·021	·059	·096	·133	·170	·207	·244	37	119	29
30	17·281	·318	·355	·392	·429	·466	·503	·539	·575	·611	37	121	30
31	17·647	·683	·719	·755	·791	·827	·863	·899	·934	·969	36	122	31
32	18·004	·039	·074	·109	·144	·179	·214	·249	·284	·319	35	123	32
33	18·353	·388	·422	·456	·490	·524	·558	·592	·626	·660	34	123	33
34	18·694	·728	·762	·796	·830	·864	·897	·930	·963	·996	34	123	34
35	19·029	·062	·095	·128	·161	·194	·227	·260	·293	·325	33	124	35
36	19·357	·389	·421	·453	·485	·517	·549	·581	·613	·645	32	124	36
37	19·677	·709	·741	·773	·805	·836	·867	·898	·929	·960	31	125	37
38	19·991	·022	·053	·084	·115	·146	·177	·207	·237	·267	31	125	38
39	20·297	·327	·357	·387	·417	·447	·477	·507	·537	·567	30	124	39
40	20·597	·627	·657	·687	·717	·746	·775	·804	·833	·862	29	124	40
41	20·891	·920	·949	·978	·007	·036	·065	·093	·121	·149	29	124	41
42	21·177	·205	·233	·261	·289	·317	·345	·373	·401	·429	28	121	42
43	21·457	·485	·513	·541	·568	·595	·622	·649	·676	·703	27	120	43
44	21·730	·757	·784	·811	·838	·865	·892	·919	·946	·972	27	119	44
45	21·998	·025	·052	·078	·104	·130	·156	·182	·208	·234	26	118	45

Abstract of the Discussion.

Mr. H. E. Melville, in opening the discussion, said the author had brought before the members a new and very interesting method of approximate valuation, based on the well-known fact that nearly all the actuarial functions in common use were approximately of the second order over short ranges, and involving a wide extension of the present methods of grouping. The author set out to find an average valuation function that could be applied to the data for a series of consecutive age groups, and as a first step he made the convenient assumption that the function proceeded by a constant first difference. On that assumption the average function was, of course, the normal function for the average age, and the problem then became one of finding the average age. He accordingly weighted the age with the sum assured, totalled the individual weighted ages, divided by the total sums assured in the group, and thus obtained his average age and hence his average valuation factor. That factor had been obtained on an erroneous assumption, and some adjustment was necessary to allow for the fact that the ordinary factor was nearly of the second order. The author pointed out that the proper correction on the assumption of a constant second difference would be

$$\left(\frac{\Sigma_0^{t-1} x^2 S}{\Sigma S} - \omega^2\right)\frac{\delta^2}{2}$$

where x was measured from the lowest age of the group, t was the range of the group, and ω was the average age already found.

This expression took the form

$$\left(\frac{\Sigma_{-\frac{t-1}{2}}^{+\frac{t-1}{2}} x^2 S}{\Sigma S} - \omega^2\right)\frac{\delta^2}{2}$$

where x was measured from the central age of the group, and ω was the difference between the average age already found and the central age.

It was obviously very difficult to employ x^2S in the calculations, and the author, therefore, made the convenient assumption that the sums assured at each age were constant. On that assumption ω^2 in the second expression became 0, and

$$\left(\frac{\Sigma_{-\frac{t-1}{2}}^{+\frac{t-1}{2}} x^2 S}{\Sigma S}\right)\frac{\delta^2}{2} \text{ became } \frac{t^2-1}{24}\delta^2$$

the correction given by Mr. King.

In practice the sums assured were not constant at each age, but were generally increasing up to a point and thereafter decreasing and although the author had shown by his numerical results that

his method was sufficiently accurate for all practical purposes, yet it was interesting to consider alternative assumptions—for example, what result would be obtained if it was assumed that the sums assured progressed by a constant first difference. As already mentioned

$$\frac{\sum_{-\frac{t-1}{2}}^{+\frac{t-1}{2}} x^2 S}{\sum S} \frac{\delta^2}{2} \text{ became } \frac{t^2-1}{24} \delta^2$$

if S were assumed constant, but it also gave this value if S were either increasing or decreasing by a constant first difference. On the former assumption, however, ω became 0. but on the latter assumption there remained a correction of $\omega^2 \frac{\delta^2}{2}$ and if this correction were made, a better result would be expected in most cases than if the author's assumption were adopted. At the same time, however, ω was usually small, the average age differing but little from the central age, and $\omega^2 \frac{\delta^2}{2}$ often negligible, and although his experience was that a slight improvement in accuracy could generally be gained by making the further correction suggested, yet for practical purposes the author's method was sufficient, particularly in view of the fact that any departure therefrom would render the labour involved in the valuation much heavier and would prevent the use of the author's tables in their present simple form.

The author next proceeded to consider the application of his method to the valuation of premiums and bonuses. The method was applicable to premiums and bonuses if a special age was obtained for each group, but the advantages were obvious of obtaining an age from the sums assured alone, making some small modification to that, and applying that age to the premiums and bonuses. Mr. King obtained an approximate value of the rate of increase in the rate of premiums and the rate of bonuses, and having obtained that rate of increase he again made the assumption that the sums assured were constant at each age, and obtained very neatly a correction to the valuation age. Another method of procedure would be to obtain the rate of increase in the rate of premium, and combine it with the valuation factor to form a new valuation factor, which would for all practical purposes have a constant second difference again, and which could be applied in very much the same way as the author applied his method. But whatever method were adopted for dealing with the premiums and bonuses, he thought some simplification could be made in practice, because the difference between the valuation age for the sum assured and the valuation age for the premiums would vary very little from year to year. Having found the difference at the beginning of a quinquennium, one could, he thought, quite safely use that difference and apply it throughout the remainder of the quinquennium to the valuation age obtained from the sums assured alone.

In the application of the method to endowment assurances, it was necessary to have an average maturity age for the whole of the group, and the author obtained that by summing the Z's and the sums assured for the whole of the group, and using the ratio in the ordinary way with the Z table to find the maturity age. Incidentally with reference to Mr. Elderton's suggestion (*J. I.A.*, vol. xlviii, p. 14), that a company could pass from a valuation by the Z method with the net premiums for the true maturity ages to a valuation by his (Mr. Elderton's) method, by finding an average maturity age for the whole existing business and putting on the new business with the net premiums for that maturing age, the author had given a neat method of finding the average maturing age for the existing business. He (the speaker) had tested the method and found it gave remarkably good results, even applied to the whole of the business of the office, and not merely to a small group.

Passing to the practical application of the method, the first point for consideration was what value should be given to t, the range of the group. The factor $\frac{t^2-1}{24}$ in the adjustment given by the author suggested 7 or 11 as suitable numbers, but for obvious reasons the author preferred 10—used, of course, in conjunction with special class books. The saving of labour if those special class books were employed was very great, and even with the present class books it would be substantial. Another point to be borne in mind was that that saving did not involve any appreciable loss of accuracy, although he thought loss of accuracy was not after all such a serious thing. If a method of valuation was convenient, gave adequate reserves and showed exactly what those reserves were, then he thought the mere fact that it did not give a very close approximation to a net premium valuation was no serious objection. It seemed to him that an approximate valuation if used ought to give consistent results one year with another, and the same insight into the working of the business as they had at present. If those two conditions were not fulfilled then he thought an approximate valuation except as a check valuation could not be depended upon. The author's method, in his opinion, complied with both those conditions. It certainly would give consistent results in normal circumstances one year with another, and it gave the same insight into the working of the business as they had at present: it did not disturb the interest or loading margins, and it might be made to give the expected mortality and death strain. There would be some error in using the valuation age obtained from the sums assured for the number of policies and the bonuses, but that was not a very serious matter. It was more important that one should be able to get a very good idea of the expected strain. The death strain was a function not only of the sums assured but also of the premiums, the rate of mortality and the annuity value, and consequently if the author's method was to be used it was necessary to have a combined valuation factor of the rate of mortality and the annuity value. That combined factor should strictly be used separately

in conjunction with the sums assured, the bonuses and the premiums, but he thought that inasmuch as any method of obtaining the death strain must be more or less approximate, because some rather wide assumptions had to be made in practice as to the distribution of the new business and the withdrawals, it was quite sufficient to use the age obtained from the sums assured alone, to take the total premiums in the group, and to use the formula $(P+d)(1+a)\times q$ modified to suit the valuation circumstances. That meant that the total premiums in the group would be added to d times the sums assured and bonuses, and the total multiplied by the new factor of q times the annuity value for the age obtained from the sums assured alone.

There was one minor point of criticism he desired to make. He asked the author to reconsider his use of the word "weight." At one time he spoke of weighting the age with the sum assured, and that seemed to him (the speaker) exactly what ought to be done; at another time he spoke of weighting with the age; while yet again he referred to the sum assured multiplied by the age as a weight, *i.e.*, the constant which was recorded in his class book. He thought it would be an advantage if the author would make that small correction, and call that third function a constant instead of a weight.

Mr. S. E. MACNAGHTEN agreed with Mr. Melville that one great advantage of the new method of valuation proposed by Mr. King was that the weights were independent of the valuation factors, so that when they had once been calculated the method could be used for other purposes besides actual valuations; for example, it could be used for determining the profit or loss on Claims, Surrenders, and Lapses.

If the valuation particulars had been tabulated in the ordinary way, *i.e.*, whole-life assurance according to years of birth, and endowment assurance according to years of maturity, it would be a comparatively simple matter to make a valuation by the weighted-year method, and undoubtedly there would be a considerable saving of labour at a time when every minute was of value; but although that was important, the labour involved in actually making a valuation was very small compared with the work entailed in collecting and balancing the various particulars before the valuation itself could be proceeded with. That was well known to anybody who had to do with valuations. Where there was a large number of books and classes to be dealt with, the work was enormous: in fact, a continuous valuation was almost impossible. In his opinion, the new method would be found most valuable in the case of an office which kept a continuous record of the valuation particulars, and made yearly investigations into the sources of profit and loss for each of the main classes of business. The grouping of ten sections into one would simplify the book-keeping and greatly facilitate the balancing of the particulars. If, therefore, it could be shown that the results were reliable, he thought there were decided advantages to be gained by the employment of the more extended groupings.

If a table of whole-life annuity values were differenced, it was found that a_x could be very closely represented by a rational integral function of x of the second degree; that the second differences were very small throughout; that they were all negative up to about age 59 (O^M 3 per-cent), and that thereafter they were all positive. From that it would seem to follow that actuaries should always in practice obtain quite good results by the employment of the method described in paragraph 10 of the paper. He had taken out the values of the whole life sums assured on that basis, and the total error in the final result was under £100, the total positive deviations being about £1,000, and the total negative deviations about £900, the maximum error in any one group being less than 1 in 800—results which in the ordinary way would be considered quite good approximations. Mr. King had, however, still further improved matters by bringing in a correction to allow for second differences. It was quite clear that the amount of the correction must depend on the distribution of the sums assured throughout the group; for example, if the sums assured were all concentrated at any one single point of the group no correction would be required. On the other hand, if the sums assured were equally distributed at the two ends of the group, the correction required would be a maximum. The author had adopted for his correction an average value $\frac{t^2-1}{24}\delta^2 A_\omega$ which would clearly be the correct value to employ if the sums assured were equal for all ages in the group, but it was obviously also approximately true in a large number of other cases; it was simply an average value. In most offices if the whole-life sums assured at successive ages were plotted out on cross-ruled paper they would take the form of a bell-shaped curve, the sums assured being smallest at the youngest and oldest ages. It could be shown mathematically that if the sum assured could be represented by a function of the first degree, the factor $\frac{t^2-1}{24}\delta^2 A_\omega$ would always be too large; but unless the rate of increase or decrease in the sums assured age by age was great, the error introduced by taking an average correction of $\frac{t^2-1}{24}\delta^2 A_\omega$ for all groups was quite small. Taking a group of ten ages, if the gradient were 1 in 10, the error was only ·16 of the second difference, and even if it were as much as 1 in 1 the error was only 1·125 of the second difference. Since the second differences changed sign in the middle of the whole life table about age 59, the errors above and below that age would tend to cancel one another. He thought that they could always rely on obtaining good results from that method for the valuation of whole life assurances.

In the case of Endowment Assurances the second differences of temporary annuities, differenced according to duration, were at the short durations larger than the second differences of whole life annuities; also they were all negative; therefore, with groups of the same size, *i.e.*, groups of 10, they could not expect to obtain

quite such close approximations for endowment assurance valuations as they did for whole life. But compensating errors would still be found to occur, and he thought results would always be obtained which were quite accurate enough for all practical purposes. If greater accuracy were required they could always reduce the range of the groups. There was no magic in 10; if 5 were preferred they could take groups of 5, but they would then lose the advantage of the tables supplied by Mr. King. In any group valuation, whether the groups consisted of one or ten years, certain assumptions had generally to be made, and if through some abnormality facts were contrary to what had been assumed errors would be introduced into the results; but once the principle of grouping was admitted he saw no reason why they should confine themselves to groups of one year. The advantages to be gained by reducing the number of groups were so great that the subject was well worth the attention of actuaries.

Mr. ALFRED HENRY referred to the author's suggestion that the whole of the policies of one class might be included for valuation purposes in a single group by means of a frequency curve. He wished to indicate briefly a method by which that might be effected. He found on inspection of A_x, that it could be represented with quite remarkable fidelity from age 20 up to 80 by a parabolic curve of the form of $A+Bx+Cx^2+Dx^3$. By means of that approximation the value of the sums assured for the whole group could be readily obtained by taking the first, second and third moments of the sums assured, multiplying them respectively by B, C, and D, and adding them to A times the total. In applying the method practically one would, of course, make use of the fact that the moment for any particular year of birth could be found from the moment for any other year by simple adjustment. The first, second and third moments of the sums assured, calculated for some convenient year, would be entered once and for all in the class books, and from the totals the moments at each valuation date would be obtained. By that simple method the sums assured could be valued in a single group, and it would simply be necessary to make the corrections in the aggregate. That applied only to sums assured, but he thought it followed from the relation between the reversion and the annuity that a similar curve would hold good for the annuity value. He had had neither the opportunity nor the data for working out a complete valuation, but taking the author's ten year groups, which could be expected to give only a very rough result, and applying the method he had suggested, he had found that it was possible actually to reproduce the value of the sums assured within $\frac{1}{2}$ per-cent.

Mr. R. E. UNDERWOOD suggested that the time saved by the use of the author's method might be utilized to make a select valuation. Having reduced the number of separate age-groups in the proportion of 10 to 1, they could afford to have 10 or 11 durations in place of the omitted ages, and by weighting separately for each duration they would obtain a select valuation with little

more work than was at present required for an aggregate valuation. Additional labour would be entailed in calculating the constant weights, but that was a comparatively small matter. There would also probably be more incomplete groups, but that did not seem to present much difficulty, and it could probably be dealt with in the way suggested by the author. It would, of course, be necessary to calculate extended tables of the weighted functions. It seemed to him that the expected claims could be calculated quite well by the new method, both for the purpose of comparison, and for analysis of the surplus. He knew it was not likely that valuations by Select Tables would become general at present, but if at some future time all the facts could be grouped into a single group, select valuations would then become a practical possibility.

Mr. W. PALIN ELDERTON enquired, with reference to Mr. Henry's remarks, whether the author had tried valuing whole-life assurances or any other class in one group, employing as factor a single A' (in the author's notation), or whether in the alternative he had tried two groups, one, as it were, based on the up grade of the sums assured and the other based on the down grade? The only objection he saw at the moment to Mr. Henry's suggestion was that it would need three constants for the sums assured—proportional respectively to the year of birth, the year of birth squared, and the year of birth cubed. Unless some convenient way could be found of passing from the sums assured to the bonuses and the net premiums, as Mr. King suggested in the paper, it would need a similar number of constants, he thought, for those other factors as well.

Mr. G. J. LIDSTONE said that on his first appearance at the Institute the author came forward with a paper which as to form successfully followed the very best models, while as to substance it was quite evident it was of the utmost value; and he would like to offer Mr. King his personal congratulations.

Approximate group valuations gave rise to errors of two classes. There was one class that would happen even if they were multiplying by first difference functions, owing in the first place to the facts being irregularly distributed, and in the second place, so far as they represented an inherently regular general distribution, to that distribution being nothing like uniform. He thought the great feature of the author's paper was that by one very simple method it at once got rid of the whole of that error by a simple system of weights, and as Mr. Macnaghten had pointed out, in the aggregate that alone led to very accurate results. There was, secondly, the error due to the fact that the functions were not first difference functions. Whether the second difference correction was necessary depended to a great extent upon the function which was being dealt with. He thought, therefore, it might be desirable to give a little attention to that second difference correction. It was certainly not very important in the case of the A_x and the a_x involved in a whole life valuation, partly because their second differences were not very big in themselves, and partly because, as Mr. Macnaghten pointed out, they

changed sign in the middle of the range and therefore tended to counteract each other. It had already been pointed out that that was not so to the same extent with endowment assurances. It would be so to a much less extent with compound functions like $\mu_x a_x$ or $\mu_x A_x$, which, as Mr. Melville had pointed out, would be required for the death strain, or with μ_x itself or q_x which would be required for the expected deaths. Therefore he thought, although the author's excellent results might suggest that no further correction was required, the investigation that Mr. Melville had given was one of great value. He had looked into the matter himself on similar lines, and arrived at a similar conclusion to that of Mr. Melville, who had pointed out the very simple form the correction took if it was assumed that the data were distributed in a straight line. He wished to add to that statement that instead of having a correction to the function A or a, it might be convenient to throw it into the form of a correction of the mean age, which could be very simply done. That might be considered in relation to Mr. Macnaghten's remarks as to the distribution of the data. If the data were set out in a curve they would be in the form of a curve of frequency ; or to those who found that rather a fearsome expression he might say they were practically in an oblique triangle, with the top of the triangle and the ends nicely rounded off. He thought Mr. Melville would agree that his results were applicable whether the straight line was going up or going down, and consequently they could deal independently with the upward slope of the triangle and the downward slope, and very accurate results indeed could thus be obtained by Mr. Melville's process. Those results, he thought, might be useful for functions which had a greater curvature than the A and a applicable to whole life groups.

The whole of the reasoning for a method of that kind, which led to functions based on mean or weighted ages, was, of course, materially helped so far as it was possible to do away with the irregularities in the sums assured and premiums as distinct from the general underlying form of the curve. In that connection he desired to repeat a suggestion he made many years ago (*J.I.A.*, vol. xxxv, p. 31) with regard to re-assurances. In practice it was the very large policies which threw out one group as compared with another. Why should not actuaries, for all purposes except one, deal in their cards and in their classification registers with the net sums assured ? That was, in fact, what they needed for all practical purposes. Smaller and more regular figures were thus obtained, and therefore a much more favourable set of conditions for the application of a method of that kind. In any case the re-assurances had to be recorded separately. The distinction between the two methods was that at present the gross figures were valued, the re-assurances were deducted, and the net figures obtained. By the method he suggested the net figures would be valued, which gave the results really required, and for all purposes between valuations or for annual valuations that would be enough in itself. But when they came to the periodical valuations, when they had to bear in mind the requirements of the

Board of Trade, they could then value the re-assurances and add them to the net figures to produce the gross. That, he considered, would really be a useful thing in any case, but, more than that, he thought it was a great help to any method of the kind under discussion.

He desired very cordially to support what Mr. Macnaghten had said in qualification of the author's remark that the great advantage of the method was at the finish. He thought, on the contrary, if he were asked to say where the greatest advantage was, he would put it, as Mr. Macnaghten did, in the simplification of the classification, always the more anxious and the more difficult part of the valuation, rather then in the simplification of the last few hours of the calculation. He also wished to say that he entirely agreed with the remarks made by Mr. Underwood. Actuaries had occasion more and more to feel that a select valuation, if it could be obtained with a reasonable amount of labour, would be a very good thing. It would keep them clear from what might be called an artificial surplus arising, not really from the difference between the actual and expected mortality, using the word "expected" in the proper sense, but rather from the difference between actual and provided-for mortality, which was a very different thing. Unless the new business was exceedingly regular that difference was not at all regular; and when they added to the disturbance so caused the additional disturbance which was often obtained by passing large and varying blocks of business from an O^{M} to an $O^{M(5)}$ standard, the result was that the surplus was very often considerably disturbed. Further, the actuary did not have the same view of the facts as to the actual and expected mortality that he would get with a select valuation. The system of grouping in ten years, as Mr. Underwood pointed out, meant that with practically the same number of groups as in an ordinary valuation without extended grouping a select valuation could be produced. He wished to add to that that he thought in practice it would very probably be a smaller number of groups, because taking the years of assurance separately from 0 to 10, the range of ages, or at any rate the effective range of ages, would be very much smaller than it was in the whole distribution of aggregate facts. It might very well be that in each of those separate years it would be possible to do with a very much smaller number of groups, even possibly, as Mr. Elderton suggested, one or two groups, though, on the other hand, fascinating as this was, it was perhaps a thing that would hardly be likely to be adopted as a practical measure for the whole of a business embracing many millions sterling. Alternatively, if they had no predilection for the select valuation they could very much reduce the number of the groups and have that as a net gain.

He thought that although the author's method might present itself to some minds as giving merely a new, a better, and more powerful check valuation, in the future it would be considered from the point of view of practical politics, as it was known that one office was already considering it, for the actual valuation. It was for that

reason he thought discussions of comparatively unimportant details might be valuable, because they might lead those who had to consider the question to see, first, that a greater degree of accuracy could be obtained if it were wanted, and, secondly and eventually, that it was not wanted at all. If, however, the investigation were not made they might distrust the method from the start. It would be rash to predict that the author's method would in the future displace the ordinary detailed valuations, but should it be found that it could properly do so, and that the only difficulty in the way was the requirements of the Assurance Companies Act, 1909, as to setting out details age by age, then in that case he thought the profession would very seriously have to consider whether they should not press for an alteration to be made, with a view of having those requirements removed.

Mr. F. T. M. BYERS said that group valuations for whole life assurances were, he believed, usually made in quinary groups, and the author had submitted a method with the object of making more extended groupings which he claimed might be substituted for the more detailed existing methods. The author's corrected functions in effect treated the elements for valuation in each group as being equally distributed at each age on either side of the weighted age, whilst the ordinary uncorrected function at the weighted age treated the total of the group as concentrated at that age. The author's method might therefore be expected to give the more accurate results in each group, seeing that the valuation factors were not functions of the first degree. As to the practical results of the method, the experiments he had made confirmed the results given by Mr. King and also what Mr. Macnaghten had stated earlier in the discussion. In the case of whole life assurances with a liability of over £2,600,000, the approximate method showed an excess of only £34 over the valuation made for separate ages, and the differences in each group were quite insignificant. In the case of endowment assurances, with a liability of £300,000, the results of the grouped method showed an excess of £21 only. In that case there were four groupings, the differences in the second and third being quite insignificant. Those in the first and last groups were somewhat larger and in opposite directions, hence the close agreement in the aggregate; but even had those differences been in the same direction, the difference in the aggregate would only have been about ·1 per-cent of the net liability.

In the days when each endowment assurance was valued separately that class absorbed a very large proportion of the time spent in valuation. The group methods of Mr. Lidstone and Mr. Elderton, together with the extended grouping now suggested by Mr. King, seemed to reduce the labour required for the valuation of endowment assurances to even smaller limits than that required for whole life assurances. The author had given an interesting method of obtaining the weighted age for bonuses and premiums from that for the sums assured, but as the rate of increase required in the application of the method would have to be obtained he was

rather doubtful whether any saving of time over the actual weighting for the bonuses and premiums would be effected; the time occupied in weighting on the positive and negative sides of an ordinate near the centre of the group was really extremely small. He thought it would be somewhat risky to adopt the suggestion which he understood Mr. Melville to make, namely, that the difference in the age for the bonuses and premiums, when once calculated, could be carried on from year to year. With groups extending over as many as 10 years, the figures that would have to be dealt with, certainly in the case of net premiums, would be large, and the deviations in value on account of changes might consequently be greater than would be desirable.

The PRESIDENT suggested that it might perhaps be well if the title of the paper were amplified. In these modern days of complex card systems and Hollerith sorting machines, some of the members might have thought that a paper was to be read that evening dealing with a system of valuation whereby the result of a valuation could be found by means of very delicately-poised scales, the cards having had the necessary holes punched in them by the machine prior to being weighed. There was no doubt, however, that Mr. King had placed in their hands an extremely valuable tool, and nothing but well-merited praise had been meted out to it by the various speakers. Mr. Lidstone mentioned the detailed valuation for the Board of Trade Returns. If the author's method or any other short cut was to be used for a detailed valuation, the particulars of which had to be given in the Board of Trade Returns, it of course followed that the actuary must satisfy himself that the results from it were as accurate as those obtained from the method at present adopted in his office, because he had to state in the Board of Trade Returns the exact methods and the rate of interest he adopted; and he must also satisfy his directors that that was the case. Taking a broad view, there was perhaps a slight danger, if approximations were to be admitted for detailed valuations. Approximations might be a source of danger in the hands of unprincipled people. That was the only possible objection he could see to the adoption of such a system as the author had set forth.

Mr. A. E. KING, in replying to the discussion, said that he would touch very shortly upon a few of the points that had been raised. As Mr. Macnaghten had said $\frac{t^2-1}{24}\delta^2 f_\omega$ was to be regarded as an average value of the adjustment, and although it was possible with a little more labour to obtain a more accurate adjustment to suit the circumstances obtaining in any particular group, it seemed to him an advantage to make a general assumption. The expression would then do service for all classes of policies and at all times. The adjustment $\frac{t^2-1}{24}\delta^2 f_\omega$ seemed to him also the simplest one to adopt for the purpose of tabulation. He was not so sure that a curve representing the particulars in the case of endowment assurances would resemble the curve in the case of whole-life assurances, or that

the particulars could be said to proceed by first differences. With reference to the remarks by Mr. Byers and Mr. Melville as to the adjustment to be made to the weighted age found for the sums assured when valuing the bonuses and premiums, he was in agreement with Mr. Byers that it might not be entirely safe to make use of the differences between the weighted ages, found at the time of making a quinquennial valuation, throughout the next five years; as stated in the paper, there were many ways of attacking that Problem, but the method suggested seemed to him to keep more in touch with things as they were at the time of the valuation. In reply to Mr. Elderton's query as to valuation in one group, he had made a rough shot at the value of the sums assured of a company where the liability in respect of sums assured was about £4,700,000. The resulting error by taking one group was only £22,000—about five per mille. He then assumed that the sums assured curve rose by first differences to a maximum about age 55 and then fell by first differences, and he found a correction to allow for that; the error was brought down to £7,000, only 1½ per mille. There was, he thought, a disadvantage in valuing the business in one group, because, if the actuary wished thoroughly to analyze the mortality profit, he could not make the comparison that would have been possible if he had employed eight or nine groups, because he would not easily be able to compare the profit or loss from mortality at the younger ages with that at the older ages.

Graduation by Reduction of Mean Square of Error. By W. F. Sheppard, Sc.D., LL.M.

I.

1. THE data for tables of mortality, &c., usually contain irregularities. In some cases these irregularities are, or may be assumed to be, due to "errors" in the technical sense. The present paper is concerned only with these cases. We assume, *e.g.*, that our data do not present difficulties such as those which arise from a tendency to misstatement of age in a definite direction; though casual misstatements due to imperfect recollection might properly come under the description "errors."

2. We denote our data by $u_1, u_2, \ldots u_N$, and the fundamental assumption is that these correspond to true values $U_1, U_2, \ldots U_N$, from which they differ by errors $e_1, e_2, \ldots e_N$, so that

$$u_r = U_r + e_r.$$

Some of the errors may be positive and others negative.

3. The methods of dealing with the data so as to produce a more regular table fall into two classes.

(i) If we know the law which connects all the values of U, and if this law can be stated by expressing U_r explicitly in terms of r, with some unknown constants, we can find values for these constants by "fitting" the expression to the data as a whole.

(ii) If we do not know the law, we can deal with each value separately by combining it in some way with adjacent values, so as to give an opportunity for positive and negative errors to neutralise one another. It is this latter kind of method alone that is considered in the present paper. The "graduation"—I use the word under protest—of a table in this way consists essentially of a set of operations performed on each term separately; and we have to see under what conditions this process is legitimate and—for a special class of cases—what is the best formula to use for altering each term.

4. Under what conditions then is it legitimate to replace (say) u_3 by such an expression as

$$v_3 \equiv b_0 u_3 + b_1(u_4 + u_2) + b_r(u_5 + u_1);$$

it being assumed that we know nothing about the real relation between U_1, U_2, U_3, U_4, and U_5 (or at any rate between U_3, $U_4 + U_2$, and $U_5 + U_1$)? The answer is that, if we know nothing, even approximately, about this relation, the process is not legitimate under any conditions. In replacing u_3 by v_3 we are really replacing U_3 by

$$V_3 \equiv b_0 U_3 + b_1(U_4 + U_2) + b_2(U_5 + U_1)$$

and the error e_3 by

$$f_3 \equiv b_0 e_3 + b_1(e_4 + e_2) + b_2(e_5 + e_1);$$

and this is not legitimate if we do not know that the difference between U_3 and V_3 may be ignored. But, even if we have no exact knowledge of the relation between the U's, the data may give us some approximate knowledge; for they may show that differences of U_3 after (say) the 2nd are so small

as to be negligible in comparison with the necessary inaccuracies of our ultimate results. Now*

$$V_3 = b_0U_3 + b_1(2U_3 + \delta^2U_3) + b_2(2U_3 + 4\delta^2U_3 + \delta^4U_3)$$

$$= (b_0 + 2b_1 + 2b_2)U_3 + (b_1 + 4b_2)\delta^2U_3 + b_2\delta^4U_3;$$

and therefore, in this case, it is sufficient that we should have

$$b_0 + 2b_1 + 2b_2 = 1, \ b_1 + 4b_2 = 0.$$

Provided that these two conditions are satisfied, we can choose the b's so as to satisfy some third condition. It must of course be remembered that "negligible" is a relative term; δ^4U_3 has some value, and therefore the value given to b_2 must not be so great as to make the product $b_2\delta^4U_3$ cease to be negligible.

The first principle therefore is that (except in special cases) *in order to know whether it is legitimate to replace any* u *by a linear compound of itself and other* u*'s it is necessary to know whether there is a definite order of differences beyond which all differences of the* U*'s may be neglected.* This order of differences will be denoted throughout by j; it need not be the same throughout the whole range of values of u.

5. Suppose then that we replace u_r by a linear compound of m u's, *e.g.* (in most cases) by a symmetrical expression

$$v_r \equiv b_0u_r + b_1u_{r\pm 1} + b_2u_{r\pm 2} + \ldots + b_nu_{r\pm n},$$

where, as usual, $u_{r\pm t}$ means $u_{r+t} + u_{r-t}$; and suppose that differences of the U's above those of order j are negligible. Then v_r can be written in the form

$$v_r = c_0u_r + c_2\delta^2u_r + c_4\delta^4u_r + \ldots + c_{2n}\delta^{2n}u_r;$$

and, if $j = 2k$ or $2k+1$, we may take the b's to have any values we please, provided that

$$c_0 = 1, \ c_2 = 0, \ c_4 = 0, \ \ldots \ c_{2k} = 0.$$

* I use my central-difference notation, in which, if h is the interval between successive values of x,

$$\delta f(x) \equiv f(x + \tfrac{1}{2}h) - f(x - \tfrac{1}{2}h), \quad \mu f(x) \equiv \tfrac{1}{2}\{f(x + \tfrac{1}{2}h) + f(x - \tfrac{1}{2}h)\}.$$

The notation has been explained by Dr. J. Buchanan in *J.I.A.*, vol. xlii, p. 369.

In other words, taking v_r in the second form, $c_0, c_2, \ldots c_{2k}$ must have the above values, and we may give any values we please to

$$c_{2k+2}, c_{2k+4}, \ldots c_{2n}.$$

6. In order to choose the c's (or the b's) so as to give as good a graduation as possible, we must first fix on a criterion of goodness. This is quite arbitrary; and a criterion which is a proper one in one case will not necessarily be a proper one in another case. The criterion used in the present paper is the smallness of the mean square of error of v_r. A definite set of u's being involved, one formula is regarded as being better than another if it gives a less mean square of error; and that formula is regarded as the best—though of course it may not be the most convenient for practical use—which makes the mean square of error a minimum. For any case in which this is not a proper criterion, the results given below do not apply, though the formulæ obtained may still be used as "correct" formulæ—*i.e.* as satisfying the conditions stated in §§ 4 and 5—if they are found to be convenient.

7. If

$$v_r = b_0 u_r + b_1 u_{r+1} + b_2 u_{r+2} + \ldots,$$

the error of v_r is

$$f_r = b_0 e_r + b_1 e_{r+1} + b_2 e_{r+2} + \ldots,$$

and the mean square of error of v_r is

$$\begin{aligned} R^2 = b_0^2\pi_{0,0} &+ 2b_0b_1\pi_{0,1} + 2b_0b_2\pi_{0,2} + \ldots \\ &+ b_1^2\pi_{1,1} \quad + 2b_1b_2\pi_{1,2} + \ldots \\ &\qquad\qquad + b_2^2\pi_{2,2} \quad + \ldots \\ &\qquad\qquad\qquad\qquad + \text{\&c.}, \end{aligned}$$

where $\pi_{s,s}$ is the mean square of error of u_{r+s} and $\pi_{s,t}$ is the mean product of errors of u_{r+s} and u_{r+t}. Our object is to determine the b's so as to make R^2 a minimum; and in order to be able to do this we must know the π's, or at any rate their mutual ratios. Hence, according to our definition of "best", *it is impossible to determine the best graduation-formula until we know how the mean square of error varies along the range of values and how the errors are correlated.*

8. We therefore limit ourselves to the most simple cases, namely, those in which the errors of the u's all have the same

mean square and are independent, and we describe these as being of the "standard" type. Cases which belong absolutely to this class are rare; but (1) there are methods, which lie outside the scope of this Paper, for reducing some other cases to the standard type, (2) it is necessary to consider the simpler cases before we proceed to the more complex, and (3) the criterion mentioned above, in the particular form which it takes in cases of the standard type, has in fact been used before now in the *Journal* as a criterion for comparing different formulæ, though it would seem that these formulæ are intended to be applied to cases in which, on account (more particularly) of the correlation of errors, the criterion in this particular form, is meaningless.

9. The problem, then, is as follows. Taking

$$v = \Sigma bu,$$

where Σ denotes summation for a specified group of u's, we have to determine the b's so as to make

$$R^2 \equiv \Sigma b^2$$

a minimum, subject to the condition that v shall differ from a specified u by differences of u of order exceeding j. This problem is discussed in three papers mentioned in the footnote*, and referred to as "Reduction", "Fitting I", and "Fitting II", respectively; the last of the three has not yet been published. The general result is that the solution of the problem—or of the wider problem in which the quantity under consideration is not one of the u's but is any linear compound of the u's and/or their differences—is the same solution that would be obtained by fitting a polynomial of degree j to the specified group of u's by the method of least squares. The solution is obtained in several different forms, which are capable of further modifications for practical use. We are in the first instance only concerned with the case in which we are obtaining an improved value for u_r by compounding it with the n u's on each side of it. The resulting expression v_r is symmetrical about u_r, so that it only involves u_r and its central differences of even order; and the formulæ are the same for $j=2k+1$ as for $j=2k$.

(i) The formula for v_r in terms of u_r, $\delta^2 u_r$, $\delta^4 u_r$, . . . $\delta^{2k} u_r$, where $j=2k$ or $2k+1$, is obtained in

* "Reduction of Errors by means of negligible differences", *Fifth International Congress of Mathematicians*, Cambridge, 1912, ii, 348–384; "Fitting of Polynomial by method of least squares," *Proceedings of the London Mathematical Society*, 2nd series, xiii, 97–108; and a continuation of the latter, communicated to the London Mathematical Society in February 1914.

"Reduction", § 15, pp. 359–362, and is also given in "Fitting I", § 7 (i), p. 102, by putting $t=0$ in (18) of the latter. The values of the coefficients for values of $m\equiv 2n+1$ up to 19 are given for $k=0, 1, 2$, in "Reduction", Table I, pp. 378–380, under the heading A(i). The formula is not convenient when m is large.

(ii) We know that the method of least squares gives the same result as the method of moments, and that the unknown constants in the polynomial can be expressed in terms of the moments. Since the moments themselves can be expressed in terms of successive sums, it follows that v can be expressed in this way. The solution in this form is given in "Fitting I", § 9, p. 104, by putting $t=0$ in (32). The special feature of this formula is that it is not necessary that the sums should be constructed specially for each v; they can be obtained for the whole series of u's, and the formula is such that the arbitrary constants involved in their construction disappear. The notation is that of central sums, in which σ is the inverse of δ, so that (*e.g.*) the difference of $\sigma u_{3\frac{1}{2}}$ and $\sigma u_{4\frac{1}{2}}$ is u_4, and the difference of $\sigma^2 u_3$ and $\sigma^2 u_4$ is $\sigma u_{3\frac{1}{2}}$. For $2n+1$ terms from u_{-n} to u_n, for instance, the sums with which we are concerned, for $j=2$, would be as follows:

$$\begin{array}{llll}
\sigma^3 u_{-n-\frac{3}{2}} & & & \\
 & \sigma^2 u_{-n-1} & & \\
\sigma^3 u_{-n-\frac{1}{2}} & & \sigma u_{-n-\frac{1}{2}} & \\
 & \sigma^2 u_{-n} & & u_{-n} \\
\sigma^3 u_{-n+\frac{1}{2}} & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots \\
\sigma^3 u_{n-\frac{1}{2}} & \vdots & \vdots & \vdots \\
 & \sigma^2 u_{n} & \vdots & u_{n} \\
\sigma^3 u_{n+\frac{1}{2}} & & \sigma u_{n+\frac{1}{2}} & \\
 & \sigma^2 u_{n+1} & & \\
\sigma^3 u_{n+\frac{3}{2}} & & &
\end{array}$$

In the formula quoted above, r_0 is expressed in terms of $\sigma u_{n+\frac{1}{2}}$, $\sigma^2 u_{n+1}$, $\sigma^3 u_{n+\frac{1}{2}}$, . . . and $\sigma u_{-n-\frac{1}{2}}$, $\sigma^2 u_{-n-1}$, $\sigma^3 u_{-n-\frac{1}{2}}$, . . . ; but the coefficients are smaller if we use $\sigma u_{n+\frac{1}{2}}$, $\sigma^2 u_n$, $\sigma^3 u_{n-\frac{1}{2}}$, . . . and $\sigma u_{-n-\frac{1}{2}}$, $\sigma^2 u_{-n}$, $\sigma^3 u_{-n+\frac{1}{2}}$, . . . The values of the

coefficients of the σ's for values of $2n+1$ up to 31 are given for $j=2$ or 3 in Table 1 (appended to the present Paper), cols. (1)–(4), and for $j=4$ or 5 in Table 2, cols. (1)–(6); for $j=1$ the formula for v is of course the same as for $j=0$, *i.e*, it is the arithmetic mean of the u's. In each case the coefficients are fractional; the common denominator M is given in col. (1) of each table, and the numerators (alternately of opposite signs) in the subsequent columns; and the value of 1/M, by which the ultimate results have to be multiplied, is given in col. (7) of Table 1 and col. (10) of Table 2.

(iii) If we are graduating a long table, the successive sums will become very large. We can meet this difficulty by using the sum-formula for finding the first few v's only, and obtaining the remainder by successive additions of differences. Taking, for instance, $j=2$ or 3, $2n+1=13$, we have from Table 1,

$$143v_7 = -2(\sigma^3 u_{12\frac{1}{2}} - \sigma^3 u_{\frac{1}{2}}) + 11(\sigma^2 u_{13} + \sigma^2 u_1) - 11(\sigma u_{13\frac{1}{2}} - \sigma u_{1\frac{1}{2}}),$$

$$143v_8 = -2(\sigma^3 u_{13\frac{1}{2}} - \sigma^3 u_{2\frac{1}{2}}) + 11(\sigma^2 u_{14} + \sigma^2 u_2) - 11(\sigma u_{14\frac{1}{2}} - \sigma u_{1\frac{1}{2}}),$$

and so on. Hence by taking the successive differences of the v's as calculated we shall have

$$143\delta^3 v_{8\frac{1}{2}} = -2(u_{14} - u_3) + 11(\delta u_{14\frac{1}{2}} + \delta u_{2\frac{1}{2}}) - 11(\delta^2 u_{15} - \delta^2 u_2),$$

and so on. It is therefore sufficient, for $j=2$ or 3, to calculate the first three v's from the sums, derived from the first $2n+3$ u's; the remainder can then be obtained by successive additions. The formula for $\delta^3 v$ can be expressed in various ways; for instance,

$$143\delta^3 v_{8\frac{1}{2}} = -23(u_{14} - u_3) + 33(u_{15} - u_2) - 11(u_{16} - u_1).$$

But, with a little care as to the signs of the differences, the calculation is simpler with the coefficients given in Tables 1 and 2. We can avoid

the alternations of the signs of the coefficients by writing

$$143\delta^3 v_{8\frac{1}{2}} = 2(u_3 - u_{14}) + 11(\delta u_{2\frac{1}{2}} + \delta u_{14\frac{1}{2}}) + 11(\delta^2 u_2 - \delta^2 u_{15}).$$

(iv) We can express v_r in terms of the u's themselves. If, for $j=2k$ or $2k+1$, we write v_r in the form

$$v_r = b_o u_r + b_1 u_{r\pm1} + \ldots + b_n u_{r\pm n},$$

then b_t is a polynomial in t^2 of order k, *i.e.*,

$$b_t = b_o + \tfrac{1}{2}t^2\delta^2 b_o + \frac{1}{24}t^2(t^2-1^2)\delta^4 b_o + \ldots \text{(up to } \delta^{2k}b_o).$$

The values of b_o, $\delta^2 b_o$, . . . , multiplied by M, are given for $j=2$ or 3 in col. (5) and (6) of Table 1, and for $j=4$ or 5 in cols. (7)—(9) of Table 2. The meaning of $\lambda_{0,0}$, $\lambda_{0,2}$, . . . is explained in "Fitting II."

(v) We use R^2 to denote the ratio of the mean square of error of v to that of u. The value of R is given for $k=1$ in col. (8) of Table 1, and for $k=2$ in col. (11) of Table 2; for $k=3$ and $k=4$ the values of R for values of $2n+1$ up to 31 are given in "Reduction", Table II, p. 384.

10. As an example, take the rates of infantile mortality from causes other than diarrhœal diseases. Taking u to be the number of these deaths under one year of age for every 1000 births, the values of u for the 42 years from 1870 to 1911 are*

137, 137, 131, 131, 133, 138, 128, 124, 132, 127, 130,
118, 128, 125, 126, 127, 129, 127, 125, 128, 135, 136,
133, 131, 125, 133, 127, 125, 123, 123, 126, 119, 118,
114, 115, 107, 101, 105, 100, 96, 92, 94.

As the basis of enumeration of deaths is not the year of birth but the year of death, there is probably some negative correlation between consecutive values, but it can hardly be very great. In other respects the methods under consideration appear to be suitable, since the actual numbers are sufficiently large to make the mean-square criterion appropriate, and the mean square of error of u may be taken to vary very slowly from year to year.

* Seventy-fourth Report (1911) of the Registrar-General: Cd. 6578 (1913), p. xxxiv.

(*a*) To see what orders of differences may be neglected, we can take the u's in groups of 6. We get the following:

Period	Sum of u's	1st Diff.	2nd Diff.	3rd Diff.
1870–1875	807			
		−48		
1876–1881	759		+51	
		+3		−28
1882–1887	762		+23	
		+26		−81
1888–1893	788		−58	
		−32		+33
1894–1899	756		−25	
		−57		−29
1900–1905	699		−54	
		−111		
1906–1911	588			

The effect of the grouping is to multiply the U's and their 1st, 2nd, and 3rd differences by (roughly) 6, 36, 216, and 1296. We see that there is an appreciable 2nd difference of U, amounting (numerically) at some points to perhaps ·3, but that the 3rd difference is hardly greater (numerically) than ·05. As we expect to introduce another significant figure into the u's, we must take 2nd differences into account, but the 4th and higher differences may be neglected. We are not concerned at present with the 3rd difference.

(*b*) Now, denoting the u's by $u_1 \ldots u_{42}$, let us improve $u_7 \ldots u_{36}$ by using 13 consecutive u's for each r. The formula has already been given in § 9 (iii); the working is set out in full on pages 183–185, in two portions marked Part I and Part II.

(i) We take $y \equiv u - 110$, to prevent the sums from being too great, and we denote the improved values of u and of y by v and z.

(ii) Part I shows the calculation of $z_7, z_8, \ldots z_{12}$, each multiplied by 143. The y's are given in col. (1), and the successive sums in cols. (2)–(4). For reasons connected with the calculation (not dealt with in the present paper) of $z_1, z_2, \ldots z_6$, the sums derived from the first 13 y's are obtained by the Hardy method* of continued subtraction, subsequent sums being obtained by the ordinary method of continued addition; the Hardy sums in cols. (2) and (4), if headed σy and $\sigma^3 y$, are really negative, and are therefore printed in italic type. The upper term in each column of sums is ignored in forming the next column, since the formula for v_7 involves $\sigma u_{\frac{1}{2}}$, $\sigma^2 u_1$, and $\sigma^3 u_{1\frac{1}{2}}$.

* W. P. Elderton, *Frequency-curves and Correlation* (1906), p. 19.

Cols. (5)-(7) give the results of taking terms in pairs from cols. (2)-(4). In the headings of these columns, [ ] denotes algebraical difference, and { } algebraical sum; but, since the upper terms in cols. (2) and (4) are negative, the arithmetical sum is taken in each case. Each row in the 3 columns can be written down by covering a portion of cols. (2)–(4) with a movable piece of paper, as shown by the heavy lines; a set of entries is printed in heavy type in order to show how they correspond.

Cols. (8)–(10) are subsidiary columns, the entries being respectively $\{\sigma^2 y\} - [\sigma y]$, 11 times this, and $2\,[\sigma^3 y]$; and col. (11) gives the difference of the last two, which is $143z = -2\,[\sigma^3 y] + 11\,\{\sigma^2 y\} - 11\,[\sigma y]$. Only 3 values are required; but 6 are given, as a check on the next part of the work.

(iii) Part II shows the calculation of $\delta^3 z$, as explained in § 9 (iii), up to years 1903/4, and the resulting values of $10v$ up to year 1905. (For proper spacing, it has been necessary to spread this over two pages, and this involves some repetition of figures). The process for obtaining col. (10) is the same as for col. (11) of Part I, except that, as differences take the place of sums, the movable piece of paper has to be inverted, and that there are some changes of sign. For cols. (4) and (6) the lower term in col. (1) or (3) is subtracted from the higher; col. (7) is the sum of cols. (5) and (6); and col. (10) is the sum of cols. (8) and (9). Introducing the values obtained in Part I into col. (13), differencing towards the left, and then summing successively towards the right from col. (10), we complete col. (13); and the values of $10v$, as shown in ordinary type in col. (14), are obtained by multiplying by ·0699300699 and adding 1100. The explanation of the values in italic type is left over for a subsequent Paper.

If the original mean square of error is a^2, the new mean square of error is R^2a^2, where $R = \sqrt{(25/143)} = {\cdot}418$. By Woolhouse's method, which uses 15 u's instead of 13, the value of R would be $\sqrt{({\cdot}179264)} = {\cdot}423$.

TABLE 1.—*Improvement of central terms* ($j=2$ or 3).

$$v_r = A(\sigma u_{r+n+\frac{1}{2}} - \sigma u_{r-n-\frac{1}{2}})$$
$$+ B(\sigma^2 u_{r+n} + \sigma^2 u_{r-n})$$
$$+ C(\sigma^3 u_{r+n-\frac{1}{2}} - \sigma^3 u_{r-n+\frac{1}{2}})$$
$$= b_0 u_r + b_1(u_{r+1} + u_{r-1}) + \ . \ . \ . \ + b_n(u_{r+n} + u_{r-n}).$$
$$b_t = b_0 + \tfrac{1}{2} t^2 \delta^2 b_0.$$
$$R^2 = b^2{}_0 + 2b^2{}_1 + 2b^2{}_2 + \ . \ . \ + 2b^2{}_n.$$

$M = 2n+1$	$M =$ multiplier	Mv_r in terms of Sums			Mv_r in terms of u's		1/M	$R = \sqrt{\lambda_{0,0}}$
		MA	MB	MC	$Mb_0 = M\lambda_{0,0}$	$M\delta^2 b_0 = M\lambda_{0,2}$		
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		−	+	−	+	−		
3	1	0	1	2	1	2	1·	1·
5	35	3	15	10	17	10	·[1] 285714286	·697
7	21	2	5	2	7	2	·[1] 476190476	·577
9	231	21	35	10	59	10	·[2] 432900433	·505
11	429	36	45	10	89	10	·[2] 233100233	·455
13	143	11	11	2	25	2	·[2] 699300699	·418
15	1105	78	65	10	167	10	·[3] 904977376	·389
17	323	21	15	2	43	2	·[2] 309597523	·365
19	2261	136	85	10	269	10	·[3] 442282176	·345
21	3059	171	95	10	329	10	·[3] 326904217	·328
23	805	42	21	2	79	2	·[2] 124223602	·313
25	5175	253	115	10	467	10	·[3] 386473430	·300
27	1305	60	25	2	109	2	·[3] 766283525	·289
29	8091	351	135	10	629	10	·[3] 123594117	·279
31	9889	406	145	10	719	10	·[3] 101122459	·270

NOTE.—In col. (7) the number in [ ] is the number of 0's after the decimal point.

TABLE 2.—*Improvement of central terms* ($j = 4$ *or* 5).

$$
\begin{aligned}
v_r = {} & \mathrm{A}(\sigma u_{r+n+\frac{1}{2}} - \sigma u_{r-n-\frac{1}{2}}) \\
& + \mathrm{B}(\sigma^2 u_{r+n} + \sigma^2 u_{r-n}) \\
& + \mathrm{C}(\sigma^3 u_{r+n-\frac{1}{2}} - \sigma^3 u_{r-n+\frac{1}{2}}) \\
& + \mathrm{D}(\sigma^4 u_{r+n-1} + \sigma^4 u_{r-n+1}) \\
& + \mathrm{E}(\sigma^5 u_{r+n-\frac{3}{2}} - \sigma^5 u_{r-n+\frac{3}{2}}) \\
= {} & b_0 u_r + b_1(u_{r+1} + u_{r-1}) + \ \ldots \ + b_n(u_{r+n} + u_{r-n}).
\end{aligned}
$$

$$b_t = b_0 + \frac{1}{2} t^2 \delta^2 b_0 + \frac{1}{24} t^2(t^2 - 1)\delta^4 b_0.$$

$$\mathrm{R}^2 = b^2_0 + 2b^2_1 + 2b^2_2 + \ \ldots \ + 2b^2_n.$$

≡ +1	M = multiplier	$\mathrm{M}v_r$ in terms of Sums					$\mathrm{M}v_r$ in terms of u's			1/M	R = $\sqrt{\lambda_{0,0}}$
		MA	MB	MC	MD	ME	$\mathrm{M}b_0 = \mathrm{M}\lambda_{0,0}$	$\mathrm{M}\delta^2 b_0 = \mathrm{M}\lambda_{0,2}$	$\mathrm{M}\delta^4 b_0 = \mathrm{M}\lambda_{0,4}$		
)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		+	−	+	−	+	+	−	+		
5	1	0	0	1	3	6	1	2	6	1·	1·
7	231	5	35	140	189	126	131	112	126	·[2] 432900433	·753
9	429	15	70	155	135	54	179	88	54	·[2] 233100233	·646
1	429	18	63	98	63	18	143	46	18	·[2] 233100233	·577
3	2431	110	308	371	189	42	677	154	42	·[3] 411353353	·528
5	46189	2145	5005	4928	2079	378	11063	1876	378	·[4] 216501764	·489
7	4199	195	390	325	117	18	883	116	18	·[3] 238151941	·459
9	7429	340	595	430	135	18	1393	146	18	·[3] 134607619	·433
1	260015	11628	18088	11543	3213	378	44003	3766	378	·[5] 384593197	·411
3	2185	95	133	76	19	2	337	24	2	·[3] 457665904	·393
5	30015	1265	1610	833	189	18	4253	256	18	·[4] 333166750	·376
7	930465	37950	44275	20930	4347	378	121943	6286	378	·[5] 107473145	·362
9	445005	17550	18900	8225	1575	126	54251	2422	126	·[5] 224716576	·349
1	29667	1131	1131	456	81	6	3381	132	6	·[4] 337074864	·338

NOTE.—In col. (10) the number in [ ] is the number of 0's after the decimal point.

Example 1.—*Infantile Mortality.*

$u \equiv$ number of deaths under 1 *year of age, from causes other than diarrhœal diseases, for every* 1000 *births.*

Graduation of u by 13 *terms* $(j=2)$.

Part I—(*Commencement of central graduation*).

Year	y	σy	$\sigma^2 y$	$\sigma^3 y$	$[\sigma y]$	$\{\sigma^2 y\}$	$[\sigma^3 y]$	...	...	...	$143\,z$
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		264									
1870	27		1403								
		237		*4822*							
1	27		1166								
		210		*3656*							
2	21		**956**								
		189		**2700**							
3	21		767								
		168		*1933*							
4	23		599								
		145		*1334*							
5	28		451								
		117		*880*							
6	18		337		261	1403	4822	1139	12529	9644	2885
		99		*543*							
7	14		238		252	1166	3656	914	10054	7312	2742
		85		*305*							
8	22		153		**241**	**971**	**2700**	730	8030	5400	2630
		63		*152*							
9	17		90		237	813	1948	576	6336	3896	2440
		46		*62*							
1880	20		41		235	693	1395	458	5038	2790	2248
		26		*18*							
1	8		18		229	615	1035	386	4246	2070	2176
		18		0							
2	18		0								
		0		0							
3	15		0								
		15		**0**							
4	16		**15**								
		31		15							
5	17		46								
		48		61							
6	19		94								
		67		155							
7	17		161								
		84									

EXAMPLE 1—(*continued*).

PART II—(*Graduation and final Table*).

Year	u	δu	$\delta^2 u$	$-[u]$	$\{\delta u\}$	$-[\delta^2 u]$				143 × $\delta^3 z$	143 × $\delta^2 z$	143 × δz	143 × z	$10v$
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1870	137													*1365*
		0												
1	137		− 6											*1354*
		− 6												
2	131		+ 6											*1343*
		0												
3	131		+ 2											*1332*
		+ 2												
4	133		+ 3											*1322*
		+ 5												
5	138		−15											*1312*
		−10												
6	128		+ 6										2885	1302
		− 4										−143		
7	124		+12								+ 31		2742	1292
		+ 8		+ 6	− 5	− 6	−11	−121	+12	−109		−112		
8	132		−13								− 78		2630	1284
		− 5		+ 5	+ 1	+ 5	+ 6	+ 66	+10	+ 76		−190		
9	127		+ 8								− 2		2440	1271
		+ 3		+ 6	+ 4	+ 6	+10	+110	+12	+122		−192		
1880	130		−15								+120		2248	1257
		−12		+ 9	+ 3	+ 3	+ 6	+ 66	+18	+ 84		− 72		
1	118		+22								+204		2176	1252
		+10		+ 1	−12	−20	−32	−352	+ 2	−350		+132		
2	128		−13								−146		2308	1261
		− 3		− 1	− 1	+ 2	+ 1	+ 11	− 2	+ 9		− 14		
3	125		+ 4								−137		2294	1260
		+ 1		+ 4	+15	+18	+33	+363	+ 8	+371		−151		
4	126		0								+234		2143	1250
		+ 1		− 8	− 4	− 9	−13	−143	−16	−159		+ 83		
5	127		+ 1								+ 75		2226	1256
		+ 2		− 6	0	+ 7	+ 7	+ 77	−12	+ 65		+158		
6	129		− 4								+140		2384	1267
		− 2		−15	−14	−11	−25	−275	−30	−305		+298		
7	127		0								−165		2682	1288
		− 2		− 3	+ 4	+ 8	+12	+132	− 6	+126		+133		
8	125		+ 5								− 39		2815	1297
		+ 3		0	+ 5	+ 1	+ 6	+ 66	0	+ 66		+ 94		
9	128		+ 4								+ 27		2909	1303
		+ 7		− 7	− 5	0	− 5	− 55	−14	− 69		+121		
1890	135		− 6								− 42		3030	1312
		+ 1		0	− 1	0	− 1	− 11	0	− 11		+ 79		
1	136		− 4								− 53		3109	1317
		− 3										+ 26		
2	133		+ 1										3135	1319
		− 2												
3	131		− 4											
		− 6												
4	125		+14											
		+ 8												
5	133		−14											
		− 6												
6	127		+ 4											
		− 2												
7			0											

EXAMPLE 1—(*continued*).

PART II—(*Graduation and final Table*)—(*continued*).

Year	u	δu	$\delta^2 u$	$-[u]$	$\{\delta u\}$	$-[\delta^2 u]$				143 ×				$10v$
										$\delta^3 z$	$\delta^2 z$	δz	z	
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1885			+ 1											
		+ 2												
6	129		− 4											
		− 2												
7	127		0											
		− 2												
8	125		+ 5											
		+ 3												
9	128		+ 4											
		+ 7												
1890	135		− 6											
		+ 1								− 11				
1	136		− 4								− 53			
		− 3		+ 4	0	− 1	− 1	− 11	+ 8	− 3		+ 26		
2	133		+ 1								− 56		3135	1319
		− 2		+ 4	− 2	− 7	− 9	− 99	+ 8	− 91		− 30		
3	131		− 4								− 147		3105	1317
		− 6		+ 2	+ 1	+ 10	+ 11	+ 121	+ 4	+ 125		− 177		
4	125		+ 14								− 22		2928	1305
		+ 8		+ 2	− 4	− 1	− 5	− 55	+ 4	− 51		− 199		
5	133		− 14								− 73		2729	1291
		− 6		+ 16	+ 6	+ 7	+ 13	+ 143	+ 32	+ 175		− 272		
6	127		+ 4								+ 102		2457	1272
		− 2		+ 18	− 3	− 11	− 14	− 154	+ 36	− 118		− 170		
7	125		0								− 16		2287	1260
		− 2		+ 19	− 2	+ 5	+ 3	+ 33	+ 38	+ 71		− 186		
8	123		+ 2								+ 55		2101	1247
		0		+ 16	− 10	− 1	− 11	− 121	+ 32	− 89		− 131		
9	123		+ 3								− 34		1970	1238
		+ 3		+ 18	− 12	− 14	− 26	− 286	+ 36	− 250		− 165		
1900	126		− 10								− 284		1805	1226
		− 7		+ 32	+ 12	+ 23	+ 35	+ 385	+ 64	+ 419		− 449		
1	119		+ 6								+ 165		1356	1195
		− 1		+ 22	− 11	− 15	− 26	− 286	+ 44	− 242		− 284		
2	118		− 3								− 77		1072	1175
		− 4		+ 25	− 6	+ 4	− 2	− 22	+ 50	+ 28		− 361		
3	114		+ 5								− 49		711	1150
		+ 1		+ 27	− 6	− 6	− 12	− 132	+ 54	− 78		− 410		
4	115		− 9								− 127		301	1121
		− 8										− 537		
5	107		+ 2										− 236	1083
		− 6												
6	101		+ 10											*1055*
		+ 4												
7	105		− 9											*1026*
		− 5												
8	100		+ 1											*998*
		− 4												
9	96		0											*970*
		− 4												
1910	92		+ 6											*942*
		+ 2												
1	94													*914*

The Medico-Actuarial Investigation of the Mortality of American and Canadian Life Assurance Companies.

[Communicated by Mr. Arthur Hunter, A.I.A., F.F.A., Chairman of the Central Bureau.]

VOLUME IV* of the Report on the Medico-Actuarial Mortality Investigation deals with the effect on mortality of:

1. Defects in physical condition, which were found to exist at date of application for insurance.
2. Defects in personal history.
3. Defects in family history.

The term "Medical Impairment" or "Impairment" is used in the Report to cover all of these.

The classes dealt with in the volume consist solely of policies issued on the lives of men.

One of the main points to be kept in mind in reading the Report is that all cases which by reason of physical condition or of personal history had been treated as sub-standard, were excluded from the investigation; but policies were included under which the insured were treated as sub-standard on account of family history alone. Cases were considered as sub-standard if the policies were issued with an extra premium, or with a lien, or were placed in a special dividend class; but those which were simply limited to Endowment insurance were not considered as sub-standard.

If any of the "Required" or "Optional" occupations (see Volume III) were recorded on the mortality card, the case was excluded from the classes on which a report is now made, as such occupations were generally expected to show a higher mortality than the normal. It may therefore be assumed that in the impairment cases there was no hazard from occupation. Again, on a small proportion of the cards there were two or more impairments recorded, but these were of minor importance, as otherwise the risks would have been treated as sub-standard. Such cases were excluded.

Furthermore, in most of the impairment classes a mortality heavier than the normal was expected among lives which in

* For Abstracts of the contents of the three preceding Volumes see *J.I.A.*, vol. xlvi, pp. 381–394, and vol. xlvii, pp. 414–421 and 539–548.

other respects were "standard." In order to offset the effect of any given medical impairment and so maintain an approximately normal mortality, standards of fitness in other respects higher than the average must have been maintained.

For these reasons the mortality ratios in the classes recorded in Volume IV cannot be regarded as measuring the effect or the absence of effect of the medical impairment upon the mortality. In some groups the mortality ratio is less than 100 per-cent, but the causes of death indicate that the impairment was prejudicial to longevity.

In many of the classes a higher standard of selection was maintained by the Companies on account of the impairment, and this had an important effect on the mortality. It should therefore be remembered that Volume IV records the mortality experience of classes which are generally superselect except for the impairment in question. Thus, the vast majority of persons with a history of blood-spitting within five years of application were undoubtedly refused acceptance as standard risks; among the comparatively few cases accepted the mortality was 151 per-cent of the normal, but it would be wrong to assume that an extra premium or a lien to cover this excess mortality would justify the free acceptance of such lives in the future.

It is advisable to read the comments of the Committee before drawing deductions from the data, otherwise errors may be made. For example, it is stated in the comments that the class of those with irregular pulse consisted largely of insured having this impairment at the first, but not on subsequent examinations, and the low mortality was undoubtedly due to this cause. Another illustration is that of albumen in the urine. In this impairment the condition was apparently temporary, as in the majority of cases it was only found in one of several tests.

The expected deaths were calculated by the M.A. Table, which represents the average mortality of the forty-three companies in the investigation. This table was based on policies issued on "standard" lives during the years 1885 to 1908 inclusive—the same period as was covered by the investigation of medical impairments.

The standard or normal death rates for the various causes of death were based upon the records of 17,792 policies terminated by death (Vol. II, pp. 26–33). The death rate from each cause to 10,000 exposed to risk is used, and not the proportion of deaths from each cause to the total deaths.

The following summary of the results under the principal impairments is given :

Syphilis.—There were four classes of insured who had a history of syphilis, but two of them were consolidated. The statistics were divided according to time elapsed since the attack. A summary of the results in the three groups is given in the following table :

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Syphilis, surely; thoroughly treated; two years' continuous treatment and one year's freedom from symptoms...	100	53·3	188%
Syphilis, surely; not thoroughly treated, or no details of treatment given ...	174	100·1	174%
Syphilis, doubtful...	67	48·7	138%

In commenting on the results of the foregoing, the Committee mentions that the experience of a large American company on its underaverage business issued in recent years, shows a distinctly lower mortality than in the foregoing classes. An investigation made, however, by Dr. Gollmer of the Gotha Insurance Company, confirms the Medico-Actuarial Experience. On 1,778 men insured in that Company from 1852 to 1904, the relative mortality was 168 per-cent (actual deaths 487, expected 290) among those who had received treatment for syphilis.

Albumen or Sugar in the Urine. An investigation of those cases in which either albumen or sugar was found on examination, or in which there was a history within ten years of application, shows a relative mortality slightly in excess of the normal. This was due to the fact that as the companies expected a mortality higher than the normal, a small and carefully selected proportion of the applicants only were accepted with either of these impairments. The custom of the companies was generally to decline cases in which sugar or albumen was found on examination unless only one out of several specimens had a trace, and unless there was no history in the past of sugar or albumen having been found.

The Committee reported that a company doing a large under-average business had a mortality of 50 per-cent above the normal in cases in which albumen was found in more than one

specimen at date of examination, and fully double the normal when sugar was found. The distinction between the experience in this company's underaverage business, and that in the Medico-Actuarial Investigation is that the former referred to groups with *persistent* albumen or sugar, and the latter to groups in which the impairment was intermittent or temporary in most cases.

It is significant that the death rate from diabetes in the M.A. class, in which sugar was found on examination, or where there was a history within ten years of date of application, was six times the normal death rate from that cause; and in the class in which albumen was found on examination, or where there was a history within ten years of application, the death rate from Bright's disease was more than three times the normal.

Blood Spitting without a Distinct History of Tuberculosis of the Lungs.

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
One attack within five years of application	46	30·4	151%
One attack within five to ten years of application	63	48·2	131%
One or more attacks, the last one at least ten years prior to application ...	223	219·6	102%

In the first group mentioned above the death rate from tuberculosis of the lungs was fully five times the normal; in the second group, three times; and in the third group, twice the normal. The great majority of people with a history of blood-spitting within a few years of date of application were undoubtedly refused insurance as standard risks, and the foregoing mortality is therefore the result of accepting a small and very carefully selected proportion of the applicants.

Habits as to Alcohol.

There were several classes relating to habits as to alcohol, and each class was divided into a number of groups, but a synopsis only is now given :

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Occasional excessive use of alcohol ...	289	190·5	152%
Steady, free use of alcohol	2,423	1,834·7	132%
Reformed from intemperate habits without treatment	377	286·0	132%
Took cure for alcoholic habits, total abstainer since cure...	79	58·4	135%

The statistics in the second class were divided into two sections, according to whether a conservative or a liberal interpretation of the expression "Steady, free use" had been adopted. In the latter section were placed the risks from those companies which used a standard as high as or higher than Anstie's limit of two ounces of alcohol per day; and in the former, all the companies which used less than Anstie's limit. (This limit is the equivalent of about one pint of champagne, five tumblersful of beer, or three glasses of strong wine.) The following are the results of this investigation:

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Conservative interpretation = steady but moderate users of alcohol	1,725	1,460·3	118%
Liberal interpretation = steady, free, but not immoderate users of alcohol ...	698	374·4	186%

It is evident from the foregoing that anything but a very moderate use of alcohol is detrimental to longevity.

In the foregoing classes the death rate was generally high from cirrhosis of the liver, Bright's disease, pneumonia, and suicide; and also from diabetes in several of the sub-divisions.

Pleurisy other than Purulent.

Certain of the groups have been combined in the following summary:

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
One attack within five years of application	490	334·5	146%
One attack more than five years prior to application	781	797·4	98%

In the past a history of pleurisy has not been looked upon as important by the companies, and, accordingly, little attention has been paid to it; but the foregoing results may change the practice in this respect.

The death rate from tuberculosis of the lungs in the first group was three times the normal; twice the normal if pleurisy had occurred between five and ten years prior to date of application; and normal when the attack occurred at least ten years prior to date of application. This is excellent evidence that there is a direct connection between pleurisy and tuberculosis.

Acute Articular Rheumatism (Muscular Rheumatism Excluded).

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
One attack within five years of application	972	807·4	121%
Two or more attacks within five years of application	163	138·9	118%

The cases in the second group were undoubtedly more carefully selected than in the first group.

The death rate from organic disease of the heart was markedly above the normal, especially at the younger ages of entry, where it was three times the standard.

Fistula in Ano, with or without Operation.

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
One attack within five years of application	229	178·7	128%
One attack between five and ten years of application	98	97·8	100%

The data were divided into overweights and underweights, the former showing little effect of fistula on the mortality. Among the insured more than 15 per-cent underweight, however, there was a higher mortality than would appear among a corresponding group of underweights without the impairment in question. Out of 30 deaths among these light weights, 13 were due to tubercular disease.

Pulse—Irregular, Intermittent, or Rapid.

In the two classes of those who had irregular or intermittent pulse on examination, the mortality of the former was 95 per-cent, and of the latter 113 per-cent of the standard. An investigation of the records showed that in the great majority of cases the irregular or intermittent pulse had been found only in the first of two or more examinations for the same policy. The classes largely consisted, therefore, of cases in which the condition was temporary. The company already referred to as insuring large groups of underaverage lives had a mortality of 50 per-cent in excess of the normal among persons who had a *persistently* irregular or intermittent pulse.

A summary of the results in two groups of insured with high pulse rate follows :

High Pulse Rate.

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Pulse rate 90 to 100 beats per minute, found on examination	332	193·0	172%
Pulse rate over 100 beats per minute, found on examination or within ten years of examination	49	23·9	205%

In the above groups the death rate from heart disease and pneumonia, and especially from tuberculosis of the lungs, was high.

Asthma.

A summary of two groups in this class is given :

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
One attack within five years of application	98	86·9	113%
Two or more attacks within five years of application	176	140·3	125%

The death rate was slightly higher than the normal from pneumonia and other respiratory diseases, but practically normal from tuberculosis. The deaths from asthma and pulmonary emphysema were 6 per-cent of the total—many times the normal.

Appendicitis Without Operation.

In this class the deaths were distinctly less than 100 per-cent of the expected, and this was probably due to extreme care in selection. A significant fact is that 10 per-cent of the total deaths were from appendicitis and typhlitis—three times the normal death rate from these causes.

Stricture of the Urethra.

There are three groups in this class, but only a summary is now presented :

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
One attack within ten years of application	194	170·6	114%

The relative mortality was distinctly higher at the older than at the younger ages of entry.

Suppurative Middle Ear Disease, Otorrhœa, Discharge from Ear.

The relative mortality of those in the Specialized Mortality Investigation who had had otorrhœa was 95 per-cent of the present standard, but the material was not divided according to number of attacks, or time elapsed since last attack. In the present investigation there was not a large difference between the mortality of three groups, divided as to time elapsed since attack, and, accordingly, these groups have been combined in this synopsis :

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
One attack within five years of application	138	180·6	76%
Two or more attacks, the last within two years of application	46	39·6	116%

It is not to be inferred from the foregoing that those who have had one attack of otorrhœa are much better risks than the average. A very careful selection probably accounts for the favourable experience.

Abdominal Girth greater than Chest Expanded.

This large class was divided into the following groups:

Group	Abdominal Girth greater than Chest expanded	Group	Departure from Average Weight at age 37
A	0 to 1 inch	0	− 5% to + 5%
B	$1\frac{1}{4}$,, 2 inches	1	+ 5% ,, +15%
C	$2\frac{1}{4}$,, 3 ,,	2	+15% ,, +25%
D	over 3 ,,	3	+25% ,, +35%
		4	+35% ,, +50%
		5	over 50%

The following shows the results of the sub-division by weight, the four abdominal girth groups being combined:

Build Group	Approximate Weight	Ratio to M.A. Table
0	Between 5% underweight and 5% overweight	98%
1	,, 5% and 15% overweight	114%
2	,, 15% and 25% ,,	129%
3	,, 25% and 35% ,,	136%
4	,, 35% and 50% ,,	165%
5	More than 50% overweight	228%

The smallest of the foregoing groups contained 97, and the largest 986 deaths.

In order to determine the effect of large abdominal girth, expected deaths were calculated by a special table, giving effect to overweight. In Volume II. there appears the relative mortality of overweights, irrespective of whether the abdominal girth is greater or less than the chest expanded. All the cases in the present class are included in the statistics of overweights in Volume II. Unfortunately, this process only shows the excess mortality among those with large abdominal girth, over a group of lives of the same ages and with the same percentage of overweight, among whom there is an unknown proportion with abdominal girth greater than chest expanded. (In weight groups 3 and 4 this proportion was estimated to be 13 per-cent.) The results of this part of the investigation are:

Ratio of Actual to Expected Deaths by "Special" Table, giving effect to Weight.

Build Groups 2, 3, 4 and 5		Girth Groups A, B, C and D	
Girth Groups A and B ...	105%	Build Groups 0 and 1 ...	108%
,, ,, C ,, D ...	109%	,, ,, 2 ,, 3 ...	106%
		,, ,, 4 ,, 5 ...	107%

The following deductions were drawn from the various tables in the report on this class :—

First, the relative mortality of those with large abdominal girth is greater than the already heavy mortality found to exist among the general body of those of corresponding weight ;

Second, the relative mortality increases slightly with increasing excess of abdominal girth, notwithstanding the fact that the greater such excess the more rigid was the selection by the companies ;

Third, the excess mortality over that among overweights as a whole was practically a constant percentage of the latter.

In a company which issued sub-standard policies with a lien or an extra premium, and which made no more rigid selection with the very heavy weights than with those of average weight, the mortality was markedly heavier among insured with excessive abdominal girth than among those of the same weight whose abdominal girth was not more than that of the chest expanded. This result was decidedly different from the M.A. experience on "standard" lives, and confirms the conclusion that the selection was very rigid with persons of large girth.

A study of the causes of death shows the same conditions as were found in the investigation among overweights (Volume II). The heavier the relative weight, generally speaking, the smaller is the death rate from tuberculosis of the lungs ; while the death rate from diabetes, apoplexy, and pneumonia increases with the percentage overweight. The death rate from heart disease and from Bright's disease increased very rapidly with the increase in weight. In the heaviest weight group, for example (No. 5), it was four times as great as in the average weight group (No. 0). The death rate from heart disease and cancer increases with the increase of abdominal girth.

Renal Colic, Gall Stones, &c.

The companies were first requested to prepare classes of (1) Renal colic or calculus, and (2) Gall stones or hepatic colic; but a further subdivision was subsequently made, a class with gravel being separated from (1), and a class of bilious colic from (2). The details are given according to time elapsed since the attack, and number of attacks, but a summary only appears in the following:

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Renal Colic or Calculus:			
One attack	365	346·0	105%
Two or more attacks	52	44·9	116%
Gravel:			
One attack	91	90·3	101%
Gall Stones or Hepatic Colic:			
One attack	136	104·8	130%
Two or more attacks	27	13·8	195%
Bilious Colic:			
One attack	36	28·1	128%

The relative mortality among men who have had a history of one attack of gravel is practically the same as among those with a history of renal colic or calculus; while the mortality among men with a history of one attack of bilious colic is practically the same as among those with a history of one attack of gall stones or hepatic colic.

In the foregoing classes the death rate from Bright's disease was much higher than the normal.

Defective Family History.

There were investigated four classes, involving two or more deaths in the family from heart disease, insanity, apoplexy or cancer. The results of the two largest classes appear in the following summary:

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Apoplexy or Paralysis in Family Record:			
Two or more deaths from apoplexy or paralysis in family record ...	374	347·3	108%
Heart Disease in Family Record:			
Two or more cases of heart disease in family record	233	207·0	113%

The death rate from cerebral hæmorrhage, apoplexy, and suicide was twice the normal in the first class; and from organic disease of the heart was distinctly above the normal in the second class.

Volume IV does not cover all the impairments under investigation. In the succeeding volume will appear the results of two large and important classes—a personal history of malaria, and a family record of tuberculosis. There are fully half a million cases in these two classes, which will justify sub-division of the malarial cases according to habitat, and of the cases with tubercular family history according to weight, height, and plan of insurance. This volume will probably also include reports on mortality in certain Southern States, and on Joint Life Policies.

LEGAL NOTES.

By William Charles Sharman, F.I.A., *Barrister-at-Law.*

Claim to Statutory Deposit on Liquidation of Company.

THE question as to whether in the winding-up of an insurance company the claims of the company's creditors under its fire and accident policies are payable in priority to the claims of general creditors out of the deposit of £20,000 required from every insurance company by s. 2 (1) of the Assurance Companies Act, 1909, was decided in the case of *In re The British Union and National Insurance Company* (*Limited*), 30 T.L.R. 290.

This was a summons taken out by the liquidator of the British Union and National Insurance Company (Limited), which was registered on 24 November 1909, for the purpose of carrying on every kind of insurance business, including the granting of annuities. In pursuance of the requirements of the Life Assurance Companies Acts, 1870—1872, £20,000 was paid into Court in the name of the Company as security for the life policyholders.

An order having been made for the winding-up of the Company, it appeared that claims of life assurance policyholders had been satisfied or provided for, and that the liquidator of the company took out this summons to determine whether the claims of the company's creditors under its fire and accident insurance policies ought to be satisfied out of the deposited

fund in priority to the claims of the general creditors of the company.

For the fire and accident policyholders it was contended that the object of the deposit was the general security of the policyholders. Sections 31 (*d*) and 32 (*c*) of the Assurance Companies Act, 1909, exempted companies from paying in further money in respect of fire and accident insurance where a deposit of £20,000 had once been made, but could not be intended to take away all security and all interest in the fund from fire and accident policyholders. Had it been intended by the Legislature that only life policyholders should have that interest, the Act of 1909 would have limited the obligation to make this deposit to companies doing life business, but in fact, by section 2 of the Act, all companies were liable.

For the general creditors, it was argued that the Assurance Companies Acts of 1870 and 1872 had given no interest under this deposit to fire and accident policyholders, and that the Act of 1909 had not in any way modified these provisions. It was clear that the Legislature contemplated varying obligations of increasing value under life policies, obligations which could not be determined at the will of either party in the same way as in the case of fire and accident policies. Sections 30 (*c*) and 31 (*b*) of the Act of 1909, showed that this deposit was for the security of life policyholders alone.

Astbury, J., said : "that in his opinion, the fire and accident "policyholders were not entitled to their claims for priority "over the general creditors. Looking at the general intention "of the Acts, it was seen that an existing company which had "made the deposit in respect of life assurance, need not deposit "any other sum in connection with the fire and accident busi-"ness, but the sum of £20,000 so deposited must be kept to "the credit of the life fund. He could find nothing in the Act "of 1909 which said that that particular fund should be subject "to any charge in favour of the fire and accident policyholders."

Covenant in Marriage Settlement to settle policies which may subsequently be effected.

The case of *In re Hewett Hewett* v. *Eldridge*, reported in the *Law Journal* of 3 January 1914, is concerned with the question as to whether, when a settlor has covenanted to settle any other policy of insurance which he might effect, and subsequently effects a policy upon his own life under the Married Women's Property Act,

1882, the trustees of which are also the trustees of the settlement, volunteers claiming under the settlement can insist upon the trustees holding the policy monies upon the trusts of the settlement, the trusts of the policy having failed. The material facts were as follows :—

On 30 January 1893, the plaintiff made a marriage settlement and thereby settled some existing endowment policies upon trusts to receive such of the policy monies as should become payable during his life, and to invest the same and pay the income to himself during the rest of his life, and after his death to his wife for life, and after her death upon trusts for issue of the marriage, with remainders over in favour of the plaintiff's sisters. The settlement contained a covenant by the plaintiff to settle any other policy of insurance he might effect.

On 16 October 1895, the plaintiff effected an endowment policy on his own life under the Married Women's Property Act, 1882, for £1,000 with profits payable on 4 September 1910 to the plaintiff or the trustees appointed by him. By the conditions of the policy a trust was created in favour of the wife of assured for her separate use if she should survive him, and subject thereto the policy monies were to form part of the estate of assured. In 1910, after the £1,000 and profits assured under this policy had become payable, the plaintiff appointed the same gentlemen who were the then trustees of his marriage settlement to be the trustees to receive the policy monies, and the sum of £1,280 was paid to them. The plaintiff's wife died on 3 October 1912. There was no issue of the marriage. The plaintiff took out a summons for a declaration that the £1,280 should be held by the trustees in trust for himself and not subject to the trusts of the marriage settlement.

Astbury, J., decided that, as the moneys in question came into the hands of the trustees upon the trusts declared by the policy, there was no right in volunteers claiming under the settlement to insist on the trustees changing the character in which they held the money. The £1,280 must be declared to be held for the plaintiff absolutely.

Misstatement of material facts in proposal. Knowledge of Agent is knowledge of Company.

The case of *Thornton-Smith* v. *Motor Union Insurance Company, Limited*, 30 T.L.R. 139, although concerned with Motor Car Insurance, deals with the suppression or mis-statement of material facts in the proposal, such facts being within the knowledge of the agent of the

Company, and is therefore of sufficient interest for inclusion in these notes. The material facts are as follows :—

The plaintiff had a Metallurgique car, which he had insured with the Car and General Insurance Corporation against the risk of accidents and mechanical breakdowns. He had made a number of claims under the policy, and when it expired in February 1912, the Corporation declined to renew it. These facts were mentioned to a Mr. Lambert, since deceased, who was an agent of the Motor Union Insurance Company, and who offered to effect an insurance for the plaintiff with that Company. The form of proposal included, *inter alia*, certain questions relating to previous insurances, and the plaintiff telephoned to Lambert, and asked him what he was to do about these questions. Lambert replied :— " Send it along, and I will make it right." The proposal was afterwards sent by Lambert to the defendants, and when it reached them, contained answers in the negative to the questions relating to previous insurances. Subsequently, the plaintiff insured a Siddeley car with the defendants, but in this case, although the proposal form was signed, the spaces for the answers to all the questions were left blank.

The plaintiff sued the defendant company for a declaration that the two policies of insurance were valid and binding.

The case came before Channell, J., who held that the defendants were liable under both policies, and observed that with regard to the Siddeley policy, the most that could be said was that something had been concealed. In the course of his judgment he said : " If Lambert had been here, there might " have been some explanation, and he would not impute any-" thing fraudulent to him. On the evidence he found that " Lambert was acting as agent for the defendants, and that " the plaintiff had made full disclosure to him. He could not " conclude that the plaintiff had sent the form in blank to " Lambert in order that false statements should be inserted in " it, and that being so he felt there was no further difficulty " left in the case. The fact was disclosed to the agent, with " whom the plaintiff was not acting in any sort of collusion."

Restriction against assignment except by "operation of law."

In re Birkbeck Permanent Benefit Building Society, Official Receiver v. *The Licenses Insurance Corporation* (1913), 2 Ch. 34, deals with the case of a policy issued to guarantee the payment of a mortgage, such

policy containing a condition that the insurance should cease if the interest of the insured in the mortgaged property should pass from the insured otherwise than by operation of law, unless certain specified conditions were fulfilled. It was decided that the exception in the prohibition of assignment or alienation of a transfer "by operation of law" covered the assignment by a person such as a liquidator to whom the property passed by operation of law and who was under an obligation to assign it.

The material facts are as follows: The Official Receiver was appointed liquidator of the assets in the winding up of the Birkbeck Permanent Benefit Building Society, and an order was made vesting all the property of the Society in him. Part of the Society's property was a mortgage, the repayment of which was insured with the Licenses Insurance Corporation by a policy which defined "the insured" as "the trustee for "the time being of the Birkbeck Permanent Benefit Building "Society", and contained the following condition:

"This policy shall cease to be in force if the whole or any "part of the interest of the insured in the mortgaged property, "or any part thereof shall pass from the insured otherwise than "by will or operation of law, unless notice thereof in writing is "given to the Corporation, and the insurance be declared to be "continued to a successor in interest by a memorandum made "on the policy by or on behalf of the Corporation, and the "expression 'the insured' shall include a successor in interest "to whom the insurance is so declared to be or is otherwise "continued."

The Official Receiver desired to dispose of and transfer the mortgage, together with the insurance policy, to complete the liquidation of the Society's affairs, but the Corporation claimed the right to refuse their consent to any assignment. The Official Receiver therefore applied to the Court for a declaration that he was entitled to transfer the policy without the consent of the Corporation.

Neville, J., in the course of his decision, said: "In my "opinion, the condition was not intended to prevent an assign-"ment of the policy, but was intended to ensure notice of the "assignment being given to the insurers; and, further, that "the terms of the notice given to the insurers should be endorsed "upon the policy so as to be beyond dispute in case of necessity. "In my opinion, if the policy were assigned, and notice were "given to the insurers, and the policy produced for endorsement,

"it would be the duty of the insurers to endorse it, a duty which "would be enforced if they refused to perform it.

"There is another point of view which I think equally affects "the condition of the insured, and it is this. *Doe* v. *Bevan* "(3 M & S. 353) clearly shows that, *primâ facie*, a contractual "restriction of assignment does not apply to the assignment by "a person on whom the property has devolved by operation of "law, and who is under an obligation to assign. Of course you "may have so contracted as to include such a case, but the mere "condition against assignment does not, *primâ facie*, include "that state of things. And that this condition was not in-"tended to include it is, I think, to be found not only from the "general construction which ought to be placed upon such a "condition, but also from the fact that we have in the conditions "a direct elimination of the case of the passing of a policy by "operation of law. I do not think, therefore, it was ever "intended to apply to an assignment by a person in whom the "policy became vested, not by a voluntary act of the insured "but by the operation of law, and who then was under an "obligation to assign to somebody else. I do not think it was "intended in any case, even if I am wrong in the construction "of the rest of the clause, that the insurers should be in a position "to say, 'We will not give our consent.' I think, therefore, "that this policy can be assigned, and the Official Receiver is "entitled to the declaration asked for."

The Bankruptcy and Deeds of Arrangement Act, 1913.

By the Bankruptcy and Deeds of Arrangement Act, 1913, which came into force on 1 April 1914, several important alterations are made in the Law of Bankruptcy. The following sections are likely to be of particular interest to those who have to adjudicate upon titles to insurance policies:

Section 10 is as follows:—

"A payment of money or delivery of property to a person "subsequently adjudged bankrupt, or to a person claiming by "assignment from him, shall, notwithstanding anything in the "enactments relating to bankruptcy, be a good discharge to "the person paying the money or delivering the property if the "payment or delivery is made before the actual date on which "the receiving order is made (except in cases where the re-"ceiving order is made under sub-section (5) of section 103 "of the principal Act) without notice of the presentation of a

"bankruptcy petition, and is either pursuant to the ordinary "course of business or otherwise *bonâ fide*."

Section 103 (5) of the Bankruptcy Act, 1883, referred to above, enables the Court to make a receiving order on a judgment summons.

The doctrine of the relation back of the title of the trustee in Bankruptcy has been dealt with by Mr. Barrand in his papers on Legal points arising in Life Assurance Practice, *J.I.A.*, vol. xxxiii, p. 217, and vol. xli, p. 165, and it is sufficient to mention that by Section 49 of the Bankruptcy Act, 1883, any payment of money to a trustee for benefit of creditors was not invalidated, provided that the payer had not at the time of payment notice of an available act of bankruptcy, and that the payment was made before the date of the receiving order. As the assignment for benefit of creditors was in itself notice of an act of bankruptcy, it was not safe to make any payment to a trustee for benefit of creditors until three months had elapsed from the date of the deed. The 1913 Act, however, makes such a payment valid provided it is made *bonâ fide*, and without notice of a bankruptcy petition, and before the actual date on which a receiving order is made. As anything in the previous enactments relating to Bankruptcy is specifically excluded by the section, it would appear that notice of an available act of bankruptcy such as a deed of assignment for benefit of creditors, would not in itself make the payment invalid. On the other hand, opinions have been expressed that the fact that the office has notice of an act of bankruptcy on which a bankruptcy petition could be presented, might render it advisable to have a search made for a bankruptcy petition before taking advantage of the section.

Section (11) (1) deals with the question of after-acquired property. (See *J.I.A.*, vol. xli, p. 168.) According to the Bankruptcy Act, 1883, all property, whether real or personal, which may be acquired by or devolve on a bankrupt before his discharge, vests in the trustee in bankruptcy. To this the Courts created an exception in deciding the case of *Cohen* v. *Mitchell* (1890), 25 Q.B.D. 262, by laying down that transactions by a bankrupt with any person dealing with him *bonâ fide*, and for value, in respect of his after acquired property, even with notice of the bankruptcy, shall if completed before any intervention by the trustee be valid against the trustee. This was known as the *Cohen* v. *Mitchell* doctrine. In the case of *In re The New Land Development Association* v. *Gray* (1892), 2 Ch.

138, it was decided that this doctrine did not apply to land, but *In re Clayton and Barclay's Contract* (1895), 2 Ch. 212, it was held to apply to leaseholds. The *Cohen* v. *Mitchell* rule has now been given statutory effect, and it has also been made to apply to realty as well as to personalty, and also to apply retrospectively to transactions as to real property completed before the commencement of the Act, in any case where there has not been any intervention by the trustee before that date. The material part of the subsection reads as follows:

"All transactions by a bankrupt with any person dealing "with him *bonâ fide* and for value in respect of property, whether "real or personal, acquired by the bankrupt after the adjudica-"tion shall, if completed before any intervention by the trustee, "be valid against the trustee, and any estate or interest in "such property which by virtue of the enactments relating to "bankruptcy is vested in the trustee, shall determine and pass "in such manner, and to such extent as may be required for "giving effect to any such transaction."

"This sub-section shall apply to transactions with respect "to real property completed before the commencement of this "Act in any case where there has not been any intervention by "the trustee before that date."

Administration revoked on subsequent discovery of will. Position of purchaser from administrator.

The case of *Hewson* v. *Shelley* (1913), 29 T.L.R. 699, already referred to in these notes, *J.I.A.*, vol. xlvii, p. 653, dealt with the position of a purchaser from an administrator when the letters of administration are subsequently revoked. In view of the interest aroused by the decision in this case, owing to its effect upon existing titles, it may be noted that the Court of Appeal has now unanimously reversed that decision. The case will be dealt with more fully in these notes when it appears in the Law Reports.

REVIEWS.

Bunyon on the Law of Life Assurance. Fifth Edition. By J. V. Vesey Fitzgerald, K.C., Arthur Rhys Barrand, F.I.A., *and* Cecil A. Hunt, M.A., LL.B.

London: C. & E. Layton.

Bunyon's Law of Life Assurance was first published in 1854, and the completion of its sixty years' existence has been marked by

the issue of a fifth edition containing a number of features distinguishing it from the earlier volumes. In the first place, those portions of the subject that most frequently come before the actuary in his general office practice have been considerably revised and amplified. Questions to which no reference was made in former editions have been dealt with, and certain sections, such as those on Assignments, Notice, Agents, Voluntary Settlements, Claims, and kindred subjects, have from the practical point of view been rendered more useful. To ascertain the legal principles affecting points arising out of these branches of his work, the actuary has hitherto depended to a great extent on several excellent papers to be found in the pages of this *Journal* or other actuarial publications. Legal text books, produced in the ordinary way, not infrequently fail in the direction of information on practical points, and there is sometimes a want of proportion in the attention given to various parts of the subject. In this respect, however, the present volume forms an exception. The inclusion as one of the editors of a member of the actuarial profession, well acquainted with the practical as well as the legal aspects of the subject, has resulted in the production of a text book which is not only serviceable to the lawyer, but also in line with life office requirements. A second noticeable feature of the new edition is the reduction in the size of the book. In most works of this class the volumes tend to increase in bulk and become unwieldy, with the multiplication of statutes and judicial decisions; but whereas the previous edition of the present work contained 735 pages, the new one has 524, and this curtailment has been effected not by the exclusion of any valuable matter, but by the elimination of portions of the text that could be dispensed with. Certain sections that had historical interest but no practical value have been omitted. The discussion of obsolete law has been abandoned; and while the principles underlying the judicial decisions are mentioned and a reference to the case is always given in the footnotes, the reports of the cases have generally been left out. Further, questions of more important concern to the actuary have been elucidated by the exclusion for the first time of a good deal of matter relating to subjects that do not properly come within the scope of a text book on the Law of Life Assurance. Chapters on Guarantee and Issue Policies, for example, included in former editions, have been dropped. In the remarks on Mortgages, several pages relating to principles of a general nature that had no special reference to life assurance companies' mortgages have been omitted. A chapter on friendly societies has also been excluded, though some of the matter that formerly appeared in it has been relegated to other portions of the volume. As the result of the elimination just referred to, the new edition has gained both in clearness and usefulness. Moreover, a comparison of the two editions shows that a good deal of new and valuable matter has been substituted for that excluded. One or two points may be mentioned to illustrate this. In the Chapter on Mortgages, for example, the form of mortgage of a life policy was not given in

earlier editions but has been inserted in the latest. Then, among other questions, the duty of a company to answer enquiries as to notices is fully discussed; and the necessity for a vendor of a reversion being properly advised, and the terms of equitable relief in case of a sale of a reversion being set aside are noticed. A number of questions affecting the payment of claims have also been dealt with for the first time, and Voluntary Settlements are discussed at much greater length.

The value of a legal text book is, of course, considerably diminished if it is not brought up to date from time to time to allow for changes in legal principles due to new statutes and the most recent judicial decisions. In the case of the work under review such a revision has, of course, been made. New statutes affecting the law of Life Assurance since 1904 are discussed in the text, and the principal changes due to them pointed out. In an Appendix are given the provisions of the Assurance Companies Act, 1909, and other important statutes to which an actuary finds it useful to refer. Judicial decisions within the last ten years turned on many points of office practice, and some of them may be mentioned to illustrate their nature. Several cases related to the subject of disclosure of material facts in the proposal. In *Joel* v. *Law Union*, the point was as to whether the proposer's answers to the questions put by the Company's doctor were the basis of the insurance; *Holdsworth* v. *L. and Y. Insurance Co.* dealt with the insurers' liability for the knowledge of their agent as to a description of the occupation of the proposer: while in a third case, *Pearl Life Co.* v. *Johnson*, it was held that the company could not rely on misstatements in the proposal not signed by or with the authority of the proposer, and that the liability of the company depended on the terms of the policy without any reference to the statements contained in the proposal. Questions involved in the well known case, *Roberts* v. *Security Co.* came under consideration in *Equitable Fire Office* v. *Ching Wo Hong*, 1907, showing that the earlier case would not necessarily be followed in other somewhat similar cases. On the subject of Proof of Age, there was the case, *Hemmings* v. *Sceptre Life* in 1905, where the company had to pay the sum assured without deduction, as it had received two premiums after the mistake as to age was known. There have been a number of new cases on the question of indisputability of a policy. The return of the premiums on a policy declared void was dealt with in *Sparenborg* v. *Edinburgh Life Assurance Co.*, 1912, in which Foreign Residence was also involved. In a case in the Scottish Courts, *Ingram Johnson* v. *Century Insurance Co.*, the time at which a contract for surrender of a policy was complete was decided. One of the most important cases relating to the subject of Notice was *re Weniger's Policy*, 1910, involving many points of practical importance. In that case it was held that in default of notice the order of date of charges regulates priority; that no principle has been established that a subsequent mortgagee must give notice to a prior mortgagee; that a mortgagee without the policy is fixed with constructive notice

of a prior mortgage, and that an agreement to assign is not an assignment. Two other cases affecting notice appear for the first time; *re Russell*, in which it was held that notice at any time before payment of claim is effectual, and *Bateman* v. *Hunt*, showing that this rule applies even to notice given after the death of the assignor. On the question of priority we have *ex parte Cavendish*, where it was held that the necessity for notice exists even if the first assignee is unaware of the assignment, and therefore guilty of no neglect in not giving it. In the Chapter on Voluntary Settlements a number of cases given in detail in the earlier editions have been summarized, some parts have been revised, and new and useful matter on the question of intent to defraud creditors has been added. On the payment of Claims there are two cases that appear in the work for the first time; *Dockray* v. *Refuge*, in which it was held that the Company was not entitled to any covenant for production of the assignment under which payment was made; and *re Palmer*, in which it was decided that the Company was not entitled to the assignees' documents of title.

The volume concludes with three chapters on Stamps, Death Duties and Income Tax. In that on Stamps the text has been rendered clearer by the relegation to the footnotes of remarks on Acts not now in force, but apart from this, there are not many changes to note in this portion of the work. Two cases, *Northumberland* v. *Commissioners*, 1911, as to the duty on policies in settlement, and *Inland Revenue* v. *Oliver*, 1909, as to the substitution in a settlement of one description of security for another, are referred to. In conclusion, there is no doubt that as the result of the improvements of which we have given some indication above, Bunyon's Law of Life Assurance will occupy a much more important place as a work of reference for members of our profession than it has done in the past.

J. R. H.

The Census of 1911. *Report on the Graduation of Ages. By* MR. GEORGE KING, F.I.A., F.F.A.

IN this Report are given the results of the graduation, for both sexes and according to conjugal condition, of the population of England and Wales according to the Census of 1911. There are also given separately the unadjusted and adjusted distribution for the County of London and for a sample population derived from specially selected districts (in this last mentioned distribution the total enumerated males are 177,359 and females 205,240). For the first time, the population enumerated at the Census has been given for each separate age, and it is possible to make an examination of the returns with a view to deciding what grouping, if any, may be expected to give the most satisfactory results.

A well-marked peculiarity of the statistics is pointed out in the

earlier paragraphs of the Report, viz.: that there is a great fondness on the part of the population for even numbers, and an objection to odd, with the exception of the digit 5; for instance, the following figures extracted from the Report for the numbers enumerated for ages 39 to 49, show clearly a decided preference for digits of age 0, 2, 5 and 8.

Age	Male	Female
39	234,517	252,247
40	262,690	282,281
41	198,344	208,303
42	226,889	242,412
43	196,204	216,351
44	190,949	208,188
45	202,458	213,063
46	184,881	199,948
47	176,713	191,863
48	189,271	207,318
49	172,779	187,295
50	195,197	209,676

In view of this preference for particular digits it has been thought of interest to draw up a comparison between male and female for the whole table. For each sex, the numbers living from ages 10 to 99 inclusive have been summed according to digits of age and then reduced to the ratio of the numbers for digit of age 9. The results are given below, in tabular form and also on diagram 1.

Ages	Males	Females
10, 20, 30 . . . 90	1·349	1·347
11, 21, 31 . . . 91	1·154	1·137
12. 92	1·203	1·206
13. 93	1·130	1·143
14. 94	1·126	1·138
15. 95	1·111	1·112
16. 96	1·087	1·087
17. 97	1·016	1·023
18. 98	1·066	1·074
19. 99	1·000	1·000

To remove these irregularities, quinquennial age groupings in which digits of age 1 and 6 are central, were adopted; it was found that this grouping gave more satisfactory results on graduation than any other; the age groupings are then, 4–8, 9–13, 14–18, &c.

Such a grouping separates digit of age 0 and 8, which are the two where there appears to be the largest misstatement in excess, and further provides that in the group with the digit of age 0, where there is the greatest misstatement of age in excess, there

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Diagram I.

Population according to Digits of Age

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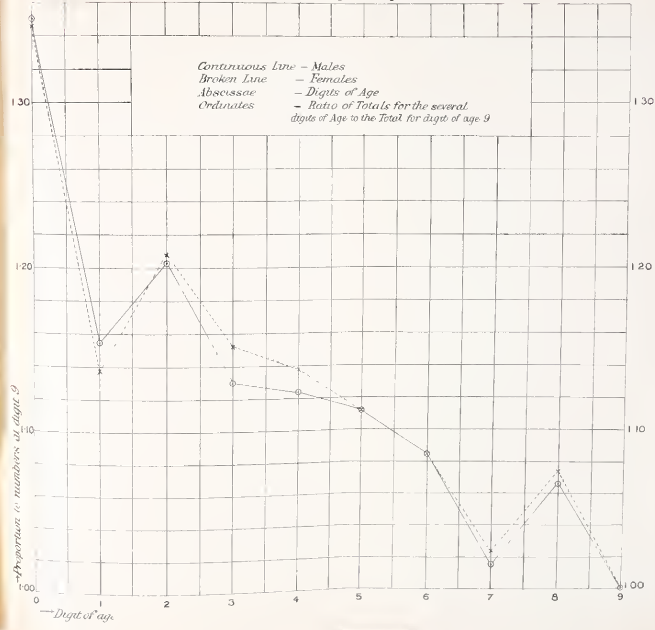

should be three odd digits of age as compared with two in the group with digit of age 8.

The method of graduation employed was that described by Mr. King in his paper, *J.I.A.*, vol. xliii, p. 109. In the application of this method, as is well known, graduated quinquennial values are first obtained from the quinary groups by means of a difference formula, correct to either third or fifth differences, and the intermediate values are then supplied by osculatory interpolation. The actual formulæ used are the third difference formulæ :

(i) $u_7 = \cdot 2w_5 - \cdot 008\Delta^2 w_0$ for the quinquennial values,

(ii) $$\left.\begin{aligned} \delta u_5 &= \cdot 2\Delta u_0 + \cdot 12\Delta^2 u_0 - \cdot 016\Delta^3 u_0 \\ \delta^2 u_5 &= \cdot 04\Delta^2 u_0 - \cdot 016\Delta^3 u_0 \\ \delta^3 u_5 &= \cdot 024\Delta^3 u_0 \end{aligned}\right\}$$ for the intermediate values,

given on pp. 115 and 120 respectively of the paper mentioned above.

By these means graduated values for the population are obtained from age 14 upwards for each age : the graduation of the infantile ages is effected by Milne's method, working on the Census figures in default of better material, in spite of obvious deficiencies in the total numbers enumerated ; the young ages are joined to the adult, obtained as above, by a third difference formula.

The results for the Males in quinary groups from age 14 are shown in Table 1. The graduated numbers are very close to the ungraduated, the total deviations from age 14 upwards being only 8,462 positive and 8,339 negative. For all Males, including those at ages 9 to 13, the total deviations are 10,549, positive and negative, and the accumulated deviations have their greatest value, 2,237, at age 14.

The first differences of the graduated results and of the enumerated numbers, in quinary groups, are shown in columns (5) and (3) of Table I, and also by the continuous and the dotted lines respectively in diagram 2.

This diagram, and the deviations shown in columns (7) to (9) show how closely the data are reproduced by the graduated results. Apart from accidental errors, present in any set of statistics, the most important error in these age returns is that shown by the preference for particular digits, and Mr. King considers that this error is almost entirely eliminated by the particular quinquennial groups he has adopted. As stated in the Report, if the misstatements are within the limits of the age-groups employed, the elimination is practically complete ; if they extend to the age-group on either side, the correction for second differences in formula (i) has great effect.

Granting that the grouping, together with the second difference correction, has eliminated this source of error, the graduation must be considered to have attained its first object of reproducing the original facts with as little disturbance as possible.

TABLE 1.

Ages	Enumerated		Graduated			Deviations		
	Numbers living	$-\Delta$	Numbers living	$-\Delta$	Δ^2	$-$	$+$	Accumulated
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
14–18	1,674,934	146,228	1,672,820	140,391	+73,149	2,114	...	+2,237
19–23	1,528,706	62,270	1,532,429	67,242	− 2,430	...	3,723	−1,486
24–28	1,466,436	71,384	1,465,187	69,672	−33,798	1,249	...	− 237
29–33	1,395,052	101,130	1,395,515	103,470	−69,212	...	463	− 700
34–38	1,293,922	175,278	1,292,045	172,682	− 1,164	1,877	...	+1,177
39–43	1,118,644	174,372	1,119,363	173,846	+27,143	...	719	+ 458
44–48	944,272	143,872	945,517	146,703	−13,416	...	1,245	− 787
49–53	800,400	162,814	798,814	160,119	+25,498	1,586	...	+ 799
54–58	637,586	133,170	638,695	134,621	+11,985	...	1,109	− 310
59–63	504,416	123,246	504,074	122,636	+ 7,764	342	...	+ 32
64–68	381,170	113,679	381,438	114,872	− 2,936	...	268	− 236
69–73	267,491	119,369	266,566	117,808	+35,332	925	...	+ 689
74–78	148,122	82,139	148,758	82,476	+40,217	...	636	+ 53
79–83	65,983	41,816	66,282	42,259	+23,633	...	299	− 246
84–88	24,167	18,801	24,023	18,626	...	144	...	+ 102
89 & over	6,201	...	6,099	...	...	102	...	0
...	12,257,502	...	12,257,625	...	...	8,339	8,462	...

First Differer
quinary Gro

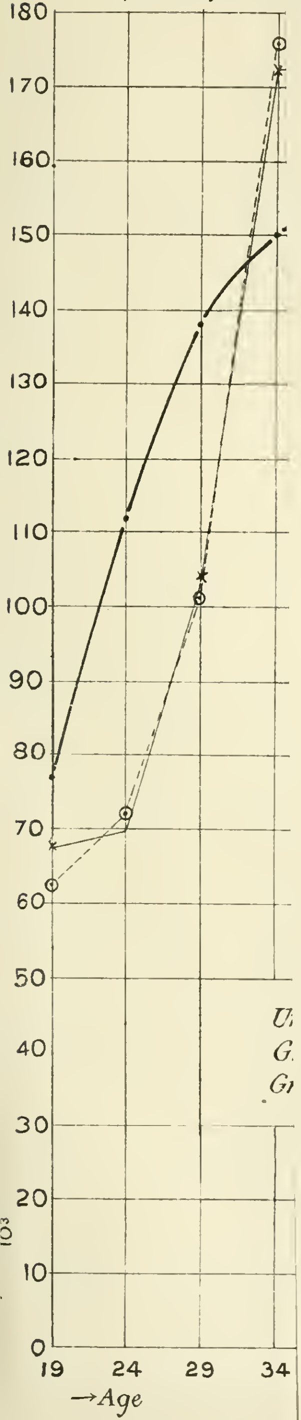

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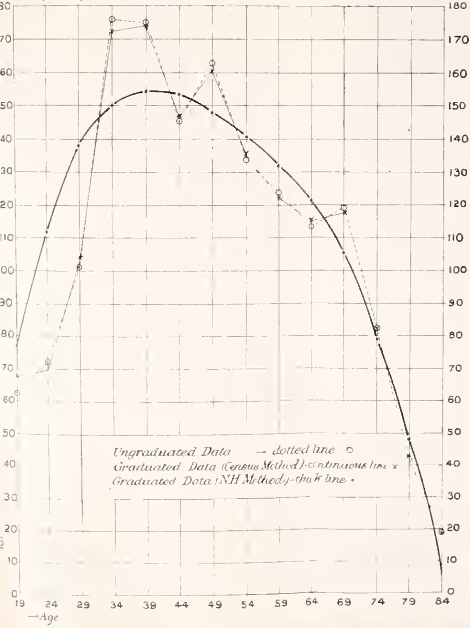
Diagram 2.

First Differences of Ungraduated and Graduated, summed in quinary Groups, 19-23, 24-28 &c



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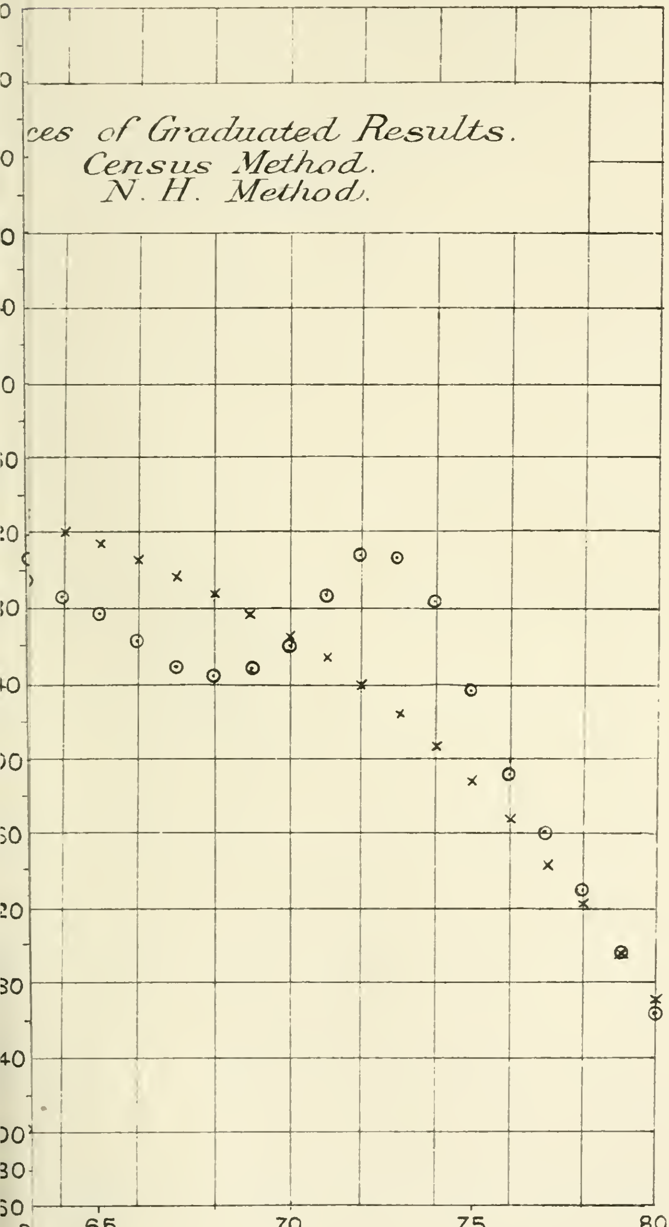
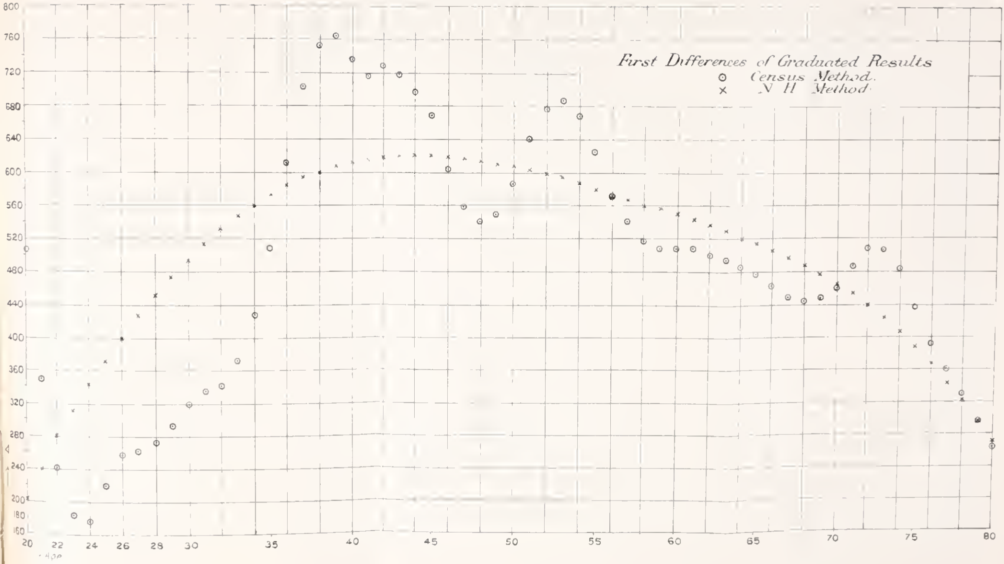


Diagram 3.



First Differences of Graduated Results
Census Method.
N H Method
800
760
720
680
640
600
560
520
480
440
400
360
320
280
240
200
180
160
20
22
24
26
28
30
35
40
45
50
55
60
65
70
75
80

With respect to its second object, of producing results passing smoothly from one value to the next, diagram 3 has been prepared, where the small circles show the first differences of the graduated "Census" results from ages 20 to 80. It will be seen that these differences vary considerably from one group to the next for a large section of the table.

It may be pointed out that the age-groups used by Mr. King can be readily detected from this diagram by the variations in curvature, for $\Delta^3 u_0$ has the same sign as $\delta^3 u_5$, *i.e.*, the second difference of the "small-circle" curve will have the same sign as the third difference of the quinquennial values determined by equation (1). Hence, considering the various parts of this curve, and remembering that Δ is negative, the sections that are concave will be derived from graduated quinquennial values whose third differences are positive, and those that are convex from values with negative third differences; further, when the third differences do not change sign in passing from one value to the next, there will be a break in the smoothness, as at ages 26, 31 and 41.

In view of the fact that the result of a graduation of material similar to that worked on by Mr. King has been published in the Report of the National Health Insurance Commissioners for 1912–1913, and reprinted in *J.I.A.*, October 1913, p. 548 *et seq.*, it has been thought that it would be interesting to obtain results by the method there described working on the 1911 Census data employed by Mr. King.

The method adopted by the Actuarial Advisory Committee was to graduate Log ΣL_x by the expression $a + bx + c\dfrac{x(x-1)}{2} + mr^x$ *i.e.*, practically to assume that the population can be represented, for all adult ages, by an expression analogous to Makeham's (modified) Law. (Here L_x is used to represent the actual population living.) This method appeared to give good results, although it does not follow that a graduation, satisfactory for Log ΣL_x, will be as satisfactory for L_x; in addition the difference in the ultimate aim in the preparation of the table has to be borne in mind.

Working on similar lines to those described in the last mentioned Report, but on data for Males from age 14 upwards, so as to use the same grouped data as those used by Mr. King, the following constants are obtained:

Decennial intervals.	Annual intervals.
$r = \log^{-1}(0{\cdot}375)$	$r = \log^{-1}(0{\cdot}0375)$
$m = -{\cdot}0025365$	$m = -{\cdot}0025365$
$c = -{\cdot}0313891$	$\delta^2 = -{\cdot}000313891$
$b = -{\cdot}1277039$	$\delta = -{\cdot}01135789$
$a = +7{\cdot}0907456$	$a = 7{\cdot}0907456$

The value of r, as given by the data, is $\log^{-1}$ (0·3757); but more suitable values for the constants are given by $\log^{-1}$(0·375)

than by $\log^{-1}(0\cdot376)$; the former is also the value adopted by the Actuarial Advisory Committee.

The results of the graduation of Log ΣL_x are shown in Table 2: in Table 3 (on the next page) are given, in quinary groups, the graduated values of L_x, their first and second differences, and the deviations and accumulated deviations of the ungraduated from the graduated figures.

Table 2.

Males—Log ΣL.

Above Age	Ungraduated	$(r=\log^{-1}0\cdot375)$ Graduated	Deviations × $(10)^5$	
			+	−
14	7·08840	7·08821	...	19
24	6·95683	6·95703	20	...
34	6·79186	6·78969	...	217
44	6·57747	6·57964	217	...
54	6·30859	6·31139	278	...
64	5·95092	5·94813	...	279
74	5·38823	5·40263	1,440	...
84	4·48242	4·66802	...	1,440
94	...	2·65373	...	...
...		...	1,955	1,955

The difference between the results of the two methods is at once apparent; the "N.H." method is much smoother, with only one change of sign in the second differences, and although the dotted and continuous lines in diagram 2 show the effect of the respective graduations on the first differences of the statistics when taken in quinary groups, the first differences of the graduated figures for each age shown in diagram 3 follow similar paths; there are, for the "Census" method, pronounced minima at ages 24, 48 and 68, with maxima at ages 39, 53 and 72.

The deviations from the unadjusted figures are much greater, although when reduced to percentages, in column (7), it is seen that the deviation is not so very material; while the accumulated deviations are larger than those obtained by the "Census" method, and in addition, the value for the constant a is too large, giving 5,443 in excess for the graduated numbers above age 14; there is, too, a diminution in the slope of L_x from ages 22 to 14, which is revealed by the sudden dropping of the curve in diagram 2 for the graduated differences of L_x.

These two last-mentioned features are also present in the graduated results given on pp. 557, 558 of *J.I.A.*, vol. xlvii, and it seems that the values given by this method cannot be employed for ages younger than 20. An alternative would be to start the graduation at one group younger, or preferably to join on at about age 20 to an independent expression for the younger ages.

TABLE 3.

Ages	Enumerated	Graduated			Deviations				
		Numbers living	−	Δ^2	−	+	Accumulated	Per-cent −	Per-cent +
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
14–18	1,674,934	1,612,805	31,435	−45,690	62,129	...	−62,129	3·85	...
19–23	1,528,706	1,581,370	77,125	−34,959	...	52,664	− 9,465	...	3·34
24–28	1,466,436	1,504,245	112,084	−23,906	...	37,809	+28,344	...	2·51
29–33	1,395,052	1,392,161	135,990	−13,636	2,891	...	+25,453	0·21	...
34–38	1,293,922	1,256,171	149,626	− 5,026	37,751	...	−12,298	3·00	...
39–43	1,118,644	1,106,545	154,652	+ 1,369	12,099	...	−24,397	1·10	...
44–48	944,272	951,893	153,283	+ 5,371	...	7,621	−16,776	...	0·80
49–53	800,400	798,610	147,912	+ 7,362	1,790	...	−18,566	0·23	...
54–58	637,586	650,698	140,550	+ 8,554	...	13,112	− 5,454	...	2·02
59–63	501,416	510,148	132,096	+10,692	...	5,732	+ 278	...	1·12
64–68	381,170	378,052	121,404	+16,312	3,118	...	− 2,840	0·83	...
69–73	267,491	256,648	105,092	+25,315	10,843	...	−13,683	4·23	...
74–78	148,122	151,556	79,777	+32,121	...	3,434	−10,249	...	2·26
79–83	65,983	71,779	47,656	+28,236	...	5,796	− 4,453	...	8·10
84–88	21,167	24,121	19,420	...	46	...	− 4,499	0·19	...
89 & over	6,201	5,257	...	...	944	...	− 5,443	18·16	...
...	12,257,502	12,252,059	...	...	131,611	126,168	...	...	

Both methods of graduation show a diminution in the first differences of the graduated numbers living, for ages 19 to 25, and Mr. King, in his Report, attributes this to an influx of immigrants, from Scotland and Ireland in the case of England and Wales, and from the country in the case of the County of London. The same feature appears in the case of the graduated sample populations (taken from selected districts in the Metropolitan Borough of Hampstead, the County Boroughs of Burnley and South Shields, the Urban District of Rhondda, and the Rural Districts in the Counties of Cambridge and the Isle of Ely), and in all three cases this feature is more pronounced in the female tables.

It hardly seems that migration can be regarded as a completely adequate explanation ; for if so, it would follow that there was more migration amongst females than amongst males, while the migration, if it took place as suggested in the Report, would have counteracting effects on the rural and urban portions of the sample populations. The customary explanation of serious misstatement for these ages, as regards the female sex at any rate, is not mentioned in the Report.

Before passing to the graduation of the data relating to conjugal conditions, it may be remarked that the largest percentage deviation given by the "N.H." graduation of the 1911 Census is for the group 69–73 ; a similar feature is reproduced by the "Census" graduation, and it has been suggested that this feature is due to the operation of the Old Age Pensions Act of 1908.

In graduating the data relating to conjugal conditions (married, single, and widowed, for both sexes), Mr. King remarks that apparently mathematical formulæ are quite unsuitable. The method finally adopted was, for ages above 39, similar to that for the total males and females ; for ages below 39, the unadjusted data for each sex were multiplied by the ratio, at each age, of the total graduated to the total enumerated. It is pointed out in the Report that if errors of age were similar for married, widowed and single, this plan would be satisfactory ; as it is, the effect is to produce a graduated table of some roughness, with a transference, to a small extent, of numbers from one marital state to another.

S. J. G.

Grondbeginselen der Levensverzekerings - Wiskunde (Elements of Actuarial Science). By P. Schouten, A.I.A., *Actuary of the Arnhem Life Assurance Company.*

W. L. & J. Brusse, Rotterdam.

This book—an elementary treatise on actuarial theory and practice —will be of interest to British actuaries, as the work of a Dutch actuary who received his early training from our Institute. The International or Institute notation is employed throughout, and many of the formulæ closely resemble those of the Text-book, so that, even if ignorant of Dutch, English students can readily follow

the demonstrations. The different portions of the book are not separated by numbered chapters, and present rather the appearance of a series of notes printed consecutively, and this is perhaps a feature that might be improved in subsequent editions. The general arrangement of the work, however, is quite individual. After a short description of mortality tables and their uses, Mr. Schouten deals with *single* premiums for various simple benefits, combining, broadly speaking, Chapters VII and XVI of the Text-book so far as applicable to single premiums; and he then proceeds, in a separate section, to deduce the corresponding *annual* premiums.

In the next section, two-life probabilities are taken up, leading to a deduction of formulæ for the various kinds of joint life assurances and annuities, reversionary annuities, including pensions for widows and orphans, educational annuities, and the ordinary survivorship assurance on two lives. The next small section, or chapter, dealing with the question of loadings, is quite worthy of notice. A percentage addition to the premiums appears to be recommended by Mr. Schouten, and is defended by him as equitable, provided initial expenses are determined in relation to the single premium for the benefit, instead of in relation solely to the sum assured. He further shows how under this condition a percentage addition can be derived applicable to all classes of assurance. While the business soundness of this as a whole may not be conceded, a few neat ideas are brought forward.

The question of reserves is taken up in the following section, mainly on well-known lines; formulæ allowing for initial expenses, and the subject of negative values are also discussed in this section. Mr. Schouten then shows how various changes and transformations of policies may be effected, including the calculation of surrender, loan, and paid-up values, deferred premium payments and suspense forfeitures; and the work concludes with a dissertation on the notation, a list of symbols, and a well selected assortment of numerical examples.

Although as shown, strictly elementary in its scope—finite differences, the calculus, summation formulæ, or any advanced problems being entirely ignored—Mr. Schouten's work is essentially practical in its aims, and the requirements of daily business are never lost sight of. Thus every section or chapter is preceded by an interesting description or discussion of the nature and object of the following demonstrations, of which those anent loadings and reserves respectively may be specially mentioned. The book appears to have been well received, as a second edition has been issued.

G. W. R.

THE INSTITUTE OF ACTUARIES.

Resolution of Congratulation to Sir G. F. Hardy, K.C.B.

At the meeting of the Council of the Institute of Actuaries, held at Staple Inn Hall on 13 January 1914, it was unanimously Resolved :

> That the Council offer their warmest congratulations to Sir George Francis Hardy on his appointment to be a Knight Commander of the Bath. They recognise that the originality of Sir George Hardy's contributions to actuarial science and the inspiring influence of his teaching have won for him, in an exceptional measure, the esteem and regard of the profession, and they believe that the members of the Institute feel a personal pleasure and pride in the signal honour which has been conferred upon him by His Majesty the King.

ERRATUM.

J.I.A., vol. xlviii, p. 98, line 7 from bottom :

For "30 per-cent" read "20 per-cent."

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

On the Valuation of Benefits dependent upon promotion to a higher status. *By* ERNEST C. THOMAS, F.I.A., *an Assistant Actuary of the Gresham Life Assurance Society.*

[Read before the Institute, 23 February 1914.]

PART I.—THE PROBLEM.

1. THE suggestion has been made to me by Mr. George King, to write a paper upon an interesting problem which recently occupied our joint attention.

A brief statement of the case is as follows:

2. In a great public service there are certain well-defined ranks, the upper ones of which are exclusively recruited by promotion from the lower ones. There are limiting ages of retirement in each rank, and in the event of an officer reaching the fixed age without being promoted he is retired on a pension. The scale of pension depends mainly on length of service, varying of course with each rank, and, as the majority enter in the lowest rank between comparatively narrow limits of age, it is sufficient to deduce an average rate of pension at each age in the various ranks. The problem is to find what amount of annual contribution, uniform throughout future service, should be made in respect of each officer now in the service, of whatever age or rank, in order to provide for the ultimate charge for pensions.

3. In the particular case presented there were six ranks to be dealt with, and it was understood that the calculations

were to be made at 3 per-cent compound interest. I trust I may be excused for this vague statement of the case, but I could hardly be more explicit without being too definite. I shall refer to these ranks throughout as first, second, third ranks, &c., where the first is understood to mean the highest.

4. The problem is not unlike that of an ordinary pension fund where the members are entitled to retirement benefits dependent upon the salary earned during service, or, in other words, upon the extent of the promotion enjoyed; and it might be thought at first that the whole service could be satisfactorily lumped together, ignoring promotions and new entrants in the higher ranks altogether, *i.e.*, treating the service of the promoted ones as continuous. This idea was considered only to be summarily dismissed. In the first place there is considerable over-lapping of the ages in the various ranks, and while a man of 50, say, in one rank may have nothing before him but immediate compulsory retirement, a man of the same age in the next higher rank may be able to look forward to many active years of service in his own and higher ranks. In the second place such a method would hardly have answered the question put, and, when the final results were obtained, the great variations in the contributions required for officers of the same age but different ranks, demonstrated that we were fully justified in trying other methods.

5. The simplest case, of course, is that of the first rank. Here the issue is narrowed down to the calculation of the uniform annual contribution required to provide for certain retirement allowances, varying only with the age of retirement. It is, in fact, the simplest variety of pension fund. The basis of the calculations would be a service table containing the following columns:

l_x = numbers living and remaining in service.

d_x = numbers dying.

r_x = numbers retiring.

6. Having ascertained the average annual retiring allowance at each age, which we may call R_x, we next require a scale of annuity-values for valuing the pensions when entered upon. Having no experience of the service to fall back upon and having no reason to consider the pensioners, at the higher ages, at least, as materially differing from

normal lives, we chose the O^{am} Ultimate Table for this purpose (with a modification for the younger ages).

We proceed as follows:

Let
$$r_x \times v^{x+\frac{1}{2}} = {}^r\bar{C}_x$$
$${}^r\bar{C}_x \times R_x = {}^r\bar{C}^R_x$$
$${}^r\bar{C}^R_x \times (\tfrac{1}{2} + a_{x+\frac{1}{2}})^{O^{am}}_{\text{ult.}} = {}^{ra}\bar{C}^R_x$$
$$\Sigma\, {}^{ra}\bar{C}^R_x = {}^{ra}\bar{M}^R_x$$

Then $\frac{{}^{ra}\bar{M}^R_x}{D_x}$ = present value of pension for an officer in the first rank now aged x (where $D_x = l_x \times v^x$). We will call this function F^r_x.

7. To equate this cash sum to an annual payment payable continuously, we require the continuous service annuity at age x.

Let
$$N_x = D_{x+1} + D_{x+2} \ldots$$

Then
$$\frac{N_x}{D_x} = a_x \text{ (First payment a year hence),}$$

and $\frac{1}{2} + a_x$ or $\bar{a}_x$ = the corresponding continuous annuity. This is the same as would be obtained by recording
$$\bar{D}_x = \frac{D_x + D_{x+1}}{2}$$

and summing from x to the end of the table.

8. The formula $\frac{F^r_x}{\bar{a}_x}$ gives us the annual contribution (payable continuously) required to provide for the pension.

9. So far we have provided only for the simplest case, that of the first rank where there are no promotions and where every officer's service must be terminated either by death or retirement.

10. When we come to the second and the lower ranks it appears on the face of it that a similar procedure for each of them will be a step in the right direction. We can at least by these means assess the value of the pension of an officer consequent upon retirement in his present rank, and from that also deduce the annual payment required *during his service in that rank* to provide for this charge. We therefore proceed to form service tables for each of these lower ranks. We no longer have only two modes of exit to deal with, but are now concerned also with the rate of promotion.

Our service table consists of four columns:

l_x = numbers living and remaining in rank.
d_x = „ dying.
r_x = „ retiring.
P_x = „ promoted to a higher rank.

I have used the capital "P" to avoid confusion with p_x (the probability of living).

11. This table resembles the more usual type of pension fund service table, where the column P_x corresponds (as far as its effect on the particular rank is concerned) with w_x, the numbers withdrawing. There is this difference between the two, that whereas the withdrawals in a pension fund go right out of observation, and cease to have any influence on the fund from the moment of exit, the promotions in the present case continue in the service with an increasing influence on the whole finances of the service.

12. I may here interpolate a few remarks as to the method of obtaining the service tables. On this occasion (as on several others) we adopted the device of using central rates of death, retirement, and promotion instead of the ordinary yearly rates. These were obtained by giving all of the exits, from whatever cause, one-half of a year's exposure only in the year of exit instead of a full year, *i.e.*, the "numbers exposed to risk" obtained in the usual way are decreased by $\frac{1}{2}(d_x + P_x + r_x)$. The great advantage of this method is that each of the separate functions can be graduated independently, whereas if the ordinary yearly rates are used, the graduation of each respective function can only be done in conjunction with, and by constant reference to, the others. For instance, at the later ages the rate of retirement is high, and the mortality rate ${}^dq_x$ is thereby reduced, and at the final age, where all who do not die are retired, the effect on the mortality is practically to reduce it by 50 per-cent, as the time of exposure is, on the average, very little in excess of half a year. All this difficulty and inconvenience can be avoided by the use of central rates. The actual construction of the service table is a little more tedious and complex, but it is well worth the extra trouble.

13. Being free, therefore, to consider each function quite independently, and having only scanty material in the shape of actual deaths in each rank, we lumped the whole service together for the purpose of deducing a satisfactory central

rate of mortality, and were rewarded by finding that the graduated rate when applied to deduce the expected deaths in each separate rank almost exactly reproduced the facts in every case.

14. For the central rates of promotion and retirement it was, however, absolutely necessary to deal with each rank separately. The labour of deducing and graduating these was not, however, so great as one might suppose. The retirements in the upper ranks were mainly observed among the more advanced ages, but in the lower ranks there were quite a number going off at the younger ages, many of these taking no retirement allowance and reducing the average pension to that extent. The promotions were generally observed in well defined zones of ages, sometimes continuing to the end and sometimes ceasing altogether some years before the limiting age in each rank.

The different central rates of exit we distinguished as follows:

$^{d}m_x$ = central rate of death

$^{r}m_x$ = " " retirement

$^{P}m_x$ = " " promotion

$^{t}m_x$ = total central rate of exit

and the formula used was:

$$p_x = \frac{2 - {}^{t}m_x}{2 + {}^{t}m_x}$$

Having found $\log l_x$ from a summation of $\log p_x$ and taken out the antilogarithm $= l_x$, we insert the column $l_{x+\frac{1}{2}} = \frac{l_x + l_{x+1}}{2}$ and then obtain d_x, r_x and P_x by multiplying $l_{x+\frac{1}{2}}$ by $^{d}m_x$, $^{r}m_x$ and $^{P}m_x$ respectively. The sum of these three columns gives the total exits, and these should be equal to the differences of l_x.

15. Table 1 appended hereto shows the results thus obtained in respect of each rank.

16. From the columns now obtained we readily deduce for the second and lower ranks the functions already calculated for the first rank, namely,

$\bar{a}_x$ = Continuous annuity-value at age x payable during service in present rank.

F_x^r = Present value of pension on retirement within present rank.

These, as stated, are similar to those already deduced for the first rank, but whereas for the first rank they represent the total values of benefit and annuity respectively, for the lower ranks they are only a part of the whole value.

There is, therefore, no advantage gained in finding the values of $\frac{F^r_x}{\bar{a}_x}$ for the lower ranks at this stage.

17. I may here remark that in the retirement annuity (which, as already mentioned, was the O^{am} ultimate annuity-value) we used the value of $\bar{a}_{45}$ for age 45 and all lower ages, basing this procedure on the assumption that many of the younger retiring officers would be damaged lives.

18. Tables (2) and (3) show the arrangement of the work and the resulting figures for these two functions for all the different ranks.

19. We proceed to find the complete values of the above-mentioned functions for the lower ranks, which we will call

$\bar{a}'_x$ = Present value of a payment of 1 per annum, throughout service in *present or any higher rank*; and

F'^r_x = Present value of pension on retirement at any age in *present or any higher rank*.

20. Considering first the case of the second rank, and dealing for the moment only with the simpler problem of the annuity-value. The number of men now aged x who will be promoted to first rank between ages x and $x+1$ is P_x, and we must assume that, on the average, promotion will take place at age $x+\frac{1}{2}$. At age $x+\frac{1}{2}$ they will find themselves, in rank I, subject to the altered conditions of that rank as regards possible length of service, rate of retirement, and average amount of pension. Inasmuch as these promotions have been given due effect to in deducing the probable experience of rank I, the promotions from rank II being the new entrants of rank I, we may assume that these arrivals from the lower rank are typical and normal members at and from the age of $x+\frac{1}{2}$ of rank I. At the moment of promotion, therefore, the present value of the remainder of an annuity during service is represented by the function $\bar{a}_{x+\frac{1}{2}}$ as deduced for rank I. As I have appropriated the symbol $\bar{a}'_x$ to represent the annuity value in *present and all higher ranks,* it will be convenient to apply this symbol to $\bar{a}_x$ of rank I. If, therefore, we deduce the mean values of $\bar{a}'_x$ (rank I), we have

$\bar{a}'_{x+\frac{1}{2}}$ representing the annuity-value for the remainder of service at the moment of promotion from rank II.

21. To value this prospective annuity in rank I for officers of various ages now in rank II, we proceed as follows:

22. Out of l_x men existing at age x in the second rank, P_x are promoted between ages x and $x+1$, and the value of each one's further annuity is $\bar{a}'_{x+\frac{1}{2}}$ (of rank I).*

Therefore $P_x(\text{II}) \times \bar{a}'_{x+\frac{1}{2}}(\text{I})$ = total annuity-value for the l_x men at the moment of promotion;

and $v^{\frac{1}{2}}P_x(\text{II}) \times \bar{a}'_{x+\frac{1}{2}}(\text{I})$ = present value of this total-annuity at age x;

also $P_{x+1}(\text{II})$ are promoted between ages $x+1$ and $x+2$;

and $v^{1\frac{1}{2}}P_{x+1}(\text{II}) \times \bar{a}'_{x+1\frac{1}{2}}(\text{I})$ = present value of their aggregate annuities; and so on.

Multiplying throughout by v^x and dividing by l_x we have for the present value of the prospective annuity for each man now aged x in rank II,

$$\frac{1}{l_x v^x}\{[v^{x+\frac{1}{2}}P_x(\text{II}) \times \bar{a}'_{x+\frac{1}{2}}(\text{I})] + [v^{x+1\frac{1}{2}}P_{x+1}(\text{II}) \times \bar{a}'_{x+1\frac{1}{2}}(\text{I})] + \ldots\}$$

23. We form a column of $v^{x+\frac{1}{2}}P_x$ for rank II which we may call ${}^{\text{P}}\bar{C}_x$. We then multiply this column by $\bar{a}'_{x+\frac{1}{2}}$ of rank I and call the result ${}^{\text{P}a}\bar{C}_x$. Now let $\Sigma\,{}^{\text{P}a}\bar{C}_x = {}^{\text{P}a}\bar{M}_x$ and we have as an expression for the value of the deferred annuity consequent upon promotion

$$\frac{{}^{\text{P}a}\bar{M}_x}{D_x} \text{ which we will call } \Delta\bar{a}_x.$$

24. If we add this value to the annuity-value deduced for rank II, which depends on continuation in the said rank, we obtain the total value of an annuity payable throughout service for a man now aged x in rank II.

Thus $$\bar{a}_x + \Delta\bar{a}_x = \bar{a}'_x$$

25. In order to proceed further we require to know the values of these annuities in the middle of each year of age. We, therefore, deduce $\bar{a}'_{x+\frac{1}{2}} = \frac{\bar{a}'_x + \bar{a}'_{x+1}}{2}$ and these figures represent the annuity-values during the remainder of service

* Throughout this paper the symbols (I) (II) placed after any function will indicate—where such distinction is necessary—to which rank the function applies.

in ranks II or I, in respect of those promoted from rank III, at the moment when such promotion takes place.

26. Proceeding as before, by forming the column ${}^{1}\overline{C}_x$ for rank III, combining with $\bar{a}'_{x+\frac{1}{2}}$ (II), summing and dividing by D_x, we find the value of $\Delta\bar{a}_x$, and from that $\bar{a}'_x$ for rank III.

27. This process is repeated from rank to rank until the total service annuity-values are obtained for all ranks.

28. Table 4 shows the work in detail of this part of the calculations for each rank, omitting the preliminary columns, which are computed on separate slips. From this table it will be seen that the procedure is far more simple in actual working than it appears to be in explanation, by no means an unusual experience.

29. We have now obtained

(*a*) A table of annuity-values for each rank, representing the value of a payment of 1 per annum during continuance in the said rank $=\bar{a}_x$;

(*b*) A table of factors F^r_x for each rank, representing the present cash value of the pension payable on retirement while in present rank;

(*c*) A table of complete annuity-values, $\bar{a}'_x$, representing the value of a payment of 1 per annum payable throughout service, obtained from (*a*);

and we require a further table, which we can obtain from (*b*), representing the total present value of pension payable on retirement in present or any higher rank, and this function we will call, when deduced, F'^r_x.

30. For the sake of uniformity we will from this point use the symbol F'^r_x for the function F^r_x already deduced for rank I. If we form the column $F'^r_{x+\frac{1}{2}}=\dfrac{F'^r_x+F'^r_{x+1}}{2}$ we have the value at the moment of promotion from rank II of the future retirement allowances for the remainder of service.

31. The present value of a payment of 1 on promotion from rank II between ages x and $x+1$ in respect of D_x(II) persons now aged x is ${}^{P}\overline{C}_x(\text{II})=(P_x(\text{II})\times v^{x+\frac{1}{2}})$ a column already deduced.

Therefore the present value of a payment (or benefit) of

$$F'^r_{x+\frac{1}{2}}(\text{I}) \text{ is } {}^{P}\overline{C}_x(\text{II})\times F'^r_{x+\frac{1}{2}}(\text{I})$$

and the value of the same benefit on promotion at any age for each officer now aged x is

$$\frac{1}{D_x}\left\{[{}^{P}\overline{C}_x(\mathrm{II}) \times F'^{r}_{x+\frac{1}{2}}(\mathrm{I})] + [{}^{P}\overline{C}_{x+1}(\mathrm{II}) \times F'^{r}_{x+1\frac{1}{2}}(\mathrm{I})] + \ldots\right\}$$

We therefore form the columns

$${}^{P}\overline{C}^{F}_x(\mathrm{II}) = {}^{P}\overline{C}_x(\mathrm{II}) \times F'^{r}_{x+\frac{1}{2}}(\mathrm{I})$$

$${}^{P}\overline{M}^{F}_x(\mathrm{II}) = {}^{P}\overline{C}^{F}_x(\mathrm{II}) + {}^{P}\overline{C}^{F}_{x+1}(\mathrm{II}) + \ldots$$

Then $\dfrac{{}^{P}\overline{M}^{F}_x}{D_x}(\mathrm{II})$ = present value of the pension payable on retirement at any time after promotion from rank II.
We will call this function $\Delta F^{r}_x(\mathrm{II})$.

Then $$F^{r}_x(\mathrm{II}) + \Delta F^{r}_x(\mathrm{II}) = F'^{r}_x(\mathrm{II}),$$

which represents the total value at age x of a pension on retirement in present or any higher rank.

32. As before, we require to form the mean values

$$F'^{r}_{x+\frac{1}{2}}(\mathrm{II}) = \frac{F'^{r}_x(\mathrm{II}) + F'^{r}_{x+1}(\mathrm{II})}{2}$$

and this operation gives us the function to carry down to rank III and combine with ${}^{P}\overline{C}_x(\mathrm{III})$ from which by a similar procedure to the foregoing we compute $\Delta F^{r}_x(\mathrm{III})$ and $F'^{r}_x(\mathrm{III})$.

The annual payment required to provide for the ultimate charge for pension is found from the expression $\dfrac{F'^{r}_x}{\overline{a}'_x}$. In other words the figures in Table 5 (col. 5) have to be divided by those in Table 4 (col. 5) for each age in every rank. As, however, these tables are published merely as an assistance in illustrating the methods employed, it seemed unnecessary to cumber the paper with a further table representing such a simple operation.

33. There was some idea at first of making the annual payment a varying one, increasing on promotion in some relation to the increase in salary. With this purpose in view a somewhat different procedure from that outlined above was followed in deducing the tables of complete annuity-values.

34. Instead of finding the total annuity-value after promotion in one operation we deduced:

1st. The annuity-value due to service in next rank higher than the present;

2nd. The annuity-value due to service in the rank two steps higher than the present one;

and so on.

There would be one of these annuity increments for the second rank, two for the third rank, three for the fourth, and so on, and the total of the separate increments in each case gives the value of Δa_x obtained by the previous method.

35. For rank II there is no difference in the procedure. For rank III, instead of carrying down from rank II for combination with the promotion commutation columns the whole service annuity $(\bar{a}'_{x+\frac{1}{2}})$ in one operation, we carry it down in two instalments, first, the partial annuity-value $(i_{x+\frac{1}{2}})$, and second, the continuation in rank I $(\Delta\bar{a}_{x+\frac{1}{2}})$. For rank IV the annuity-value of rank III is carried down in three instalments, and so on.

Part II.—Its Application.

36. The question as developed up to this point, while I hope of sufficient interest to justify a note in the *Journal*, may perhaps be considered to carry hardly sufficient weight to excuse the appropriation of one of the valuable sessional meetings. My reason for deciding, subject to the approval of the Council, to present the question in this guise was that an interesting and possibly profitable field of enquiry seems to be opened up if we set ourselves to consider what, if any, bearing the methods now described have upon some of the difficulties and anomalies occasionally met with in regard to Staff Pension Funds.

37. In a fund already established where the newcomers consist almost entirely of juniors in the lowest rank of about the same average age, it is quite a legitimate assumption, and, in fact, the only possible one, that they all have at the outset the same chances of ultimate promotion and, therefore, there is no injustice involved as far as they are concerned in the usual method of treating the staff as one indivisible whole. In fact, if carried out with absolute accuracy the methods above described applied to their case should give exactly the same results as the ordinary procedure.

38. Where, however, a fund is being inaugurated and imposed upon existing members of different present ages and

status the case is somewhat different. The assumption of equal chances of promotion is obviously not true. Those who have already begun to move out of the rank and file are just those who are most likely to receive further promotion, while there is certain to be a class of men who either from want of ability, industry, or good fortune, have reached a stage where they recognize themselves that they must be content to remain in a comparatively lowly position for the remainder of their service. It is, I think, clear that the ordinary methods have the effect of giving this class less value for money than those who are on the ladder of promotion, more particularly so where the basis of the calculation of pensions is the final salary. It is largely owing to the recognition of this fact that there has been an increasing tendency of late years to base the pension scale on average salary. Even this device does not, however, altogether meet the complaint, because a pension based on the average of an increasing salary entails a greater annual charge expressed as a percentage of the salary than one based on the average of a salary that is practically constant. It has been urged that in the majority of cases (*i.e.*, in cases where the employers contribute on the same scale as the employees) no injustice is done because each man gets more than the pension that his own unaided contributions would provide, that the provision made by the employers is intended for the advantage of the business as a going concern, and they have the right to say how their financial assistance should be applied. However sound this argument may appear from the employers' point of view, I cannot think it can be very convincing to the class concerned. I can understand a man retorting "you have undertaken to give me the pension "which my own contributions would purchase together with "a similar amount secured by equal contributions of your own "on my behalf. But you are not doing it; you are giving "some of it to these men in a higher rank. If you want to "give them a super-benefit in addition to what the rest of us "receive, do it from a separate fund, but don't mix it up with "our mutual fund, and by so doing give them something "which rightfully belongs to me."

39. For the sake of proceeding further, therefore, I will assume that a case can be made out for applying these methods to the circumstances of a pension fund where the benefits and contributions are functions of the salary earned.

40. We require now, in addition to the service tables for each rank obtained as described above, a scale of average

salary for each rank, and as before we will distinguish these by the suffix (I), (II), &c., to denote the different ranks concerned, thus:

s_x (I) = Scale salary at age x for an officer in rank I.

s_x (II) = Scale salary at age x for an officer in rank II, &c.

Contributions.

41. The valuation of an annuity of, say, 1 per-cent of the salary enjoyed during continuance in the present rank presents no difficulty. In view of the detailed demonstrations given by Mr. Manly and Mr. King I may perhaps be forgiven for assuming that the following expression will be at once recognized as giving the value for any given rank of such an annuity payable in advance in respect of each member now aged x.

$$\frac{\mathrm{N}^s_x}{\mathrm{D}_x} = \frac{1}{\mathrm{D}_x}(\mathrm{D}^s_x + \mathrm{D}^s_{x+1} + \ldots)$$

where $$\mathrm{D}^s_x = (\mathrm{D}_x \times \cdot 01 s_x)$$

and that $\frac{\mathrm{N}^s_x}{\mathrm{D}^s_x}$ gives the value of an annuity (within the same limits) of the whole salary in respect of every unit of salary now receivable, or (in other words) of an annuity of 1 per-cent of salary in respect of each 100 of present salary.

also that $$\frac{\bar{\mathrm{N}}^s_x}{\mathrm{D}_x} = \frac{1}{\mathrm{D}_x}(\bar{\mathrm{D}}^s_x + \bar{\mathrm{D}}^s_{x+1} + \ldots)$$

where $$\bar{\mathrm{D}}^s_x = \cdot 01 s_x \frac{\mathrm{D}_x + \mathrm{D}_{x+1}}{2}$$

and $$\frac{\bar{\mathrm{N}}^s_x}{\mathrm{D}^s_x}$$

give us the corresponding annuities payable continuously.

For rank I, as before, nothing further is required, as we have here the means of evaluating the contributions *throughout service.*

42. For the second and lower ranks we require to know the value of the continuation of salary in the upper ranks. I think it is obvious that the annuity-values deduced above in respect of each rank will in each case in some way come into the calculations for the rank immediately below, and I therefore propose to use a more convenient set of symbols:

Let $F^c_x = \dfrac{\overline{N}^s_x}{D_x}$ = value of an annuity of 1 per-cent of salary during continuance in present rank for each man now aged x.

„ ΔF^c_x = exactly similar annuity payable during remainder of service in all the higher ranks.

„ $F'^c_x = (F^c_x + \Delta F^c_x)$ = total value of such annuity throughout service.

„ ${}_sF^c_x = \dfrac{\overline{N}^s_x}{D^s_x}$ and ${}_sF'^c_x$ = corresponding annuities in respect of each 100 of salary at age x.

It will be seen later that we do not require ${}_sF^c_x$ for other than rank I, and in that case ${}_sF^c_x = {}_sF'^c_x$.

The affixes I, II, &c., distinguish the various ranks as before.

43. Now dealing with rank II, we have:
P_x = numbers promoted to rank I between ages x and $x+1$. At the mean age $x+\frac{1}{2}$ a man promoted finds himself in rank I, and from age $x+\frac{1}{2}$ to $x+1$ exchanges the salary $s_x(\text{II})$ for $s_x(\text{I})$ to be increased to $s_{x+1}(\text{I})$ at the end of the year. This is not so violent an assumption as it may sound, because when the service is divided into definite ranks it is highly probable that the actual salaries of different men of the same age in each rank will vary only within comparatively narrow limits, and will not in any case be very far from the average.

44. The value, therefore, at the moment of promotion from rank II, of an annuity of 1 per-cent of salary for the remainder of service in respect of an officer aged x will be

$$F'^c_{x+\frac{1}{2}}(\text{I}) = \frac{1}{2}\{F'^c_x(\text{I}) + F'^c_{x+1}(\text{I}).\}$$

45. If now we proceed as in the earlier problem and, reverting to paragraph 23, substitute $F'^c_{x+\frac{1}{2}}(\text{I})$ for $\bar{a}'_{x+\frac{1}{2}}(\text{I})$ in the expression ${}^{Pa}\overline{C}_x$ and call the modified column ${}^{P}\overline{C}^c_x$, we can at once write down the value for age x in rank II of the continued part of the annuity as

$$\frac{{}^{P}\overline{M}^c}{D_x} \text{ where } {}^{P}\overline{M}^c_x = ({}^{P}\overline{C}^c_x + {}^{P}\overline{C}^c_{x+1} + \ldots).$$

Now,
$$\frac{{}^{P}\overline{M}^c_x(\text{II})}{D_x} = \Delta F^c_x(\text{II}) \quad (\textit{see} \text{ paragraph } 42),$$

and
$$F'^c_x(\text{II}) = F^c_x(\text{II}) + \Delta F^c_x(\text{II}),$$

and this gives us the total value of 1 per-cent of future salary throughout service for a man now aged x in rank II.

46. If we only required to assess the rate of contribution, expressed as a percentage of salary, to secure a certain scale of pension this is all that we should need as far as this function is concerned. If, however (in a valuation for example), we desired to apply Mr. Manly's method of bringing into account the actual salaries now payable to each man it would be necessary to form the function

$$_{s}F'^{c}_{x}(\text{II}) = F'^{c}_{x} \times \frac{D_x}{D^s_x} \text{ or } F'^{c}_{x} \times \frac{1}{\cdot 01 s_x}.$$

Considering the actual salaries now payable as represented by S_x and the contribution as π per-cent of S_x and calling the actual contributions $K_x = S_x \times \frac{\pi}{100}$, we have

$K_x(\text{II}) \times {}_{s}F'^{c}_{x}(\text{II})$ = Present value of all the future contributions throughout service in respect of all officers now aged x in rank II.

47. In order to continue this operation for rank III it will be noted that we do not carry down from rank II the function $_{s}F'^{c}_{x}$ (II), or, rather, $_{s}F'^{c}_{x+\frac{1}{2}}$ (II), because it is not possible to bring into account the actual salary now payable, at any intermediate point, but only at the point at which the officer concerned has actually arrived at the time of the investigation.

We compute $F'^{c}_{x+\frac{1}{2}}(\text{II}) = \frac{1}{2}\{F'^{c}_{x}(\text{II}) + F'^{c}_{x+1}(\text{II})\}$, combine this with ${}^{\text{P}}\overline{C}_x(\text{III})$ to form ${}^{\text{P}}\overline{C}^c_x(\text{III})$, sum and divide by D_x and obtain $\Delta F^c_x(\text{III}) = \frac{{}^{\text{P}}\overline{M}^c_x}{D_x}(\text{III})$, and from that

$$F'^{c}_{x}(\text{III}) = F^{c}_{x}(\text{III}) + \Delta F^{c}_{x}(\text{III}),$$

and if necessary $_{s}F'^{c}_{x} = F'^{c}_{x} \times \frac{1}{\cdot 01 s_x}$.

Pensions.

48. In applying these methods to the valuation of pensions based upon salary, we will take the two simple cases:

(*a*) A pension scale based upon a fixed percentage of average salary throughout service for each year of service;

(*b*) A pension scale based upon a fixed percentage of final salary for each year of service.

49. (*a*) *Average Salary.*—The axiom laid down by Mr. Manly that a fixed proportion of average salary for each year of service is the same proportion of total salary holds good in this case. We will deal for the moment only with future service.

50. The first step is to obtain the value of a pension of 1 per-cent of the total future salary, on retirement within the present rank, this calculation giving us, as before, the complete solution for rank I and a partial solution for the lower ranks:

Let $$^{ra}\overline{C}_x = r_x \times v^{x+\frac{1}{2}} \times \bar{a}_{x+\frac{1}{2}}*$$

$$^{ra}\overline{M}_x = \Sigma\, ^{ra}\overline{C}_x$$

$$^{ra}\overline{M}'_x = {}^{ra}\overline{M}_x - \frac{1}{2}\, ^{ra}\overline{C}_x$$

$$^{ra}\overline{M}^s_x = {}^{ra}\overline{M}'_x \times \cdot 01 s_x$$

$$^{ra}\overline{R}^s_x = \Sigma\, ^{ra}\overline{M}^s_x$$

Then $\dfrac{^{ra}\overline{R}^s_x}{D_x}$ = Required present value in respect of each officer aged x

and $\dfrac{^{ra}\overline{R}^s_x}{D^s_x}$ = Present value of same pension in respect of each 100 of salary now enjoyed by an officer aged x. (NOTE.—This function will only be required when we come to a valuation).

These formulas have been so fully demonstrated by Mr. Manly and Mr. King that it is unnecessary for me to dilate upon them in detail (see *J.I.A.*, xxxvi, p. 209, and *J.I.A.*, xxxix, p. 129).

51. The function $^{ra}\overline{M}_x$ is modified by the deduction of $\frac{1}{2}{}^{ra}\overline{C}_x$, multiplied by $\cdot 01 s_x$, and summed from x onwards in accordance with the assumption that in the year of retirement one-half of a year's service on the average would be recorded, and that each member would receive full credit in the calculation of his pension for such fraction of a year served. In the course of a considerable experience of these funds I only remember to have met one case where it could safely be assumed that the fractional part of the last year's service would not be counted for pension, and in that case I cannot help thinking that it is partly accidental, the arrangement and wording of the rule being such as to obscure in the minds

* From the pensioners' annuity table.

of the employees, as it probably did in those of the framers of the rule, the true effect of the method adopted. I have met cases where the wording of the rule seemed to imply such an effect, but on enquiry it has transpired that in practice it is not acted upon. Even in those instances where the intention is quite clear, the probability is that if the employees properly understand the rule they will contrive to defeat it, either by not entering on a broken year's service or by arranging to complete it under special permission. It is hardly reasonable to suppose that a man who is retired in the last month of a year's service will tamely accept the position of being deprived of the credit of practically a full year's service, and that the most valuable of all as regards the effect upon the pension payable; and if the door is once opened to the consideration of eleven months' service, I can see no logical course but to allow credit for any fraction. I have enlarged upon this point because it appears to me that there is a danger in accepting such a regulation (implied or otherwise) too lightly, as the effect on the total liability is considerable.

52. Reverting to paragraph 50 we have deduced the function $\frac{{}^{ra}\overline{R}^s_x}{D_x}$ representing the value of pension in present rank. This function we will call ${}^fF^r_x$ (the letter f signifying future service). For rank (I) ${}^fF^r_x$ represents the total value and can be at once re-named ${}^fF'^r_x$, this latter function (the total value) being still to seek for the lower ranks.

53. The methods discussed in paragraph 31 afford us the means of completing the calculations for the lower ranks.

We first form

$$ {}^fF'^r_{x+\frac{1}{2}} = \frac{{}^fF'^r_x + {}^fF'^r_{x+1}}{2} \text{ for rank (I).} $$

Then let $$ {}^P\overline{C}^r_x(\text{II}) = {}^P\overline{C}_x(\text{II}) \times {}^fF'^r_{x+\frac{1}{2}}(\text{I}) $$

$$ {}^P\overline{M}^r_x(\text{II}) = \Sigma\, {}^P\overline{C}^r_x(\text{II}) $$

Then $$ \frac{{}^P\overline{M}^r_x}{D_x}(\text{II}) = \Delta\, {}^fF^r_x(\text{II}) $$

and $$ {}^fF'^r_x(\text{II}) = {}^fF^r_x(\text{II}) + \Delta\, {}^fF^r_x(\text{II}) $$

from which, by forming ${}^fF'^r_{x+\frac{1}{2}}(\text{II})$ and carrying down to rank III in the same manner, we form

$$ {}^fF'^r_x(\text{III}),\ {}^fF'^r_x(\text{IV}), \text{ \&c.} $$

54. Then $\frac{{}^{f}F'^{r}_{x}}{F'^{c}_{x}}$ for each rank gives us the percentage of salary required as an annual contribution to produce a pension of 1 per-cent of average salary for each year of future service.

If we are making a valuation we calculate the function ${}^{f}_{s}F'^{r}_{x} = {}^{f}F'^{r}_{x} \times \frac{1}{\cdot 01 s_x}$, and if $\theta =$ the percentage of average salary granted for each year of service, we obtain the total value for all members at age x in a particular rank from the expression $\frac{\theta}{100} S_x \times {}^{f}_{s}F'^{r}_{x}$ where $S_x =$ the total salaries payable at age x.

55. We further require in this connection to compute the value of the pension in respect of past service.

We may assume that the total amount of salaries paid in the past on which pension is to be calculated is known, and if we represent this function by B_x, then we have simply to find the value of an annuity on retirement, whenever it may happen, of $\frac{\theta}{100} \times B_x$.

56. Referring back to paragraph 50, we have the column

$$ {}^{ra}\overline{M}_x = \Sigma\, {}^{ra}\overline{C}_x, $$

and it is clear that this column (unmodified) divided by D_x gives us the required function in regard to service in present rank.

Let ${}^{p}F^{r}_{x} = \frac{{}^{ra}\overline{M}_x}{D_x}$ = present value of pension of 1 per-cent of average salary per year of service on retirement in present rank.

Let ${}^{p}F'^{r}_{x}$ = similar function for the whole of a member's service.

As before, for rank I we at once write ${}^{p}F'^{r}_{x}$ instead of ${}^{p}F^{r}_{x}$. To complete the calculation for the lower ranks we follow exactly the lines already discussed :

Let $\quad {}^{\mathrm{P}}_{p}\overline{C}^{r}_{x}(\mathrm{II}) = {}^{\mathrm{P}}\overline{C}_x(\mathrm{II}) \times {}^{p}F'^{r}_{x+\frac{1}{2}}(\mathrm{I})$

$\quad {}^{\mathrm{P}}_{p}\overline{M}^{r}_{x}(\mathrm{II}) = \Sigma\, {}^{\mathrm{P}}_{p}\overline{C}^{r}_{x}(\mathrm{II})$

Then $\quad \frac{{}^{\mathrm{P}}_{p}\overline{M}^{r}_{x}(\mathrm{II})}{D_x(\mathrm{II})} = \Delta^{p}F^{r}_{x}(\mathrm{II})$

and $\quad {}^{p}F'^{r}_{x}(\mathrm{II}) = {}^{p}F^{r}_{x} + \Delta^{p}F^{r}_{x}$

and so on.

The present value, therefore, of the pensions in respect of past service for all members aged x in any given rank is

$$\frac{\theta}{100}\mathrm{B}_x \times {}^p\mathrm{F}'^r_x$$

57. (*b*) *Final Salary.*—In dealing with a pension based on final salary the main procedure would be the same, but there would be a difference in the fundamental formulas. I hardly need to demonstrate that the method of carrying down the benefits of one rank to the next lower one would be unaltered. The value of pension on retirement within present rank would be obtained as follows (see the papers of Mr. Manly and Mr. King already referred to):

Let $${}^{ra}\overline{\mathrm{C}}^s_x = r_x \times v^{x+\frac{1}{2}} \times \bar{a}_{x+\frac{1}{2}} \times {\cdot}01 s_x$$

$${}^{ra}\overline{\mathrm{M}}^z_x = \Sigma\, {}^{ra}\overline{\mathrm{C}}^s_x$$

$${}^{ra}\overline{\mathrm{M}}'^z_x = {}^{ra}\overline{\mathrm{M}}^z_x - \frac{1}{2}\,{}^{ra}\overline{\mathrm{C}}^s_x$$

$${}^{ra}\overline{\mathrm{R}}^z_x = \Sigma\, {}^{ra}\overline{\mathrm{M}}'^z_x$$

Then $\dfrac{{}^{ra}\overline{\mathrm{R}}^z_x}{\mathrm{D}^s_x} \times \dfrac{\theta}{100}\mathrm{S}_x$ = present value of pensions in respect of future service.

and $\dfrac{{}^{ra}\overline{\mathrm{M}}^z_x}{\mathrm{D}^s_x} \times \overline{n|} \times \dfrac{\theta}{100}\mathrm{S}_x$ = present value in respect of past service (where $\overline{n|}$ = number of past years' service).

These are the simple formulas where only one rank is concerned.

The functions to be carried down from rank I to complete rank II would be obtained as follows:

$${}^f\mathrm{F}'^r_x = \frac{{}^{ra}\overline{\mathrm{R}}^z_x}{\mathrm{D}_x}$$

$${}^p\mathrm{F}'^r_x = \frac{{}^{ra}\overline{\mathrm{M}}^z_x}{\mathrm{D}_x}$$

and the mean values ${}^f\mathrm{F}'^r_{x+\frac{1}{2}}$ and ${}^p\mathrm{F}'^r_{x+\frac{1}{2}}$ would be the functions required.

Returns on Death or Withdrawal.

58. It would be a mere wearisome repetition to shew in detail the application of these methods to the subsidiary benefits generally met with in the shape of returns of whole

or part contributions, with or without interest, on death or withdrawal, but that they would be equally applicable to such calculations is I think sufficiently obvious.

Conclusion.

59. I am not sanguine enough to suppose that it would be often possible or even desirable to apply the principles here discussed. The cases where they might apply in some modified form, either in the simple manner of the earlier part of the paper or in the more complex way of the latter part, would be, I suppose, certain national or municipal services, civil, military, or naval. Possibly some of the larger railway services and great industrial undertakings might lend themselves to such treatment. Provided that the wish is expressed and the material can be furnished there is no insuperable difficulty for the actuary. If a thoroughly organized system cannot be presented, even a rough and ready division of a staff into two or three groups might be preferable to its treatment en bloc, and such a division might go far to allay the discontent that is sometimes caused by the usual methods.

60. It has been contended on more than one occasion that our present methods are imperfect owing to the use, more or less general, of aggregate instead of select service tables. Now select service tables are a counsel of perfection, but they are practically impossible in many cases owing to paucity of data. I do not myself feel that the use of such tables would make so much difference in the majority of cases that the want of them is any serious drawback.* But the demand for select tables for what it may be worth seems to me to a large extent met by the employment where practicable of the methods here developed.

61. The principal use of select tables would be to differentiate between the rates of retirement of men of the same present age but of varying lengths of service. Take the case of an expert official imported into the service say at age 50 for the sake of his experience and at a high salary. It is fairly obvious that he is not so likely to retire at age 55 as a

* I should like to amplify this statement as I am afraid that in its present form it is rather open to misconception. When I wrote that I was thinking for the moment more particularly of the use of these tables in a valuation. For the purposes of assessing the contributions for new entrants of advanced ages I admit their complete desirability, but it is just for that purpose that the methods here described would give practically all that select tables could provide in the way of discrimination with something additional.

man who entered at 15 and has long since reached his maximum. If, however, there is a whole class of such officials in the service there is no reason to consider him as other than a fair sample of that class and subject to the same chances of promotion or retirement as the rest.

62. There is of course nothing in this paper which can be properly described as either new or original. It is merely the restatement of old ideas and formulas in different combinations. Sometimes, however, although a principle may be quite obvious on reflection it is extremely useful to have it recorded in print in a form in which it can be referred to at any time without trouble. I have on many occasions found the printed papers of Mr. Manly and Mr. King of great value for the comparison of tables, the confirmation of formulas and other purposes, and if any Actuary in the future who may be confronted with the problem put before us, or who desires to experiment with these methods of dealing with a Pension Fund, is able to obtain any assistance from these pages it will I hope justify the appropriation of a further evening to a subject which has outlived its novelty and probably outworn its welcome.

KEY TO NOTATION.

PART I (EXCLUDING SALARY).

SIMPLE FUNCTIONS.

l_x = numbers living and remaining in present rank at age x.

d_x = ,, dying in rank between ages x and $x+1$.

r_x = ,, retiring ,, ,, ,, ,,

P_x = ,, promoted to next rank ,, ,,

${}^dm_x$ = central rate of death at age x.

${}^rm_x$ = ,, retirement at age x.

${}^Pm_x$ = ,, promotion at age x.

R_x = retirement allowance at age x.

ANNUITIES.

(*a*) *Present rank.*

$$\mathrm{D}_x = v^x l_x$$

$$\mathrm{N}_x = (\mathrm{D}_{x+1} + \mathrm{D}_{x+2} + \ldots)$$

$$\overline{\mathrm{D}}_x = \frac{\mathrm{D}_x + \mathrm{D}_{x+1}}{2}$$

$$\overline{\mathrm{N}}_x = (\overline{\mathrm{D}}_x + \overline{\mathrm{D}}_{x+1} + \ldots)$$

$$a_x = \frac{\mathrm{N}_x}{\mathrm{D}_x}$$

$$\bar{a}_x = \frac{1}{2} + a_x$$

(*b*) *Future ranks.*

$${}^{\mathrm{P}}\overline{\mathrm{C}}_x = v^{x+\frac{1}{2}}\mathrm{P}_x$$

$${}^{\mathrm{P}a}\overline{\mathrm{C}}_x = {}^{\mathrm{P}}\overline{\mathrm{C}}_x \times \bar{a}'_{x+\frac{1}{2}} \text{ (of next higher rank)}$$

(*Note.*—For first rank $\bar{a}_x = \bar{a}'_x$)

$${}^{\mathrm{P}a}\overline{\mathrm{M}}_x = \Sigma\,{}^{\mathrm{P}a}\overline{\mathrm{C}}_x$$

$$\Delta\bar{a}_x = {}^{\mathrm{P}a}\overline{\mathrm{M}}_x \div \mathrm{D}_x$$

$$\bar{a}'_x = \bar{a}_x + \Delta\bar{a}_x$$

RETIREMENT ALLOWANCES.

(*a*) *Present rank.*

$${}^{r}\overline{\mathrm{C}}_x = r_x \times v^{x+\frac{1}{2}}$$

$${}^{r}\overline{\mathrm{C}}{}^{\mathrm{R}}_x = {}^{r}\overline{\mathrm{C}}_x \times \mathrm{R}_x$$

$${}^{ra}\overline{\mathrm{C}}{}^{\mathrm{R}}_x = {}^{r}\overline{\mathrm{C}}{}^{\mathrm{R}}_x \times \bar{a}_{x+\frac{1}{2}}\mathrm{O}^{am} \text{ ult.}$$

$${}^{ra}\overline{\mathrm{M}}{}^{\mathrm{R}}_x = \Sigma\,{}^{ra}\overline{\mathrm{C}}{}^{\mathrm{R}}_x$$

$\mathrm{F}^{r}_x = {}^{ra}\overline{\mathrm{M}}{}^{\mathrm{R}}_x \div \mathrm{D}_x$ (factor for valuing allowances on retirement in present rank)

(*b*) *Future ranks.*

$${}^{\mathrm{F}}\overline{\mathrm{C}}{}^{\mathrm{F}}_x = {}^{\mathrm{P}}\overline{\mathrm{C}}_x \times \mathrm{F}'^{r}_{x+\frac{1}{2}} \text{ (of next higher rank)}$$

(*Note.*—For first rank $\mathrm{F}^{r}_x = \mathrm{F}'^{r}_x$)

$${}^{P}\overline{M}{}^{F}_{x} = \Sigma\, {}^{P}\overline{C}{}^{F}_{x}$$

$$\Delta F^{r}_{x} = {}^{P}\overline{M}{}^{F}_{x} \div D_{x}$$

$$F'^{r}_{x} = F^{r}_{x} + \Delta F^{r}_{x}$$ (factor for valuing complete retirement allowance, present and future ranks).

PART II (INTRODUCING SALARY).

s_x = scale salary at age x.

CONTRIBUTIONS.

(*a*) *Present rank.*

$$D^{s}_{x} = D_{x} \times \cdot 01 s_{x}$$

$$N^{s}_{x} = (D^{s}_{x} + D^{s}_{x+1} + \ldots)$$

$$\overline{D}^{s}_{x} = \cdot 01 s_{x}\left(\frac{D_{x} + D_{x+1}}{2}\right)$$

$$\overline{N}^{s}_{x} = (\overline{D}^{s}_{x} + \overline{D}^{s}_{x+1} + \ldots)$$

$$F^{c}_{x} = \overline{N}^{s}_{x} \div D_{x}$$ (factor for valuing contribution of 1 per-cent of salary during service in present rank)

$${}_{s}F^{c}_{x} = \overline{N}^{s}_{x} \div D_{x}$$ (above factor reduced to basis of 100 of salary at age x)

(*b*) *Future ranks.*

$${}^{P}\overline{C}{}^{c}_{x} = {}^{P}\overline{C}_{x} \times F'^{c}_{x+\frac{1}{2}}$$ (of next higher rank)

$${}^{P}\overline{M}{}^{c}_{x} = \Sigma\, {}^{P}\overline{C}_{x}$$

$$\Delta F^{c}_{x} = {}^{P}\overline{M}{}^{c}_{x} \div D_{x}$$

$$F'^{c}_{x} = F^{c}_{x} + \Delta F^{c}_{x}$$ (factor for valuing contributions of 1 per-cent of salary during service in present and future rank)

$${}_{s}F'^{c}_{x} = F'^{c}_{x} \times \frac{1}{\cdot 01 s_{x}}$$

PENSIONS BASED ON AVERAGE SALARY (FUTURE SERVICE).

(*a*) *Present rank.*

$${}^{ra}\overline{C}_{x} = r_{x} \times v^{x+\frac{1}{2}} \times \bar{a}_{x+\frac{1}{2}}$$ (from the pensioners' annuity table)

$${}^{ra}\overline{\mathrm{M}}_x = \Sigma\, {}^{ra}\overline{\mathrm{C}}_x$$

$${}^{ra}\overline{\mathrm{M}}'_x = {}^{ra}\overline{\mathrm{M}}_x - \frac{1}{2}\, {}^{ra}\overline{\mathrm{C}}_x$$

$${}^{ra}\overline{\mathrm{M}}^s_x = {}^{ra}\overline{\mathrm{M}}'_x \times \cdot 01 s_x$$

$${}^{ra}\overline{\mathrm{R}}^s_x = \Sigma\, {}^{ra}\overline{\mathrm{M}}^s_x$$

${}^{f}\mathrm{F}^r_x = {}^{ra}\overline{\mathrm{R}}^s_x \div \mathrm{D}_x$ (factor for valuing pensions based on average salary in respect of *future* service in present rank)

(*b*) *Future ranks.*

${}^{\mathrm{P}}\overline{\mathrm{C}}^r_x = {}^{\mathrm{P}}\overline{\mathrm{C}}_x \times {}^{f}\mathrm{F}'^r_{x+\frac{1}{2}}$ (of next higher rank)

$${}^{\mathrm{P}}\overline{\mathrm{M}}^r_x = \Sigma\, {}^{\mathrm{P}}\overline{\mathrm{C}}^r_x$$

$$\Delta^f \mathrm{F}^r_x = {}^{\mathrm{P}}\overline{\mathrm{M}}^r_x \div \mathrm{D}_x$$

${}^{f}\mathrm{F}'^r_x = {}^{f}\mathrm{F}^r_x + \Delta^f \mathrm{F}^r_x$ (factor for valuing pensions based on average salary in respect of future service in present and future ranks)

$${}^{f}\mathrm{F}'^r_x = {}^{f}\mathrm{F}'^r_x \times \frac{1}{\cdot 01 s_x}$$

PENSIONS BASED ON AVERAGE SALARY (PAST SERVICE).

(*a*) *Present rank.*

${}^{p}\mathrm{F}^r_x = {}^{ra}\overline{\mathrm{M}}_x \div \mathrm{D}_x$ (factor for valuing pension based on average salary on retirement in present rank, in respect of past service)

(*b*) *Future ranks.*

${}^{\mathrm{P}}_{p}\overline{\mathrm{C}}^r_x = {}^{\mathrm{P}}\overline{\mathrm{C}}_x \times {}^{p}\mathrm{F}'^r_{x+\frac{1}{2}}$ (of next higher rank)

$${}^{\mathrm{P}}_{p}\overline{\mathrm{M}}^r_x = \Sigma\, {}^{\mathrm{P}}_{p}\overline{\mathrm{C}}^r_x$$

$$\Delta^p \mathrm{F}^r_x = {}^{\mathrm{P}}_{p}\overline{\mathrm{M}}^r_x \div \mathrm{D}_x$$

${}^{p}\mathrm{F}'^r_x = {}^{p}\mathrm{F}^r_x + \Delta^p \mathrm{F}^r_x$ (factor for valuing pension based on average salary on retirement in present or future ranks, in respect of past service)

General.—The Roman numerals I, II, III, &c., placed after any given function indicate to which rank the said function applies.

TABLE 1.

Service Tables.

x (1)	l_x (2)	d_x (3)	P_x (4)	r_x (5)	Total Decrement (6)
			Rank I.		
56	10,000	149	...	...	149
7	9,851	154	...	...	154
8	9,697	160	...	...	160
9	9,537	165	...	...	165
60	9,372	172	...	...	172
1	9,200	182	...	...	182
2	9,018	205	...	...	205
3	8,813	217	...	...	217
4	8,596	120	...	8,476	8,596
			Rank II.		
44	10,000	74	...	...	74
45	9,926	82	...	...	82
6	9,844	89	...	...	89
7	9,755	95	...	...	95
8	9,660	100	...	...	100
9	9,560	105	...	...	105
50	9,455	108	...	...	108
1	9,347	112	...	...	112
2	9,235	115	...	...	115
3	9,120	117	...	...	117
4	9,003	122	...	...	122
55	8,881	126	...	...	126
6	8,755	106	3,261	...	3,367
7	5,388	68	1,070	1,070	2,208
8	3,180	26	...	3,154	3,180
			Rank III.		
38	10,000	29	...	...	29
9	9,971	34	...	...	34
40	9,937	40	...	...	40
1	9,897	46	...	...	46
2	9,851	54	...	...	54
3	9,797	62	...	...	62
4	9,735	72	...	...	72
45	9,663	80	...	...	80
6	9,583	85	419	...	504
7	9,079	84	728	214	1,026
8	8,053	77	925	309	1,311
9	6,742	66	988	460	1,514
50	5,228	50	887	865	1,802
1	3,426	33	...	1,364	1,397
2	2,029	17	...	1,301	1,318
3	711	5	...	706	711

TABLE 1.—(*continued*).

			Rank IV.		
25	10,000	10	...	...	10
6	9,990	11	...	...	11
7	9,979	12	...	...	12
8	9,967	13	...	...	13
9	9,954	14	...	...	14
30	9,940	15	...	...	15
1	9,925	16	...	...	16
2	9,909	17	...	...	17
3	9,892	18	...	...	18
4	9,874	19	...	...	19
35	9,855	20	...	...	20
6	9,835	22	...	...	22
7	9,813	25	...	...	25
8	9,788	28	...	118	146
9	9,642	32	...	145	177
40	9,465	37	...	356	393
1	9,072	40	187	933	1,160
2	7,912	41	202	809	1,052
3	6,860	41	217	651	909
4	5,951	40	494	494	1,028
45	4,923	35	1,090	272	1,397
6	3,526	21	2,344	...	2,365
7	1,161	6	1,155	...	1,161
			Rank V.		
19	10,000	5	...	...	5
20	9,995	5	...	...	5
1	9,990	6	...	...	6
2	9,984	7	...	...	7
3	9,977	8	...	...	8
4	9,969	9	...	...	9
25	9,960	10	...	70	80
6	9,880	11	...	106	117
7	9,763	12	...	115	127
8	9,636	12	...	132	144
9	9,492	13	...	136	149
30	9,343	14	...	352	366
1	8,977	14	...	377	391
2	8,586	14	...	389	403
3	8,183	14	...	371	385
4	7,798	14	...	449	463
35	7,335	14	141	141	296
6	7,039	14	892	52	958
7	6,081	13	1,580	39	1,632
8	4,449	11	1,209	27	1,247
9	3,202	9	798	16	823
40	2,379	6	1,426	237	1,669
1	710	2	...	708	710
			Rank VI.		
16	10,000	5	...	...	5
7	9,995	5	...	...	5
8	9,990	5	...	...	5
9	9,985	5	...	47	52
20	9,933	5	...	77	82
1	9,851	6	...	99	105
2	9,746	7	...	182	189
3	9,557	7	...	525	532
4	9,025	8	635	115	758
25	8,267	7	1,532	50	1,589
6	6,678	6	1,579	73	1,658
7	5,020	5	2,110	66	2,181
8	2,839	2	1,419	1,418	2,839

TABLE 2.

Service Annuity Values (present rank only).

x (1)	$D_x = v^x l_x$ (2)	N_x $(D_{x+1}+D_{x+2}+\ldots)$ (3)	$a_x = \frac{N_x}{D_x}$ (4)	$\bar{a}_x = (\frac{1}{2}+a_x)$ (5)	$\bar{a}_{x+\frac{1}{2}}$ (6)
			Rank I.		
56	1,910	12,454	6·521	7·021	6·669
7	1,827	10,627	5·816	6·316	5·951
8	1,746	8,881	5·086	5·586	5·207
9	1,667	7,214	4·328	4·828	4·432
60	1,590	5,624	3·536	4·036	3·623
1	1,516	4,108	2·710	3·210	2·778
2	1,443	2,665	1·847	2·347	1·897
3	1,369	1,296	·947	1·447	·973
4	1,296	...	...	·500	...
			Rank II.		
44	2,724	27,109	9·952	10·452	...
45	2,625	24,484	9·328	9·828	...
6	2,527	21,957	8·690	9·190	...
7	2,432	19,525	8·030	8·530	...
8	2,338	17,187	7·352	7·852	...
9	2,247	14,940	6·650	7·150	...
50	2,157	12,783	5·927	6·427	...
1	2,077	10,706	5·153	5·653	...
2	1,985	8,721	4·393	4·893	...
3	1,904	6,817	3·580	4·080	...
4	1,825	4,992	2·735	3·235	...
55	1,748	3,244	1·856	2·356	...
6	1,672	1,572	·940	1·440	...
7	999·3	572·7	·573	1·073	...
8	572·7	...	...	·500	...
			Rank III.		
38	3,252	30,735	9·449	9·949	...
9	3,148	27,587	8·762	9·262	...
40	3,047	24,540	8·054	8·554	...
1	2,945	21,595	7·333	7·833	...
2	2,847	18,748	6·586	7·086	...
3	2,748	16,000	5·822	6·322	...
4	2,652	13,348	5·034	5·534	...
45	2,555	10,793	4·224	4·724	...
6	2,460	8,333	3·388	3·888	...
7	2,263	6,070	2·682	3·182	...
8	1,949	4,121	2·114	2·614	...
9	1,585	2,536	1·600	2·100	...
50	1,192	1,344	1·127	1·627	...
1	758·9	584·7	·770	1·270	...
2	436·2	148·5	·340	·840	...
3	148·5	...	...	·500	...

TABLE 2—(*continued*).

			Rank IV.		
25	4,776	67,303	14·09	14·59	...
6	4,632	62,671	13·53	14·03	...
7	4,493	58,178	12·95	13·45	...
8	4,357	53,821	12·35	12·85	...
9	4,224	49,597	11·74	12·24	...
30	4,095	45,502	11·11	11·61	...
1	3,970	41,532	10·46	10·96	...
2	3,848	37,684	9·795	10·295	...
3	3,729	33,955	9·105	9·605	...
4	3,614	30,341	8·395	8·895	...
35	3,502	26,839	7·665	8·165	...
6	3,393	23,446	6·910	7·410	...
7	3.287	20,159	6·133	6·633	...
8	3,183	16,976	5·332	5·832	...
9	3,045	13,931	4·575	5·075	...
40	2,902	11,028·5	3·800	4·300	...
1	2,700	8,328·5	3·085	3·585	...
2	2,287	6,041·5	2·642	3·142	...
3	1,924	4,117·5	2·140	2·640	...
4	1,621	2,496·5	1·540	2·040	...
45	1,302	1,194·5	·918	1·418	...
6	905·1	289·4	·320	·820	...
7	289·4	...	...	·500	...
			Rank V.		
19	5,703	75,234	13·19	13·69	...
20	5,535	69,699	12·59	13·09	...
1	5,370	64,329	11·98	12·48	...
2	5,211	59,118	11·34	11·84	...
3	5,056	54,062	10·69	11·19	...
4	4,905	49,157	10·02	10·52	...
25	4,758	44,399	9·333	9·833	...
6	4,581	39,818	8·692	9·192	...
7	4,395	35,423	8·059	8·559	...
8	4,212	31,211	7·410	7·910	...
9	4.028	27,183	6·748	7·248	...
30	3,849	23,334	6·062	6·562	...
1	3,591	19,743	5·498	5·998	...
2	3,334	16,409	4·922	5·422	...
3	3.085	13,324	4·319	4·819	...
4	2.854	10.469·8	3·668	4·168	...
35	2,606	7,863·8	3·017	3·517	...
6	2,428	5,435·8	2·239	2·739	...
7	2,037	3,398·8	1·668	2·168	...
8	1,447	1,951·8	1·349	1·849	...
9	1,011	940·8	·930	1·430	...
40	729·5	211·3	·290	·790	...
1	211·3	...	...	·500	...
			Rank VI.		
16	6,232	53,318	8·557	9·057	...
7	6.048	47,270	7·816	8·316	...
8	5,868	41,402	7·055	7·555	...
9	5,694	35,708	6·272	6·772	...
20	5,500	30,208	5·492	5·992	...
1	5,295	24,913	4·704	5·204	...
2	5,086	19,827	3·899	4·399	...
3	4,843	14,984	3·094	3·594	...
4	4,439	10,545	2·376	2·876	...
25	3,948	6,597	1·671	2·171	...
6	3,096	3,501	1·131	1·631	...
7	2,260	1,241	·549	1·049	...
8	1.241	...	...	·500	

TABLE 3.

Present Value of Retiring Allowances in respect of existing Rank.

x (1)	${}^{ra}\overline{C}{}^{R}_{x}$ (2)	${}^{ra}\overline{M}{}^{R}_{x} = \Sigma {}^{ra}\overline{C}{}^{R}_{x}$ (3)	$F^{r}_{x} = \frac{{}^{ra}\overline{M}{}^{R}_{x}}{D_x}$ (4)	$F^{r}_{x+\frac{1}{2}}$ (5)
		Rank I.		
56	...	8,596,100	4,500·9	4,602·7
7	...	8,596,100	4,704·4	4,813·6
8	...	8,596,100	4,922·7	5,039·9
9	...	8,596,100	5,157·0	5,281·1
60	...	8,596,100	5,405·1	5,537·7
1	...	8,596,100	5,670·2	5,814·1
2	...	8,596,100	5,958·0	6,118·6
3	...	8,596,100	6,279·1	6,455·2
4	8,596,100	8,596,100	6,631·3	3 315·7
		Rank II.		
44	...	6,225,000	2,285·1	...
45	...	6,225,000	2,371·4	...
6	...	6,225,000	2,463·2	...
7	...	6,225,000	2,559·8	...
8	...	6,225,000	2,662·6	...
9	...	6,225,000	2,770·1	...
50	...	6,225,000	2,886·0	...
1	...	6,225,000	2,996·4	...
2	...	6,225,000	3,135·5	...
3	...	6,225,000	3,268·9	...
4	...	6,225,000	3,410·4	...
55	...	6,225,000	3,561·2	...
6	...	6,225,000	3,722·2	...
7	1,648,900	6,225,000	6,228·7	...
8	4,576,100	4,576,100	7,991·0	...
		Rank III.		
38	...	7,327,560	2,253·2	...
9	...	7,327,560	2,327·6	..
40	...	7,327,560	2,404·9	...
1	...	7,327,560	2,488·3	...
2	...	7,327,560	2,573·9	...
3	...	7,327,560	2,666·9	...
4	...	7,327,560	2,763·8	...
45	...	7,327,560	2,868·1	...
6	...	7,327,560	2,979·2	...
7	366,690	7,327,560	3,238·2	...
8	503,040	6,960,870	3,571·9	...
9	710,720	6,457,830	4,075·7	...
50	1,267,100	5,747,110	4,820·6	...
1	1,892,800	4,480,010	5,903·4	...
2	1,709,600	2,587,210	5,930·6	...
3	877,610	877,610	5,911·5	...

TABLE 3—*continued.*

Rank IV.				
25	...	4,444,657	930·5	...
6	...	4,444,657	959·4	...
7	...	4,444,657	989·2	...
8	...	4,444,657	1,020·0	...
9	...	4,444,657	1,052·2	...
30	...	4,444,657	1,085·2	...
1	...	4,444,657	1,119·4	...
2	...	4,444,657	1,155·0	...
3	...	4,444,657	1,191·8	...
4	...	4,444,657	1,229·7	...
35	...	4,444,657	1,269·1	...
6	...	4,444,657	1,309·8	...
7	...	4,444,657	1,352·1	...
8	77,037	4,444,657	1,396·0	...
9	138,580	4,367,620	1,434·2	...
40	382,380	4,229,040	1,457·1	...
1	1,105,900	3,846,660	1,425·0	...
2	930,680	2,740,760	1,198·4	...
3	823,380	1,810,080	940·8	...
4	642,980	986,700	608·7	...
45	343,720	343,720	264·1	...
6	...	...	...	...
7	...	...	...	...
Rank V.				
19	...	2,243,522	393·4	...
20	...	2,243,522	405·3	...
1	...	2,243,522	417·7	...
2	...	2,243,522	430·5	...
3	...	2,243,522	443·7	...
4	...	2,243,522	457·4	...
25	...	2,243,522	471·5	...
6	...	2,243,522	489·7	...
7	...	2,243,522	510·4	...
8	...	2,243,522	532·6	...
9	...	2,243,522	556·9	...
30	131,670	2,243,522	582·8	...
1	182,520	2,111,852	588·2	...
2	192,490	1,929,332	578·6	...
3	198,340	1,736,842	563·1	...
4	251,300	1,538,502	539·0	...
35	135,680	1,287,202	494·0	...
6	51,452	1,151,522	474·2	...
7	39,546	1,100,070	540·0	...
8	27,977	1,060,524	732·8	...
9	16,897	1,032,547	1,021·2	...
40	260,380	1,015,650	1,392·5	...
1	755,270	755,270	3,574·4	...

No Retirement Allowances—Rank VI

TABLE 4.

Complete Service Annuity Values (existing and subsequent ranks).

x (1)	${}^{Pa}\overline{C}_x$ (2)	${}^{Pa}\overline{M}_x$ (3)	$\Delta\bar{a}_x = \frac{{}^{Pa}M}{D_x}$ (4)	$\bar{a}'_x = (\bar{a}_x + \Delta\bar{a}_x)$ (5)	$\bar{a}'_{x+\frac{1}{2}}$ (6)
			Rank I.		
56	...	...	...	7·021	6·669
7	...	...	...	6·316	5·951
8	...	...	...	5·586	5·207
9	...	...	...	4·828	4·432
60	...	...	...	4·036	3·623
1	...	...	...	3·210	2·778
2	...	...	...	2·347	1·897
3	...	...	...	1·447	·973
4	...	...	...	·500	...
			Rank II.		
44	...	5,258·4	1·931	12·383	12·108
5	...	5,258·4	2·004	11·832	11·552
6	...	5,258·4	2·081	11·271	10·982
7	...	5,258·4	2·163	10·693	10·398
8	...	5,258·4	2·250	10·102	9·796
9	...	5,258·4	2·340	9·490	9·178
50	...	5,258·4	2·438	8·865	8·525
1	...	5,258·4	2·532	8·185	7·864
2	...	5,258·4	2·649	7·542	7·192
3	...	5,258·4	2·762	6·842	6·479
4	...	5,258·4	2·881	6·116	5·741
55	...	5,258·4	3·009	5·365	4·975
6	4,094·5	5,258·4	3·145	4·585	3·412
7	1,163·9	1,163·9	1·165	2·238	1·369
8	...	...	...	·500	·250
			Rank III.		
38	...	8,982·8	2·762	12·711	12·413
9	...	8,982·8	2·853	12·115	11·809
40	...	8,982·8	2·948	11·502	11·193
1	...	8,982·8	3·050	10·883	10·562
2	...	8,982·8	3·155	10·241	9·916
3	...	8,982·8	3·269	9·591	9·257
4	...	8,982·8	3·388	8·922	8·581
45	...	8,982·8	3·516	8·240	7·890
6	1,164·1	8,982·8	3·652	7·540	7·089
7	1,859·1	7,818·7	3·455	6·637	6·155
8	2,160·2	5,959·6	3·058	5·672	5·085
9	2,100·0	3,799·4	2·398	4·498	3·775
50	1,699·4	1,699·4	1·425	3·052	2·161
1	...	...	...	1·270	1·055
2	...	...	...	·840	·670
3	...	...	...	·500	·250

TABLE 4—*continued.*

Rank IV.					
25	...	11,033·6	2·310	16·900	16·656
6	...	11,033·6	2·382	16·412	16·159
7	..	11,033·6	2·456	15·906	15·644
8	...	11,033·6	2·532	15·382	15·117
9	...	11,033·6	2·612	14·852	14·578
30	..	11,033·6	2·694	14·304	14·022
1	...	11,033·6	2·779	13·739	13·451
2	...	11,033·6	2·867	13·162	12·863
3	...	11,033·6	2·959	12·564	12·256
4	...	11,033·6	3·053	11·948	11·632
35	...	11,033·6	3·151	11·316	10·989
6	...	11,033·6	3·252	10·662	10·326
7	...	11,033·6	3·357	9·990	9·644
8	...	11,033·6	3·466	9·298	8·998
9	...	11,033·6	3 623	8·698	8·400
40	...	11,033·6	3·802	8·102	7·887
1	579·3	11,033·6	4·087	7·672	7·693
2	570·3	10,454·3	4·572	7·714	7·746
3	555·4	9,884·0	5·137	7·777	7·786
4	1,137·6	9,328·6	5·754	7·794	7·752
45	2,240·8	8,191·0	6·292	7·710	7·552
6	4,204·4	5,950·2	6·574	7·394	6·964
7	1,745·8	1,745·8	6·033	6·533	3·267
Rank V.					
19	...	17,673·9	3·099	16·789	16·536
20	...	17,673·9	3·193	16·283	16·027
1	...	17,673·9	3·291	15·771	15·502
2	...	17,673·9	3·392	15·232	14·959
3	...	17,673·9	3·495	14·685	14·404
4	...	17,673·9	3·603	14·123	13·835
25	...	17,673·9	3·714	13·547	13·298
6	...	17,673·9	3·857	13·049	12·815
7	...	17,673·9	4·021	12·580	12·343
8	...	17,673·9	4·196	12·106	11·871
9	...	17,673·9	4·387	11·635	11·394
30	...	17,673·9	4·591	11·153	11·037
1	...	17,673·9	4·922	10·920	10·821
2	...	17,673·9	5·300	10·722	10·635
3	...	17,673·9	5·729	10·548	10·454
4	...	17,673·9	6·192	10·360	10·329
35	542·6	17,673·9	6·781	10·298	10·046
6	3,131·8	17,131·3	7·055	9·794	9·417
7	5,030·4	13,999·5	6·872	9·040	8·544
8	3,486·6	8,969·1	6·199	8·048	7·451
9	2,085·5	5,482·5	5·423	6·854	6·151
40	3,397·0	3,397·0	4·657	5·447	2·974
1	...	...	...	·500	·250
Rank VI.					
16	...	41,902·4	6·724	15·781	...
7	...	41,902·4	6·928	15·244	...
8	...	41,902·4	7·140	14·695	...
9	...	41,902·4	7·359	14·131	...
20	...	41,902·4	7·617	13·609	...
1	...	41,902·4	7·912	13·116	...
2	...	41,902·4	8·238	12·637	...
3	...	41,902·4	8·652	12·246	...
4	4,258·9	41,902·4	9·438	12·314	...
25	9,589·6	37,643·5	9·535	11·706	...
6	9,244·9	28,053·9	9·061	10·692	...
7	11,553·0	18,809·0	8·323	9·372	...
8	7,256·0	7.256·0	5·847	6·347	...

TABLE 5.

Complete Value of Retiring Allowance (present and subsequent ranks).

x	${}^{P}\overline{C}^{F}_{x}$	${}^{P}\overline{M}^{F}_{x} = \Sigma {}^{P}\overline{C}^{F}_{x}$	$\Delta F^{r}_{x} = {}^{P}\overline{M}^{F}_{x} \div D_{x}$	$F'^{r}_{x} = F^{r}_{x} + \Delta F^{r}_{x}$	$F'^{r}_{x+\frac{1}{2}}$
(1)	(2)	(3)	(4)	(5)	(6)
			Rank I.		
56	...	...	...	4,500·9	4,602·7
7	...	...	...	4,704·4	4,813·6
8	...	...	...	4,922·7	5,039·9
9	...	...	...	5,157·0	5,281·1
60	...	...	...	5,405·1	5,537·7
1	...	...	...	5,670·2	5,814·1
2	...	...	...	5,958·0	6,118·6
3	...	...	...	6,279·1	6,455·2
4	...	...	...	6,631·3	3,315·7
			Rank II.		
44	...	3,766,960	1,382·9	3,668·0	3,737·3
45	...	3,766,960	1,435·2	3,806·6	3,880·3
6	...	3,766,960	1,490·7	3,953·9	4,031·5
7	...	3,766,960	1,549·2	4,109·0	4,191·5
8	...	3,766,960	1,611·4	4,274·0	4,360·3
9	...	3,766,960	1,676·5	4,446·6	4,539·6
50	...	3,766,960	1,746·6	4,632·6	4,721·2
1	...	3,766,960	1,813·4	4,809·8	4,921·5
2	...	3,766,960	1,897·6	5,033·1	5,140·2
3	...	3,766,960	1,978·3	5,247·2	5,360·8
4	...	3,766,960	2,064·0	5,474·4	5,595·5
55	...	3,766,960	2,155·3	5,716·5	5,845·7
6	2,825,500	3,766,960	2,252·7	5,974·9	6,572·9
7	941,460	941,460	942·1	7,170·8	7,580·9
8	...	...	...	7,991·0	3,995·5
			Rank III.		
38	...	4,118,100	1,266·2	3,519·4	3,577·5
9	...	4,118,100	1,308·0	3,635·6	3,696·0
40	...	4,118,100	1,351·5	3,756·4	3,821·5
1	...	4,118,100	1,398·3	3,886·6	3,953·5
2	...	4,118,100	1,446·4	4,020·3	4,092·9
3	...	4,118,100	1,498·6	4,165·5	4,241·2
4	...	4,118,100	1,553·1	4,316·9	4,398·4
45	...	4,118,100	1,611·8	4,479·9	4,566·7
6	427,370	4,118,100	1,674·2	4,653·4	4,761·2
7	749,380	3,690,730	1,630·8	4,869·0	4,975·0
8	961,610	2,941,350	1,509·0	5,080·9	5,203·0
9	1,038,500	1,979,740	1,249·4	5,325·1	5,467·6
50	941,240	941,240	789·4	5,610·0	5,756·7
1	...	...	...	5,903·4	5,917·0
2	...	...	...	5,930·6	5,921·1
3	...	...	...	5,911·5	2,955·8

TABLE 5—*continued.*

		Rank IV.			
25	...	6,821,550	1,428·2	2,358·7	2,395·4
6	...	6,821,550	1,472·7	2,432·1	2,469·9
7	...	6,821,550	1,518·4	2,507·6	2,546·7
8	...	6.821,550	1,565·7	2,585·7	2,626·5
9	...	6,821,550	1,615·1	2,667·3	2,709·1
30	...	6,821,550	1,665·7	2,750·9	2,794·3
1	...	6,821,550	1,718·3	2,837·7	2,882·9
2	...	6,821,550	1,773·0	2,928·0	2,974·6
3	...	6,821,550	1,829·4	3,021·2	3,069·3
4	...	6,821,550	1,887·6	3,117·3	3,167·3
35	...	6,821,550	1,948·1	3,217·2	3,268·8
6	...	6,821,550	2,010·5	3,320·3	3,373·9
7	...	6,821,550	2,075·4	3,427·5	3,483·2
8	...	6,821,550	2,142·9	3,538·9	3,606·7
9	...	6,821,550	2,240·3	3,674·5	3,741·2
40	...	6,821,550	2,350·7	3,807·8	3,879·9
1	216,820	6,821.550	2,527·0	3,952·0	4,019·6
2	235,400	6.604,730	2,888·7	4,087·1	4,169·2
3	254,450	6,369,330	3,310·5	4,251·3	4,316·1
4	583,180	6,114,880	3,772·2	4,380·9	4,447·6
45	1,296,900	5,531,700	4,250·1	4,514·2	4,596·3
6	2,823,600	4,234.800	4,678·4	4.678·4	4,776·9
7	1.411.200	1,411.200	4.875·3	4.875·3	2,437·7
		Rank V.			
19	...	6,998,650	1,227·2	1,620·6	1,645·2
20	...	6,998,650	1,264·4	1,669·7	1,695·3
1	...	6,998.650	1,303·2	1,720·9	1,747·3
2	...	6,998,650	1,343·1	1,773·6	1,800·8
3	...	6,998.650	1,384·2	1,827·9	1,856·1
4	...	6,998,650	1,426·9	1,884·3	1,913·4
25	...	6,998,650	1,471·0	1,942·5	1,979·9
6	...	6,998,650	1,527·6	2,017·3	2,060·0
7	...	6,998,650	1,592·2	2,102·6	2,148·4
8	...	6,998,650	1,661·5	2,194·1	2,244·2
9	...	6,998,650	1,737·4	2,294·3	2,347·6
30	...	6,998,650	1,818·0	2,400·8	2,469·0
1	...	6,998,650	1,948·9	2,537·1	2,607·3
2	...	6,998,650	2,098·9	2,677·5	2,754·7
3	...	6,998,650	2,268·8	2,831·9	2,911·4
4	...	6,998.650	2,451·9	2,990·9	3,085·1
35	161,400	6,998,650	2,685·3	3,179·3	3,234·7
6	1,023,300	6,837,250	2,815·8	3,290·0	3,342·2
7	1,816,800	5,813,950	2,854·3	3,394·3	3,444·8
8	1,397,300	3,997,150	2,762·5	3,495·3	3,543·8
9	928,750	2.599.850	2,571·0	3,592·2	3,637·8
40	1,671,100	1,671,100	2,290·9	3,683·4	3,628·9
1	...	...	...	3,574·4	1,787·2
		Rank VI.			
16	...	6,885,280	1,104·8	1,104·8	...
7	...	6,885.280	1,138·4	1,138·4	...
8	...	6,885,280	1,173·3	1,173·3	...
9	...	6,885,280	1,209·2	1,209·2	...
20	...	6,885,280	1,251·7	1,251·7	...
1	...	6,885.280	1,300·2	1,300·2	...
2	...	6,885,280	1,353·6	1,353·6	...
3	...	6,885.280	1,421·7	1,421.7	...
4	588,980	6,885.280	1,551·0	1,551·0	...
25	1,427,600	6,296,300	1,594·8	1,594·8	...
6	1,486,300	4,868,700	1,572·5	1,572·5	...
7	2,010,900	3,382,400	1,496·6	1,496·6	...
8	1,371,500	1.371.500	1,105·1	1,105·1	...

Abstract of the Discussion.

Mr. L. E. CLINTON, in opening the discussion, said that in Part I of the paper, Mr. Thomas gave an account of the methods followed in dealing with a very interesting problem. The reasons for the departure from the ordinary methods were stated briefly in paragraph 4, but it appeared to him they became more apparent when the different circumstances of the ordinary pension schemes and those of the particular problem presented were considered. In an ordinary fund the scheme generally provided that everybody must retire at a fixed age, or at the end of a certain number of years' service subject to the attainment of an age usually about five years younger than the fixed age. This provision had the effect of confining all the retirements to a comparatively small group of ages, whereas in the case of the service under discussion the retirements were spread over nearly the whole range of ages, with distinct groupings for each rank, which overlapped at certain ages. Again, in that particular service the retirement allowances varied with each rank, and effect could only be given to those distinctive features in combination by deducing a rate of retirement applicable to each rank. Further, the reference required a separate contribution to be determined for each age according to rank, and it therefore became necessary to know the numbers passing forward from age to age in each rank, which element involved the formation of separate service tables. In that connection it might be worth while to point out that if the contribution in respect of future entrants into the lowest rank only had been required a much simpler process would have sufficed. One service table only would be necessary, but instead of a single column of retirements they would have six columns, one for each rank, rates of retirement having been deduced as a function of the total number existing in the service at each age inclusive of all ranks. The commutation columns would be constructed in the usual way and the uniform contribution at any age would be given by the formula—

$$^{ra}\overline{\mathrm{M}}^{\mathrm{R}}_x(\mathrm{I}) + {}^{ra}\overline{\mathrm{M}}^{\mathrm{R}}_x(\mathrm{II}) + \ .\ .\ .\ + {}^{ra}\overline{\mathrm{M}}^{\mathrm{R}}_x(\mathrm{VI}) \div \overline{\mathrm{N}}_x$$

The construction of the service tables and of the various commutation factors derived therefrom was very clearly explained by the author, and the processes adopted in computing the requisite contributions were easy to follow. The method of obtaining the value of the future retiring allowances and contributions contingent upon promotion to a higher rank was extremely interesting and furnished once again a striking illustration of the immense power of the commutation column. In the second part of the paper Mr. Thomas developed the formula employed in Part I to allow for pensions and contributions based upon salary, and put forward the valuable suggestion that the method might be applied to assess more accurately the contribution to be paid by men who commenced to contribute to a fund at a fairly old age with practically

little or no chance of further promotion. The author cited a characteristic case in paragraph 38, but a more typical example was perhaps that of a man promoted from the wages staff to the salary staff of a railway or municipal body at a salary which in most cases he could never hope to get beyond. Such cases frequently occurred in practice, and there could be no doubt that the use of the ordinary aggregate methods in assessing the contributions did inflict a real hardship on those men. Provided the material was available from which a reliable experience could be obtained, he thought most of them would agree with Mr. Thomas that the methods he advocated would remove a great deal of that injustice. As to the practical possibilities of the method, it seemed to him that except in the case of some of the great railway funds, where the statistics were considerable, there were remarkably few funds to which it could be satisfactorily applied.

One of the greatest difficulties with which the actuary had to contend in dealing with pension funds was to obtain sufficiently reliable data, and it frequently happened that owing to the scanty nature of the statistics furnished he had to rely almost entirely on the experience of similar funds. In order to be able to apply Mr. Thomas's suggested method, still more detailed statistics would be required, and he thought that would prove an effective bar in the majority of instances. Moreover, unless the service were of the character dealt with in the first part of the paper, a further difficulty would be experienced in satisfactorily distinguishing between the various grades of employees. Presumably Mr. Thomas advocated the application of the method more particularly for the computation of contributions, and in that case it would, if found practicable, be more generally used at the commencement of a fund. In those circumstances it would be essential to bear in mind the age of the service, for unless it were fairly old the majority of the promotions would take place from comparatively young men, which would have the general effect of deferring the retirements to older ages than would be the case if the service had reached a normal condition. He thought that point would need very careful consideration; otherwise the value of the pensions and contributions might be seriously affected.

In conclusion, he felt sure that the members of the Institute more practically concerned with that particular branch of their science would be indebted to the author for his paper, which would be most helpful should they have before them at any time a problem similar to that dealt with by Mr. King and Mr. Thomas.

Mr. T. G. ACKLAND said that the author had modestly spoken of the paper as not being very new or original, but he thought that the main point in it, viz., the introduction of the rate of promotion, was distinctly both new and original so far as the published literature of the subject went, although it was probable that the note books of some of their older and more experienced actuaries would contain cases in which this factor had been involved. The method was applicable, as Mr. Thomas had told them, and as Mr. Clinton

had also pointed out, mainly to large funds, but he thought that when they were dealing, as some of them had to do, with smaller funds also, they often felt that it was unsatisfactory that they could not in some way allow for the fact that members of the staff would be subject to a certain degree of promotion from one rank to another. It was becoming more and more common, in municipal staffs, for instance, to have a definite grading of employees, and to have a fixed scale of rises of salary going on to a maximum in each grade. Of course, it frequently happened, and must be recognized by the actuary in investigating these funds, that the officials passed from one rank to another by promotion, but in the case of small funds that would be, he would not say negligible, but comparatively a rare factor. For instance, in a borough council which had perhaps 50 officials and 200 or 300 workmen it was evident that the higher offices would be very few, and the prospects of promotion must be small. It was not altogether satisfactory to bring into account the higher salaries resulting from promotion without giving, in some way, effect to this, and Mr. Thomas had at least shown how it could be done, and it was extremely useful to have this on record. Where it was possible to get sufficient data to deduce the rate of promotion at different ages, or an assumed rate of promotion which might be considered to be reasonable in the circumstances, he thought that the method would be most usefully employed. In starting a fund they had, as a rule, to get data from some fund of a kindred character, and this rendered it necessary to make assumptions, which might after a few years' experience be found to depart widely from the facts. The introduction of the further element of promotion might, therefore, only lead them, in five years or so, farther from their original assumptions. He thought, however, that the author's method could be given effect to, within limits, and he would make some endeavour to apply it himself, as opportunity offered, in connection with some of the funds which came under his notice. It was very important, if they were dealing with these ranks as suggested, that they should be careful not only to determine the movement of salaries within each separate class, but also to satisfy themselves that the ratios between the salaries at the top of one class and the bottom of the next were such as would represent with reasonable accuracy the rate of promotion in the particular fund they had under investigation. That was a point which Mr. Thomas no doubt quite realized, but it was one which, he thought, should be emphasized.

He would now refer to the method illustrated in paragraph 14 of the paper of getting out the service table from central rates of decrement. It was a distinct improvement, he thought, to adopt the central rates of decrement (which he preferred to call the forces of decrement); and these were referred to in Mr. Manly's and his own paper on borough council funds, although the service table was set out in the familiar, but, he thought, erroneous, form which had been previously adopted by other writers, based on the assumption that the decrements for deaths, withdrawals and retirements

all took effect, uniformly and simultaneously, at the end of the year of age—an assumption which was inconsistent with the alternative assumption made in the practical calculations, that the decrements took place in the middle of the year of age. The method which the author had adopted was not, however, quite that which was familiar to himself in such cases, and he would remind him of the paper (*J.I.A.*, vol. xxi, p. 417), by Dr. T. B. Sprague, in which he dealt with the question of the decrements arising from marriage and death, and laid down the principles upon which the mortality table should be deduced in such a case. Dr. Sprague's method was further illustrated by a paper he himself had read before the Institute, involving rates of withdrawal and death (*J.I.A.*, vol. xxxiii, p. 194). There he gave a tabular illustration showing that by this method of Dr. Sprague's it was unnecessary to deduce $l_{x+\frac{1}{2}}$ for the purpose of obtaining the decrements. That, he thought, was a distinct improvement, which he had no doubt would commend itself to Mr. Thomas.

There was only one other remark he would like to make, and that was to enter an earnest but friendly protest against the form of notation adopted in the paper. Mr. Lidstone as long ago as 1905 wrote a letter to the *Journal* (vol. xxxix, p. 208) with reference to a paper written by Mr. King on the subject of pension funds, in which a similar notation was employed. Mr. Thomas had had to develop the complexities of this notation in spite of himself, and they got a capital F adorned with small letters and dashes at every possible point of its area, with the further inconvenience that these small affixes and suffixes, when set up for *Journal* purposes, were apt to drop out in the course of printing, so that the symbols did not appear correctly in a portion of the issue. That was a great inconvenience, but his principal objection was that the notation (and in particular the capital F) was not self-explanatory; and he thought personally that the suggestions made by Mr. Lidstone in 1905 were well worth consideration, including the suggestion that the whole question of notation for pension funds should be considered by a Committee of the Institute. He would further add that in adopting P_x for the rate of promotion, the author had escaped the Scylla of probabilities only to fall into the Charybdis of premiums, this symbol being that officially reserved for the premium at age x.

Mr. J. BACON said that Mr. Ackland had raised the question whether the author's method could not be applied within limits to the ordinary problems with which they had to deal. In his (the speaker's) opinion it could only be applied to the very special kind of fund dealt with by the author, and it was a question whether he had not either gone too far or not far enough, considering the data he had at his disposal. The method was obviously suitable only to a very large fund, and he thought it was probably only suitable to one where promotion was certain for a very large proportion in each rank, and where promotions took place within a very narrow range of ages. In the ordinary pension fund problems they had nearly all the retirements at a very few ages, and they

had members going out by withdrawal or resignation in comparatively small numbers at each age, and both the modifications introduced by Mr. Thomas arose from the fact that he had a very large number of promotions or of withdrawals, as the case might be, throughout the entire range of the table. If it were not for that fact, he did not think there would be a great deal to be gained by adopting the central rate of exit instead of obtaining the usual ratio of cases going out during the year to those commencing the year of age. Nor did he think it would be necessary or desirable to adopt the plan of dividing into ranks. Mr. Thomas said he adopted this method, first, because there was a considerable overlapping in the ages of the various ranks, secondly, because there were certain questions which they had to answer by the terms of their reference. He would like to know which of these was the determining factor in leading him to adopt this method. It appeared to him that if the plan of dividing into ranks was introduced and promotion did not take place within a very narrow range of ages for each particular rank the logical development of this method would be to adopt a select scale at any rate so far as salaries were concerned.

The author suggested that it was not a very violent assumption to suppose that when a man left rank V at age x and passed into rank IV at age $x+1$ he would have the average salary for that particular rank at age $x+1$. But if as was quite usual in a fund of this kind there was a minimum commencing salary for each rank, then unless the range of ages for promotion from one rank to another was very small indeed the assumption might be very far wrong, and the only logical way to deal with the matter, particularly if they had sufficient data to divide up the fund into as many as six ranks, was to use a select scale for each particular rank; and if such a scale was impossible or not considered necessary, and the range of ages during which a man passed from one rank to the next higher rank was very small, and if those who did not pass had to retire, as in the present case, then it seemed to him that probably the ordinary methods of deducing the average salary at each age passed through would be quite sufficient without introducing the question of various ranks at all. He noticed that in the particular case under discussion there was a certain amount of complication introduced. They were told in paragraph 2 that there were certain well-defined ranks, the upper ones of which were exclusively recruited by promotion from the lower ones. Judging from the tables given, that was not quite the case, because he found that the earliest age of promotion from rank VI to rank V was 24, but the L_x table for rank V commenced at 19. Similarly the earliest age of promotion from rank V to rank IV was 35, but rank IV commenced at 25, so that obviously particular cases were brought in from outside, and it was not at all probable that the men passing from rank V to rank IV at age 35—as that rank started at 24—would at once attain the average salary of the men of 35 already in that rank.

He thought, also, another reason for using select tables, apart

from the question of salary, was that the rate of promotion from one rank to the next higher rank would be to a very great extent determined by the age of promotion to that rank from the previous ranks. A man who was promoted from rank VI to rank V at a very early age was far more likely to be promoted at a far earlier age to rank IV than a man who was promoted from rank VI to rank V at a comparatively late age—possibly when he had to be promoted or retired. That emphasized what he wished to point out, that this method was applicable only to a very few funds—he would imagine to such a fund as that connected with the Civil Service, where they had a very great number of persons divided into well-defined ranks, with a certain maximum salary for each rank and a certain minimum salary at entry into another rank; but where those conditions prevailed and they were not complicated by people remaining in the lower ranks at stationary salaries for a considerable period, then probably the average salary method was quite good enough.

Mr. T. TINNER said that the method described in the paper was no doubt admirably suited to the particular fund dealt with by the author, because of the peculiar conditions of retirement in the various ranks, but he agreed with the previous speaker in doubting whether there would be much advantage in applying it to ordinary staff pension funds. In so far as such a fund had reached a stationary condition, then, subject to a remark which he would presently make, he thought an aggregate valuation would suffice. But if the fund had not reached that state it was doubtful whether the introduction of rates of promotion would really conduce to greater accuracy, because these rates—especially if obtained from an aggregate experience—were subject to alteration. One drawback to the ordinary method of valuation was that no allowance was made for the rates of withdrawal and retirement differing according to pay, although it was by no means improbable that such was the case, especially as regarded the rate of withdrawal. What was usually done was to take rates of exit observed with regard to persons and apply those rates to salaries. Whilst this practice was better than that which prevailed before Mr. Manly introduced improved methods, it was worth while to consider whether the experience of salaries might not be observed instead of the experience of persons, and this would in large measure allow for the influence of the amount of salary in the rates of exit.

Mr. G. M. REEVE referred to the author's statement that in practically every case the pension was calculated on the total service, including the final fraction of a year from the last completed year to the date of retirement. He believed that in the large majority of cases that was so. But he would call attention to the fact that in one large and important section of pension funds, namely the borough council funds, the pension was nearly always expressly stated to be in proportion to *completed* years of service, and enquiries which had been made had elicited the fact that this was intended and not accidental. At the same time he thought it very probable that in such funds no one would voluntarily retire when he had

completed, say, ten years and ten months' service, or ten years and six months' service, but would linger on, if he possibly could, until he had completed the integral number of years' service.

With regard to the subject of the salary scale, it appeared to him an unsatisfactory feature of their present methods that they did not take into account the effect of promotion. There were two recognized methods of procedure—the average salary method, and the ratio of increase of salary method. As regarded the average salary method, they had the fact that when dealing with cases in different age-groups, they might be dealing with different classes of men, and consequently it was not quite satisfactory to assume that the salaries of the men now aged $x-5$ (say) would increase up to age x in the ratios in which the average salaries were found to increase. On the other hand, when dealing with ratios of increase, they compared the salaries five years ago of the men then aged $x-5$, with their present salaries, and deduced the increase of salary from $x-5$ to x, and that seemed a more satisfactory method than the average salary method. But there was one quite serious disadvantage connected with it. Of the men aged $x-5$ five years ago some would be receiving low salaries and some reasonably high salaries, and it was quite reasonable to assume that the men with the lower salaries would be more likely to leave the service than the men with the higher salaries; the consequence of that was that in taking a period of five years, and dealing only with men who had been in service throughout that period, they were dealing very largely with those who had, on the whole, a better than average chance of getting on in the service, and therefore they might easily obtain exaggerated ratios of increase, and so produce a salary scale which would rise more rapidly than the actual facts would really warrant. If they had sufficient data to deduce a rate of promotion, and so could select those cases which were more comparable one with another as regarded salary and the probability of withdrawal, and so on, the disadvantage which at present attached to the method of deducing the salary scale by ratios of increase would be very largely removed.

A very similar point to that which he had mentioned in reference to the rate of withdrawal at the younger ages arose in connection with the rate of retirement at the older ages. An interesting table in one of Mr. Manly's more recent contributions to the *Journal* showed that if the average salary method were followed without modification, the salary scale deduced at the older ages, from 55 to 70, would be found to rise by leaps and bounds, and he had in his own experience met very similar cases. The fact was that the men who retired at 55 were mainly the lower-paid men. The higher-paid men very naturally stayed on, and the consequence was that the employees at any one age from 55 to 70 were, on the average, men of a higher status than the men at the next previous age, owing to the gradual elimination of the rank and file; consequently the salary scale went up very surprisingly. If they could obtain a proper separation into grades, the average salary method would

certainly be more reliable, but at present he must say it appeared to him an exceedingly unsafe method to adopt unless at the same time they examined the ratio of increase, although he would be quite willing to admit that the ratio of increase method at present was somewhat unsafe to adopt unless at the same time they examined the average salaries.

Mr. VYVYAN MARR understood that a somewhat similar question to that dealt with in the paper arose a few years ago in connection with the pension funds of one of the railway companies. It was the practice of the company to promote some members of its wages staff to its salary staff, and as there were separate funds for the two sections of the service it was necessary to transfer a reserve value from the one fund to the other when the promotion took place. This feature was dealt with in valuation by ascertaining a rate of promotion and treating the transfer value as if it were a benefit payable from the one fund to the other on promotion taking place.

Mr. Thomas was an advocate of the use of central rates of mortality, &c., for the purpose of forming the service table. In this he was entirely in sympathy with him, and even in ordinary mortality investigations he had frequently found it advantageous to deduce from the statistics central death rates rather than the ordinary q_x. He would like to ask if the author had ever made use of the method adopted by Mr. Chatham (*T.F.A.*, vol. ii, 388). From the central rates applicable to the different contingencies he obtained columns of the survivors in respect of each, and then the service table was formed by combining the results by multiplication. In cases which he had investigated the error seemed to be insignificant, and Mr. Chatham always maintained that the method saved labour, besides enabling one to check the results more readily and to get a better knowledge of the table.

Mr. Thomas had not given any particulars which would enable them to say what weight might be attached to his final figures, if one wanted to compare them with those relating to other funds; and the value of his tables would have been enhanced by the inclusion of a table showing the annual payment required to provide for the ultimate charge for pension. Like Mr. Thomas, he had found the papers of Mr. Manly, Mr. King and Mr. McLauchlan of great value for the comparison of tables. Some of the factors used in pension fund valuations were of such a nature that it was impossible to gauge the effect of a change in any of the elements involved without making up complete tables giving effect to the changed conditions, and it was always of interest as well as useful to have standard tables as a basis of comparison. Suppose, for instance, one wanted to examine the effect of a variation in the rate of retirement, between ages 60 and 65, in a fund in which the pension was a percentage of the total salary, the new rate being higher at some ages and less at others than the original rate. So far as accrued benefits in respect of past service and future contributions were concerned, one could readily approximate to their values as modified by the change in the

rate of retirement because the effect was similar to that caused by a variation in the annuity-value at age $x+n$ in valuing a group of deferred annuities which all became payable at age $x+n$. With regard to the value of benefits in respect of future salary the result was not so apparent, because an alteration in the rate of retirement, at any age, affected the values of $\frac{{}^{ra}\overline{\mathrm{R}}^{s}_{x}}{\mathrm{D}^{s}_{x}}$ to a different degree for different values of x, and it might well happen that the value of that function with the altered rates might be greater at some ages and less at others than it was originally, because the influence of the changed rate at any age was modified or intensified by the total percentage of salary receivable up to that age. For example, taking a difference in the rate of retirement at age 62, in arriving at the altered value of $\frac{{}^{ra}\overline{\mathrm{R}}^{s}_{20}}{\mathrm{D}^{s}_{20}}$, that difference was multiplied by

$$v^{42\frac{1}{2}}(\bar{s}_{20}+\bar{s}_{21}+\ .\ .\ .\ +\tfrac{1}{2}\bar{s}_{62})\div s_{20}l_{20},$$

whereas the value at age 40 was only affected by

$$v^{22\frac{1}{2}}(\bar{s}_{40}+\bar{s}_{41}+\ .\ .\ .\ +\tfrac{1}{2}\bar{s}_{62})\div s_{40}l_{40},$$

and the altered rate of retirement at age 62 had a different effect.

There was one function of Mr. Thomas's which for the moment he did not understand—the modification of ${}^{ra}\mathrm{M}_x$ by deduction of $\frac{1}{2}{}^{ra}\mathrm{C}_x$ and multiplication by $s_x\times{\cdot}01$, so that each member would receive full credit for such fraction of a year as was served in the year of exit. He recognized the necessity of allowing credit for such fractions if one did not err on the safe side by giving credit for a full year's service in the year of retirement, and the deduction of $\frac{1}{2}{}^{ra}\mathrm{C}_x$ allowed for that. But was it right to multiply by the same $s_x\times{\cdot}01$ as was used in arriving at D^s_x? It might be an unnecessary refinement, but in calculating ${}^{ra}\mathrm{M}^s_x$ and similar functions, he had used $s_{x+\frac{1}{2}}$, or in other words in making up a salary scale he followed the same principles as were adopted in forming columns of l_x, &c. His s_x in a denominator referred to a point of time while the $\bar{s}_x$ in the numerator of continuous functions referred to intervals of one year.

In paragraphs 37–38 Mr. Thomas referred to the question of late age entrants and his remarks recalled the interesting calculations and other evidence submitted to the Departmental Committee of the Board of Trade on the Railway Pension Funds some years ago. Of course, the increasing tendency to base pension scales on average salaries had to some degree remedied the evils. One point which had a direct bearing on the question was that it was generally the higher-paid officials who remained longest in harness. He had looked into that point in one fund, which had an experience of 740 pensions granted in recent years between the ages of 60 and 70, the total annual salaries drawn by those pensioners at the time of retirement amounting to £160,368, and it might be interesting to

submit the results of that experience. He divided it into four groups, according to rate of salary. In the first group there were 324 members in receipt of salaries below £150; in the second 306 drawing between £150 and £300; in the third 75 drawing between £300 and £500; and in the fourth, 35 drawing £500 or more. The average pension age in the first group was 62·2 years: in the second 62·5; in the third 63·1; and in the fourth 63·4. Still, he was not generally in sympathy with the usual custom of charging late age entrants an increased percentage contribution, because of the difficulty of adjusting the amount so that no hardship was imposed on the staff as a whole, and at the same time the entrant in question was not called upon to pay an excessive portion of his salary as a contribution for pension fund purposes. In cases where employers contributed from 2 per-cent to 4 per-cent of all the salaries a fairer way would seem to be to let their contribution take the form of a payment in respect of each entrant, the amount of that payment being adjusted in such a way that it represented the difference between the value of the benefits at the date of entry and the value of whatever contribution, in the nature of a percentage of salary, the employer might reasonably be called upon to pay. Such a scale might readily be obtained from a series of calculations similar to those submitted to them that evening.

In conclusion, he would say that he thought Mr. Thomas, in his concluding paragraphs, took a most modest view of the importance of his paper. He was sure it might be regarded as a further step in the development of pension fund valuations, although perhaps only as a natural sequence to the papers of Mr. Manly, Mr. King, and Mr. McLauchlan, which established the principle of recognizing in valuations rates of retirement at different ages, in place of the old method of taking merely an average age at which all members were assumed to retire.

The PRESIDENT said it was his pleasant duty to propose a vote of thanks to Mr. Thomas for an extremely able and interesting paper. He had a message from Mr. George King, who had asked him to express his regret that he was unavoidably prevented from being present to speak on the subject. Personally he had only one remark to make, for the benefit especially of the younger members, and that was, that in valuing these superannuation funds, or other funds of this description, they must not lose sight of the extreme importance of careful examination of the rules and regulations, before they began the valuation, because no two sets of rules and regulations—so far as his experience had gone—were alike. If one were asked to frame a superannuation scheme for a firm or company which at present had no rules or regulations, it was certainly the duty of the actuary to insist that such rules should be carefully prepared and formulated. Not only this, but the actuary must also insist, in his opinion, that no alteration, however trivial, should be made in these regulations without his being consulted.

Mr. THOMAS thanked the members present very heartily for the kind way in which they had received his paper. He

had put forward the suggested application in the second part quite tentatively and with some doubt as to whether it was of much more than academic interest, and he was very glad to find that Mr. Ackland not only agreed with the principle, but was prepared to make some practical use of it. Since writing the paper he had begun to feel that it was not quite so unpractical as he had at one time considered it. He thought that it would be a common experience among actuaries and those connected with assurance offices and banks that the staff, if not defined within absolute limits of rank, was divisible into quite recognizable grades. There would be a class of officers at the top and next to them heads of departments, then possibly a first-class staff and a second-class staff, with juniors at the bottom, so that even from the point of view of a comparatively small staff they had the requisite conditions if they could only get some reliable data to work upon. It might be possible again, in the case of a small railway company, to use the experience of a larger company, because, although the scales of salaries might be very different in the two cases, it would not be an unreasonable assumption that the relative positions of the various grades and the extent to which promotion operated would be similar.

With regard to the question of the notation he would be quite prepared to substitute anything which would adequately fulfil the purpose and not be open to the objections that Mr. Ackland had mentioned. The symbol F—which Mr. Ackland had criticized with especial severity—was not his own invention but had been borrowed from the paper on pension funds contributed by Mr. King. When that paper was published he had looked upon this symbol as a useful addition to the system outlined in Mr. Manly's paper, and he had considered it an improvement to employ it for the particular purpose for which it was intended. Not that it was more descriptive, but because it was short and convenient. But as regarded the other symbols they were in his opinion absolutely descriptive. When they had once familiarized themselves with the elementary symbols and with the central idea upon which the more complicated expressions were built up one would recognize the latter at first sight. There was nothing new about the use of prefixes and suffixes; they were very largely used in the notation approved of by the International Congress. At the same time he quite agreed with Mr. Ackland's suggestion that it would be an advantage that the matter should be dealt with by a Committee if the subject of pension funds was considered of sufficient general interest to warrant the setting up of a standard system of notation.

He regretted that time would not permit him to refer to more than one or two of the interesting points which had been raised by other speakers. Mr. Bacon had referred to the suitability or otherwise of his method and expressed his opinion that it was only suitable if the promotions took place within a comparatively narrow range of ages. He thought the probability was that these promotions would in fact take place in most cases within a narrow range

of ages, the usual experience being that if a man got beyond a certain age without being promoted he was definitely passed over. Mr. Bacon seemed to think also that it was a somewhat violent assumption that a man promoted from a lower to a higher rank should at once commence to enjoy the salary scale attached to the latter, but it did not seem to him that for the use to which it was put it was any less legitimate than many other assumptions constantly made; in fact, the assumption could only be said to be made in quite a limited sense, that is, to the same extent as and no further than the use of an average scale of salaries involved the assumption that every man of a given age received the same salary. Mr. Marr had referred to the function s_x, and he understood him to mean that he was in the habit of using $s_{x+\frac{1}{2}}$ in the numerator but s_x in the denominator in the expression for the value of an annuity of 1 per-cent of salary. The difference between Mr. Marr and himself seemed to be rather a question as to the method of calculating the function s_x. Mr. Marr made some further interesting remarks in reference to the fact that those who were receiving the lower salaries retired on the average at an earlier age than those in receipt of the higher salaries. He thought this was a feature which was observed in practically every case he had come across. In one case it was so marked that in making the valuation they had employed two salary scales at the higher ages. The men in receipt of less than a certain salary were assumed to retire at the average age of 61½ (and this was an assumption borne out by the facts) and those in receipt of a salary in excess of the same figure were assumed to go out on the average at age 63. They had worked out their continuation of the service table in such a way as to make these the average ages of retirement of the two classes.

On the Treatment of the Depreciation in Assets due to an enhanced rate of Interest. *By* ROBERT RUTHVEN TILT, F.I.A., *Actuary and Secretary of the General Reversionary and Investment Co., Ltd.*

[Read before the Institute, 30 March 1914.]

TWENTY years ago actuaries were fighting the effects of a falling rate of interest, and they did not then know that the battle would become more acute for the next few years, to be followed by a complete change of front on the side of their opponent. Life offices have, at the present time, chiefly in consequence of a rise in the rate of interest, large amounts of depreciation to face, amounts which in some cases are the equivalent of a substantial part of the quinquennial surplus. The question, therefore, of the proper treatment of this

depreciation, with due regard to the enhanced rate of interest secured by the lower capital values, seems to be one that may properly occupy the attention of the Institute at a Sessional Meeting.

I.—Past Changes in Valuation Rates of Interest.

It may be of advantage to consider in the first place the steps which were taken during the period of a falling rate of interest (ending in 1897) to maintain the stability of the offices and, by so doing, to retain as far as possible the bonus earning powers of the invested funds and the future premiums.

Following the publication of the H^M and $H^{M(5)}$ Tables in 1872, gradual changes in the bases of valuation had taken place; the Carlisle and other Tables had been superseded by the new Institute Tables, and the 1883 Government Annuity Tables had been largely adopted for the valuation of annuities. These changes in mortality bases had in themselves greatly strengthened the reserves of the offices and the general fall in the yield of high-class securities led many Actuaries to adopt lower rates of interest in their valuations. Valuation Reports lodged with the Board of Trade during the five years 1882 to 1886 dealing in the main with valuations at the end of the years 1881 to 1885 inclusive, and including two septennial returns lodged in 1881, show that, in respect of 55 British offices the Valuation Rates of Interest and the net Liabilities were as follows:

Number of Offices	Valuation Rate of Interest	Net Liabilities under Assurance and Annuity Contracts
		£
26	3	39,972,000
1	$3\frac{1}{4}$	3,194,000
15	$3\frac{1}{2}$	28,440,000
13	4	21,477,000
55		93,083,000

If the valuation rates of interest are weighted by the amounts of the Net Liabilities respectively the average of these valuation rates thus obtained is £3. 7*s*. 10*d*., which is the rate assumed by the offices, in the aggregate, to be earned on the invested funds representing the Liabilities.

In the Valuation Reports lodged by the same offices (reduced

in number by amalgamation to 51) during the 5 years 1897 to 1901, and including two Septennial Reports lodged in 1895, the figures were :

Number of Offices	Valuation Rate of Interest	Net Liabilities under Assurance and Annuity Contracts
		£
7	$2\frac{1}{2}$	22,079,000
4	$2\frac{3}{4}$	11,479,000
34	3	114,427,000
1	$3\frac{1}{4}$	2,014,000
4	$3\frac{1}{2}$	16,558,000
1	$3\frac{3}{4}$	520,000
51		167,077,000

One office had reduced its valuation rate by $1\frac{1}{2}$ per-cent, eight offices by 1 per-cent, one by $\frac{3}{4}$ per-cent, nineteen by $\frac{1}{2}$ per-cent, and four offices by $\frac{1}{4}$ per-cent. Weighting the rates of interest similarly the average obtained is £2. 19*s*. 5*d*. or 8*s*. 5*d*. less than before.

It may be noted that in the period 1897 to 1901 the use of the Institute and Government Annuity Tables was practically universal. In the earlier period (1882 to 1886) the principal exception, as regards Assurance Risks, was the use by five offices of the Carlisle Table. If it is assumed that the use of the Carlisle Table is equivalent to the use of the Institute Tables at a rate of interest 1 per-cent higher, the difference between the averages as given above is increased from 8*s*. 5*d*. to 10*s*. 5*d*.

[The valuation figures for the same group of offices for the period 1907 to 1911 give an average similarly obtained of £2. 18*s*. 5*d*. The net liabilities from which this average is obtained were over £250,000,000.]

II.—Past Changes in the Experience Rate of Interest.

Notwithstanding these reductions in the valuation rates of interest it does not appear that these offices, considering them in the aggregate, did more than hold their own against the diminishing earning power of their funds. The figures given in column 3 of Table I show that the average rate of interest earned by all British offices after deduction of income tax fell from £4. 4*s*. 5*d*. (in 1885) to £3. 14*s*. 2*d*. (in 1900), or a reduction of 10*s*. 3*d*.

In Table I are given for alternate years (and for 1912):—

(A) The prices of three irredeemable stocks, that is, of three securities which involve no return of capital, but are securities for income only. The prices of the 4 per-cent. stocks are reduced for facility of comparison to a 3 per-cent basis, and in column 6 are shown the average yields at the various prices. They are intended to be taken as a guide to the variations in the rate of interest obtainable from well-secured investments; conversely the prices under (2) give the corresponding fluctuations in capital value. It will be noticed that the Gas Light and Coke Debenture Stock is not a trustee investment.

(B) The average rates of interest earned by British life offices before and after deduction of income tax; these rates are calculated by the "Hardy" formula $\frac{2I}{A+B-I}$ from the summary statements at the end of the blue books after inspection and one or two necessary adjustments of the figures of the individual offices. The effective rate of income tax is (following Mr. P. L. Newman, *J.I.A.*, July 1908) taken as one-fourth of the rate current to April plus three-fourths of the rate for the remainder of the year. (The rates of column 5 include income tax and must not, of course, be used for comparison with the valuation rates of interest).

These variations in the rates of interest seem to be of sufficient importance to be shown graphically, and they are exhibited in the annexed diagram. The important features are the gradual increase, since 1900, of the margin between the valuation and the experience rates of the offices and the reduction, since 1897, of the gap between the experience rate (before deduction of tax) and the rate of the Stock Exchange Securities.

It is not to be expected that the rate of interest obtained from a large aggregate of investments of many different classes will vary as much or so quickly as the rate which is determined by fluctuations in the demand for that proportion of readily transferable capital which is available for investment from time to time but, over a series of years, the tendency of the two rates will be in the same direction. It is not surprising,

(6)	
rage Yield ıree Stocks before ction of tax	
Rate	
13	0
13	0
11	1
8	4
2	9
4	1
0	6
16	2
14	0
18	0
1	7
4	6
5	2
9	2
11	1
14	5
17	0
0	4
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DIAGRAM showing Variations in the Rates of Interest (1881-1913)

TABLE I.

(1) Year.	(2) Value of £3 per annum (before deduction of income tax) derived from: G.W.R. 4 % Deb. Stock	L. & N.W. 4 % Pref. Stock	Gas Light and Coke 3 % Deb. Stock	(3) British Life Offices. Average rate of interest earned (after payment of income tax)			(4) Effective rate of income tax	(5) British Life Offices. Average rate of interest before deduction of tax in column (4) Rate			(6) Average Yield of Three Stocks before deduction of tax Rate		
							d.						
1881	86	83	78	4	5	9	5$\frac{1}{4}$	4	7	8	3	13	0
1883	85	83	79	4	4	9	5$\frac{1}{2}$	4	6	9	3	13	0
1885	88	85	81	4	4	5	7$\frac{1}{2}$	4	7	1	3	11	1
1887	92	87	85	4	2	1	7$\frac{1}{4}$	4	4	7	3	8	4
1889	100	98	90	4	1	11	6	4	4	0	3	2	9
1891	99	97	86	4	1	4	6	4	3	5	3	4	1
1893	104	100	94	4	0	0	6$\frac{3}{4}$	4	2	4	3	0	6
1895	112	110	100	3	17	2	8	3	19	10	2	16	2
1897	117	113	105	3	16	2	8	3	18	10	2	14	0
	(Highest 119	recorded 116	price) 107										
1899	106	105	100	3	15	8	8	3	18	3	2	18	0
			(1900	3	14	2)							
1901	100	99	93	3	13	5	13$\frac{1}{2}$	3	17	9	3	1	7
1903	98	93	89	3	14	8	12	3	18	7	3	4	6
1905	95	92	89	3	15	1	12	3	19	0	3	5	2
1907	89	87	84	3	16	5	12	4	0	5	3	9	2
1909	87	83	84	3	16	11	13$\frac{1}{2}$	4	1	6	3	11	1
1911	82	80	80	3	18	9	14	4	3	7	3	14	5
1912	80	78	77	3	19	6*	14	4	4	5	3	17	0
1913	77	75	72	...	...	...	...	...	...	...	4	0	4
Average	94$\frac{1}{4}$	91$\frac{1}{2}$	87	...	...	...	...	...	...	...	...	...	...

Valuation Rates: 55 British Offices.	1881–1885 ...	3	7	10
	1896–1900 ...	2	19	5
	1907–1911 ...	2	18	5

* From Figures supplied by the Board of Trade through the good offices of Mr. Ackland.

therefore, to find that the average yield of the life office investments fell much more slowly than the yield from Stock Exchange investments during that period of rapid inflation of prices which culminated in 1897, nor is it surprising that the fall in the life office rate continued for some years after the yield of the Stock Exchange investments had commenced to rise. Old mortgages at the higher rates of interest of earlier periods would be repaid, and would have to be reinvested, and new money would be invested at a rate below the average, whilst

that average would be artificially high by the valuation of investments at the lower prices paid for them in earlier years.

I think, however, that these reasons do not fully account for the changes in the differences between the Life Offices' Experience rate and the yield of the three stocks.

There appear to me to have been three main causes for these changes:

First.—The hidden reserves of the offices created by the increase of market values over book values: these hidden reserves which gradually increased up to 1897 had the effect of maintaining the Life Offices' rate; from 1897 onwards depreciation reduced these reserves instead of increasing the published rates of interest; when the hidden reserves disappeared depreciation was written off the book value of the assets and the effect appears in the Life Offices' rate just as it is shown in the rate of the three stocks.

Secondly.—The slower movement in the rate of interest of the life funds (to which I have already referred). Up to 1897 the continuous fall in the rate of interest had been fully felt on the Stock Exchange, but not in the life office investments, and from 1897 onwards the position was the same as to the rise in the rate. (The movement in the rate of interest of the life offices is made slower artificially by the custom of adjusting values to market prices quinquennially instead of yearly. The rise in the rate of interest shown by Stock Exchange investments in any year is thus spread over the following five years of the aggregate rates of the life offices.)

The third cause is, I think, that the increase in the proportion of the life office funds invested in Stock Exchange Securities tends to bring the two rates together, and this cause seems to me the one of most importance. It indicates greater stability in the rate of interest in the future so long as investments are retained at present values, or, expressed otherwise, that a fall in the rate of interest will be accompanied in the future by a larger increase in capital value.

The following Table gives in abbreviated form, at quinquennial intervals, the percentages of investments in various groups as shown by the Board of Trade Summaries.

The figures refer to the accounts deposited with the Board of Trade in the years given in the headings.

Assets of British Life Offices (Ordinary).

Percentages of different classes of Investments.

No. of Group	Assets	1880	1885	1890	1895	1900	1905	1910
(1)	Mortgages, Loans on Policies and Loans on Personal Security ...	52	49·3	45·7	42·6	33·8	33·3	31
(2)	Agents' Balances, Outstanding Premiums and Interest, Cash, and Miscellaneous	6·7	6·1	7·6	6·9	5·3	4·8	4·7
		58·7	55·4	53·3	49·5	39·1	38·1	35·7
(3)	British, Indian and Colonial, and Foreign Government Securities	10·1	11·3	11·3	11·7	12	11	9·8
(4)	Debentures, Debenture Stocks, Shares and Stocks	12·1	13·4	16·4	20	30	30·5	33·7
(5)	Loans on Rates and Rent Charges	13·9	12·5	11	9·9	8·6	9·5	10
(6)	Land, House Property and Ground Rents... ...	3·4	5·5	6·2	6·8	7·7	8·1	8·3
(7)	Life Interests and Reversions	1·8	1·9	1·8	2·1	2·6	2·8	2·5
		100	100	100	100	100	100	100

The actual figures for the most important items are :

	1880.	1910.
Groups (1) and (2) combined ...	£85,100,000	£142,600,000
,, (3) and (4) ,, ...	32,200,000	175,500,000
Group (6)	4,900,000	33,000,000

I think that the reason for the groupings which I have adopted will be clear.

They are intended to show :

(*a*) Groups (1) and (2) amounts invested in securities representing a fixed capital either repayable at short notice or in hand.

Groups (3), (4) and (5) amounts invested either in

(*b*) Securities which do not represent a fixed capital, but are well secured incomes in perpetuity such as the three stocks of Table I, or

(*c*) Securities which form a compromise between (*a*) and (*b*), that is investments repayable either in whole, or by instalments, at fixed dates, the value of which will vary with the current rate of interest modified by the length of the unexpired term. These investments may vary from a Colonial bond repayable in a year or two to an American railroad security repayable at par in 100 years;

Group (6)—

(*d*) Land, house property, and ground rents, which must be classified with (*b*), subject in the case of ground rents to a reversion to rack rents (after Government taxes have been satisfied).

Life interests and reversions must be distributed over all the preceding classes.

It will be seen that the investment policy of insurance companies (undoubtedly by force of circumstances) has greatly departed from the two main principles enunciated by Mr. Bailey fifty years ago and nearly always repeated when investment policy is under consideration,

(1) That the first consideration should invariably be the security of the capital;

(2) That the highest practicable rate of interest be obtained, but that this principle should always be subordinate to the previous one, the security of the capital.

Mr. Bailey, indeed, himself admitted in the discussion on Mr. Mackenzie's Paper (in 1891), that whilst he was inclined to adhere to the canons he had previously laid down, circumstances had altered, and it being hardly possible to obtain sufficient investments in which the principal was secure with a reasonable rate of interest, insurance companies must go to the Stock Exchange, and be prepared to run risks of fluctuations in value.

III.—The Effect on Policy Reserves of Changes in the Rate of Interest on Term Investments and Perpetuities.

Admitting that the companies have outgrown their investment surroundings and that the principle, that engagements

to pay fixed sums must be safeguarded by assets also representing fixed sums, cannot be adhered to, it may yet be profitable to consider the effect of fluctuations in security values on the various contracts which are in existence on the books of assurance companies.

In the consideration of the various groups of investments it will be assumed that not only are the assets in the aggregate in accordance with Act of Parliament fully of the value stated in the Balance Sheet, but that each item is taken at its current market price. It is a corollary of this hypothesis that the Balance Sheet relates exclusively to existing contracts, that is, that no account is taken of the fact that the assets may remain hereafter in the company's possession as security against liabilities created by future new business.

It will be convenient, taking a concrete example of a change in the rate of interest, to assume a rise in the yield of stocks from $3\frac{1}{2}$ per-cent to 4 per-cent, and a rise of smaller extent on mortgages and loans. Let us further assume that, from an inspection of the variations in the rates of interest over a period of years the enhanced rate on the mortgages and loans is not likely to disappear for some years even if there is not a further upward movement.

What then is the position of a company, in respect of its investments, after all assets have been written down to current value consequent upon a rise in the rate of interest ?

(1) Mortgages and loans repayable at short notice.

These investments, being fully secured, have not required to be written down, and the company will have the benefit of the increased rate of interest in the future.

(2) Irredeemable debenture stocks and other perpetuities.

These investments have been written down (on the assumption of a rise in the rate of interest from $3\frac{1}{2}$ to 4 per-cent) by $12\frac{1}{2}$ per-cent. Inasmuch as the contracts of the company are for a term of years only there is an actual loss on these securities. By the reduction in book value the company has secured an extra 10*s*. per-cent per annum on the reduced capital for the term for which it requires the investment, and the actual loss is a perpetuity of 10*s*. per annum (=£12.10*s*. per-cent of capital), discounted for the term for which the investment is required by the Company. (A return of 4 per-cent per

annum is, however, assured for the full time of duration of some of the company's contracts.)

(3) Annuities for terms of years. Sums payable at the expiry of terms of years.

Four per-cent has been assured during the term, but if this term is longer than is required, there is a loss of 10*s*. per-cent per annum for the balance of the term.

Taking a general view of the position in this hypothetical case we may say that an extra 10*s*. per-cent per annum has been secured on a portion of the funds up to the time when the Company's liability will mature, and that against the losses described under headings (2) and (3), the company has the benefit of the increased income from (1).

The special circumstances of the individual company must, of course, be considered, but it seems probable that in many cases a few years' purchase of the increased income under (1) will cover the discounted losses under (2) and (3). If this position is admitted it seems to follow that the company is in as good a position at least (notwithstanding depreciation) as at the time when the experience rate of interest was 10*s*. per-cent less on stocks and at the old rate on mortgages and similar securities.

The treatment of the increased rate of interest should be considered, therefore, in respect of two classes of investments :

(*a*) The enhanced rate of interest following the writing down of perpetuities and long term debentures ;

(*b*) The increase in the rate of interest from mortgages and investments at short notice.

So long as the investments under (*a*) are kept at the lower values so long will the reserves get the benefits of the increased rate of interest ; that is, with the same valuation rate, the margin of interest profit is increased for the whole currency of the contracts.

With regard to Class (*b*) the yield will vary with the current rate of interest, but from general considerations it may be said that the effect on existing investments will be more gradual than that shown on the investment of "new money."

As the proportion of life office investments falling under Class (*a*) has, as already shown, largely increased of recent years the effect of the increased rate from these securities will be

correspondingly greater and greater stability is given to the aggregate increase in yield.

The exact extent to which the depreciation in assets affects individual companies depends upon a variety of conditions, but I think there are certain general deductions to be made from the consideration of the balance sheet.

First.—The larger the proportion of irredeemable securities and long term debentures the less the advantage of the increased rate of interest (subject to the qualification that in the event of a fall in the general rate of interest, the benefit of the increased rate will be maintained or that there will be an equivalent rise in the capital value).

Secondly.—The older the business on the books the larger should be the proportion of securities redeemable at short notice. As the contracts of the company are approaching maturity long term investments are not required, and the effect of discount will be less on the losses described under headings (2) and (3) above. The office in fact should be prepared to pay its claims from non-fluctuating securities.

In the converse the more recent the business the larger may be the proportion of long term securities. By these investments the office has provided, for a long term, interest at a higher rate than that used in the valuation. The investment of the premium income will insure that these long term securities may be utilized over a lengthened period.

It follows from these considerations that the reserves for endowment assurances will suffer more than the reserves for whole life policies from depreciation of long term investments.

IV.—The Valuation of the Assets in the Balance Sheet.

The foregoing remarks are based on the assumption that each item of the balance sheet is taken at its current market value, and it appears to me that the proper course at a periodical valuation is in the first place to make a rigid valuation of the assets. The securities for mortgages and loans should be carefully investigated, repayment or reduction being required in weak cases, and if, in any cases, these measures are ineffective, the securities should be realized or written down, and a reduced estimate should be formed of the annual income from these assets. Stocks and shares whether redeemable or not should be taken at the prices of the day with adjustment for accrued

interest. The values of house property and ground rents should be considered carefully and should, as a rule, be taken at a reduction of any estimate of the market value. Loans repayable by terminable annuities and similar securities should, strictly speaking, be valued at the rate of interest obtainable for these securities at the date of valuation, but it would be sufficient probably to alter these values when there was clearly a quarter per-cent or more difference in the rates of interest.

It must be remembered that, in a time of a rising rate of interest, the depreciation in the assets is by no means fully represented by the fall in Stock Exchange securities. Ground rents and house property will fall in value (modified in some cases by exceptional circumstances as, for instance, an increased demand for houses in the centre of a flourishing city), and the securities for mortgages and loans will show a reduced margin of value.

I am aware that this treatment of the balance sheet does not meet with favour in every quarter. I cannot put my finger on references, but I have seen it suggested that the present market value of the assets is not the value to be considered because a company will not require to realize them at the present time, and if it did prices would immediately fall and large blocks of stock could not be sold at the values put upon them. To push this argument to its limit seems to be to say that if all the world wanted to realize everything no value would attach to anything. It is an argument that may be used in respect of the balance sheets of all companies having large tangible assets.

Another plea is that as debenture bonds, terminable annuities, and similar securities are, by means of sinking funds, redeemed eventually at cost price, they may therefore be taken at their cost price. I do not think that this argument is sound. The incidence of the relationship of the invested funds to the individual policies is continuously changing. Claims arise and deplete the funds which are replenished simultaneously from premium income (either new or renewal), so that a proportion of the existing funds represents each year the investment of new money. If these investments are taken at a price above the market value, the rate of interest is artificially low, and the premium income is not being legitimately invested.

To keep a security standing above its realizable value may act as a restraint upon sale when a change of investment might be advantageously made. To sell the investment below book

value, and to write up the new security to the book value of its predecessor would be considered a questionable proceeding, but it does not appear to be worse than keeping the original security at its cost price.

If there are any offices which have retained for a period of, say, 10 or 15 years their long term investments at cost price, the balance sheets of these companies must be distinctly of an artificial nature, unless, of course, the depreciation is covered by under-valuation elsewhere. It is true that if the valuation rate has been kept constant, the margin of income on these particular securities may be much the same as after a thorough revaluation of assets and an accompanying increase in the valuation rate, but it is surely of importance (irrespective of the requirements of Act of Parliament) that the accounts of a Company should represent as nearly as possible its circumstances judged by the monetary standard of the day the books are closed.

If I may use the expression it is a "slipshod" policy to retain investments at the varying prices paid for them from time to time, and to ignore the rates of interest obtainable from similar securities at the time of valuation, whilst other investments closely reflect in their yield the true current rate of interest.

The correct method appears to me to be that which I have endeavoured to indicate in these remarks: to write the assets down rigidly at a valuation, and to make any adjustment that may be indicated to be justified by the new rate of yield, in the actuarial valuation.

The Assurance Companies Act, 1909, seems to require of the companies a rigid valuation of assets on the basis I have indicated. At each valuation "The balance sheet must state how the values "of the Stock Exchange Securities are arrived at, and a certificate "must be appended, signed by the same persons as sign the "balance sheet, to the effect that in their belief the assets set "forth in the balance sheet are in the aggregate fully of the "value stated therein less any investment reserve fund taken "into account." The only escape from a strict valuation of individual assets (to arrive at the aggregate value) seems to be to consider that the expression "Value" in the statutory form of certificate, refers to the value of the asset in relation to the estimated liability. It is true that securities valued at $3\frac{1}{2}$ per-cent are more than sufficient to meet liabilities of the same present value when discounted at 3 per-cent, but there is nothing in the Act to justify the 3 per-cent liability being taken as the

standard of value, and I think that the market price at the date of the balance sheet must be considered to be that standard.

V.—The Actuary's Valuation.

When the assets have been thus scrutinized and revalued, it is probable that there will remain under present circumstances a substantial sum to be written off as depreciation, and on the other hand there will be concurrently an increase in the yield of the investments.

I have endeavoured to show that a large proportion of this depreciation may be correctly set against the increased yield, but the case for similarly treating the balance of the depreciation is not so strong. The difference between the valuation rates and the experience rates, however, has increased so much of recent years that it appears to me that it may be legitimate in many cases to set off some part of this increase in interest profit against the balance of the depreciation. In the first section of this paper I have given the valuation figures of a group of 55 offices, showing that the average valuation rate of the period 1896 to 1900 was £2. 19*s*. 5*d*., and that for the period 1907 to 1911 it was £2. 18*s*. 5*d*. There has been, on the other hand, as shown in Table I, an increase during the 12 years ending with 1912 of 6*s*. per-cent in the aggregate earning power of the British offices, representing an increase in the surplus from interest of, say, 40 per-cent. This increase in yield is in addition to the increase to be created subsequently to 1912 by the reduced values of assets.

Against the proposition that depreciation (limited to that caused by an enhanced rate of interest) should be charged against future interest profits, it may be urged

First—That the reserves are weakened.

Secondly—That when, in process of time, a fall in the rate of interest occurs, the offices will have difficulty in recovering their position.

To the first objection the reply seems to be that the reserves have been substantially strengthened by the increased interest margin, and that a part only of the value of the increase in yield is required for the purpose of the charge for depreciation. The balance will remain to increase future bonuses. If the depreciation is provided by the use of a higher valuation rate (or by an equivalent method), and if care is taken that nothing

beyond the depreciation is released from the valuation reserves, there should be no difficulty in recovering the position when a fall in the rate occurs. On a reduction in the valuation rate following a general fall in the rate of interest, it would be legitimate to take credit for the appreciation of assets consequent on the fall in the interest rate or, in the alternative, if the old valuation rate is retained the large proportion of Stock Exchange securities now included in Assurance Funds would, if kept at the lower prices, give stability to the rate of yield shown by the accounts.

A company is no better off by keeping investments at book values and maintaining its valuation rate than by writing up investments to market values and using the increase to strengthen its reserves by a reduction in the valuation rate. The latter course may indeed be considered the better one, as new business reserves, which will gradually supplant reserves for existing business, should be taken at the lower valuation rate.

Another circumstance to be considered is the large increase, in recent years, of endowment assurances. The shorter average term for these contracts, as compared with whole life assurances, renders the reserves less affected by changes in the valuation rate of interest. As is well known and as shown later (in Tables II and III), a change in the valuation rate affects the reserves for endowment assurances policies to (roughly) one-half the extent of the effect on whole life policies.

If a company on the advice of its actuary decides that the depreciation (or a portion thereof) is to be set against the enhanced interest income of the future it does not seem to be necessary to increase throughout the calculations the rate of interest used in the valuation.

Net premium valuations vary by intervals of not less than a quarter per-cent in the rate of interest, and it may be said that whilst depreciation should be allowed for in the valuation, the amount to be provided would not justify a rise of a quarter per-cent in the valuation rate. In the Summary and Valuation form used under the Fourth Schedule of the 1909 Act, there is a provision for adjustments, following the detailed statement of the valuation of the sums assured and premiums, and there seems no reason why a deduction should not be made from the valuation totals as an adjustment for the increase in the earned rate, and in reply to Enquiry 4 of the Fourth Schedule, it may be stated that the valuation has been made on a, say, 3 per-cent

basis with an adjustment which has the effect of increasing that valuation rate to, say, $3\frac{1}{8}$ per-cent approximately.

This method would have the advantage that, used within well defined limits it would enable further adjustments to be made at subsequent valuations, so as to keep the basis of valuation more in touch with the circumstances existing at the time of the valuation. If, for example, the rate of interest still further increases (with the accompaniment of a further reduction in capital values) it may be desirable to increase the valuation rate throughout the calculations absorbing, in the process, the adjustment made at the previous valuation. If, on the other hand the rate of interest falls back, the old valuation rate will be retained, and the adjustment wiped out or reduced, the increase in capital value, consequent on the reduction in the general rate of interest, being used in whole or in part to extinguish the adjustment deduction.

There are, on the other hand, advantages in an increase in the valuation rate if the circumstances justify the corresponding release of funds to provide for depreciation.

An increase in the valuation rate involves the valuation of smaller net premiums, and a larger provision is thus made for current expenses, and for surplus from loadings, whilst if the increase in the valuation rate is well below the increase in the experience rate the equilibrium between the proportions of surplus contributed by interest and by loadings may be approximately maintained.

A larger reserve of annual premium loading would seem to conform to the present circumstances of life assurance business and would meet some of the objections urged against net premium valuations. Expenses have increased of recent years, and the Agent's remuneration has been concentrated on the first year's premium. It is therefore more in accordance with actualities to meet these expenses from premium income rather than from interest margin which takes the form of an annuity increasing with the duration of the policy.

Modern non-profit rates show but a small margin over net premiums calculated at a low rate of interest, and it is submitted that a valuation rate nearer to the rate at which premiums are calculated would have distinct advantages. A large interest margin on non-profit business seems to be unnecessary and, with present rates of premium, anomalous. If the margin of interest is more than sufficient to maintain a compound

reversionary bonus at a rate justified by the premium loading a similar anomaly arises with regard to with-profit business.

These considerations, however, must be subordinate to the requirement that no reduction in the estimate of net liabilities is to be made unless it can be clearly shown that the reduction is well within the increased reserve provided by the higher experience rate of interest on the whole funds and, in fact, does not much exceed the value of the increased experience rate provided by the amount of the depreciation.

VI.—Illustrations of Changes in the Valuation and Experience Rate of Interest.

The circumstances of offices differ so much as to the proportions of the various classes of the assets forming the invested funds, as to the method of bonus distribution, and as to the difference between the valuation and experience rates of interest, that the subject of this paper must be treated, I think, either by the use of the aggregate figures of the offices (as shown by the Board of Trade Summaries) or by means of illustrative cases.

It does not require to be demonstrated that if the rate of interest earned by the funds increases (in consequence of a general rise in the rate) the stability of the office is increased and a larger surplus arises to be distributed periodically as bonus. The cost of the insurance is reduced, and the payments by the assured being fixed there remains an increased margin of surplus.

If the valuation rate is increased to the full extent of the increase in the experience rate (a course which will have no advocates, I think) the surplus from interest will be somewhat diminished, and the increased surplus will appear as a function of the annual premium income.

If an office has to provide for a deficiency in the funds of 2 or 3 per-cent, and finds that its rate of interest has increased by, say, 7*s*. 6*d*., it may be justified in increasing the net premium valuation rate by a quarter per-cent. In the Tables which follow I have endeavoured by the use of the model offices (Mr. King's for whole life policies and Dr. Buchanan's for endowment assurances) to give illustrations of the effects of a quarter percent change in the valuation rate of interest accompanied by a change of not less than the same amount in the experience rate.

Tables II and III show the amounts (available as a charge for depreciation) released from reserves on a change from $2\frac{3}{4}$ per-cent to 3 per-cent in the valuation rate, and Tables IV and V

trace the effects on bonuses of an increase of a quarter per-cent in the experience rate, if (*a*) depreciation is met by an increased valuation rate, and if (*b*) the old valuation rate is retained, and depreciation is in this way charged against current surplus. These Tables may be of use, perhaps, as a guide under different sets of circumstances.

The effect of the change in the valuation rate indicated in the preceding paragraph is to reduce somewhat the annual surplus from interest (if no more than a quarter per-cent is the increase in the experience rate) and to increase the annual surplus from premium loading. The office premium being fixed, and the higher valuation rate requiring a lower net premium there remains an annual surplus of the difference between the two net premiums.

Mr. King's model office valuations (following the published tables of monetary values) are taken at intervals of a quarter in the rate of interest below 3 per-cent only, and I have therefore adopted $2\frac{3}{4}$ per-cent and 3 per-cent as the valuation rates. Dr. Buchanan's valuations are at intervals of a half per-cent, and I have availed myself of Mr. Elderton's suggestion (in his recent paper) for an approximate valuation of endowment assurances, and have revalued the model office at $2\frac{3}{4}$ and 3 per-cent, taking the age of maturity for all policies as 55.

In calculating the net premium income of the endowment assurance office, I have used age 58 as the maturity age, the 3 per-cent net premiums given by Dr. Buchanan being nearer to those for that age.

The results of these assumption as to ages of maturity compared with the exact valuation of the endowment assurance model office (O^M 3 per-cent) are as follows, the new business of a quinquennium being traced through its history :—

End of Quinquennium	O^M 3 % Reserves		O^M 3 % Net Premiums	
	Exact Valuation	Maturity Age, 55	Exact	Maturity Age, 58
1	109,329	108,995	43,035	42,999
2	274,012	274,004	36,525	36,466
3	419,510	419,848	30,407	30,363
4	417,252	417,383	19,522	19,492
5	270,893	270,855	9,265	9,228
6	151,580	151,769	3,869	3,815
7	49,296	49,349	990	962
8	9,124	9,143	145	138
...	1,700,996	1,701,346	143,758	143,463

The differences in these results are small, and for the purpose of comparison with a valuation at $2\frac{3}{4}$ per-cent, made on similar bases as to ages, they seem to be negligible.

In calculating the cost of a 30*s.* reversionary bonus on endowment assurance policies, at the two rates of interest, the mean valuation ages given in Table XIV of Dr. Buchanan's Paper were used.

TABLE II.

Whole-Life Policies. Model Office.
The new business of a Quinquennium followed through its history.

O^M $2\frac{3}{4}$ and 3 per-cent.

Quinquennium	Reserve at end of Quinquennium		Difference		Net Premiums	
	$2\frac{3}{4}$ %	3 %	Amount	Rate % on $2\frac{3}{4}$ % value	$2\frac{3}{4}$ %	3 %
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	25,811	24,918	893	3·46	13,105	12,767
2	57,163	55,360	1,803	3·15	10,283	10,016
3	79,384	77,119	2,265	2·85	8,486	8,261
4	93,420	91,038	2,382	2·55	6,993	6,803
5	99,511	97,258	2,253	2·26	5,658	5,500
6	97,215	95,280	1,935	1·99	4,404	4,279
7	87,529	85,996	1,533	1·75	3,273	3,175
8	72,439	71,337	1,102	1·52	2,292	2,219
9	54,905	54,190	715	1·3	1,500	1,451
10	36,912	36,503	409	1·11	886	857
...	704,289	688,999	15,290	2·17	56,880	55,328

TABLE III.

Endowment Assurance Policies. Model Office.
The new business of a Quinquennium followed through its history.

O^M $2\frac{3}{4}$ and 3 per-cent.

Quinquennium	Reserve at end of Quinquennium		Difference		Net Premiums	
	$2\frac{3}{4}$ %	3 %	Amount	Rate % on $2\frac{3}{4}$ % value	$2\frac{3}{4}$ %	3 %
(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	111,649	108,995	2,654	2·38	44,047	42,999
2	279,049	274,004	5,045	1·81	37,355	36,466
3	425,046	419,848	5,198	1·22	31,121	30,363
4	421,205	417,383	3,822	·91	20,028	19,492
5	272,998	270,855	2,143	·78	9,518	9,228
6	152,644	151,769	875	·57	3,965	3,815
7	49,576	49,349	227	·46	999	962
8	9,173	9,143	30	·38	146	138
...	1,721,340	1,701,346	19,994	1·16	147,179	143,463

The figures of Tables II and III show that on a change of valuation from $2\frac{3}{4}$ per-cent to 3 per-cent, there would be released in the case of whole life business, 3·46 per-cent of the reserves for policies at the end of the quinquennium in which they were issued, the proportion gradually becoming smaller with the duration of the business. When the policies have been in force for 45 years the proportion is reduced to 1·11 per-cent. For the whole business of the model office after 50 years existence, the proportion of the reserves released is 2·17 per-cent. The figures for endowment assurances show that the increase of a quarter per-cent in the valuation rate releases 2·38 per-cent of the reserves for policies at the end of the quinquennium of issue, the proportion when the office has reached a stationary position being 1·16 per-cent.

With regard to the reserves for reversionary bonuses the proportions released are for various ages and terms (as may be seen from an inspection of the O^M $2\frac{3}{4}$ and 3 per-cent. Tables).

Whole Life Policies		Endowment Assurance Policies at 60	
Age	Rate per-cent	Unexpired Term	Rate per-cent
40	5·6	40	7·4
50	4·3	30	5·8
60	3·1	20	4·0
70	2·0	10	2·2
80	1·2	...	...

It must be remembered that Endowment Assurance reversionary bonus has a much smaller expected duration.

In Tables IV and V there is shown, for the business of a quinquennium, the effect on bonuses (assumed to be declared annually) of an increase of a quarter per-cent in the experience rate

(*a*) If the depreciation of assets has been met by the valuation rate being increased from $2\frac{3}{4}$ per-cent to 3 per-cent;

(*b*) If the $2\frac{3}{4}$ per-cent valuation rate is retained (depreciation being charged against current surplus).

The standard for comparison is taken as a 30*s*. simple reversionary bonus, and it is assumed that a $2\frac{3}{4}$ per-cent valuation

with an interest margin of 1 per-cent on policy reserves will produce this rate of bonus at least.

The changes effected by an increase in the valuation rate from $2\frac{3}{4}$ per-cent to 3 per-cent accompanied by an increase in the experience rate from $3\frac{3}{4}$ per-cent to 4 per-cent are shown in the Tables, and the amount of surplus available for distribution in excess of the 30*s*. bonus as the effect of these changes is given (in column (8) of each Table) at the end of the quinquennium in which the policies were issued, and at intervals of five years subsequently. The rate per-cent of the increase in available surplus calculated on the cost of a 30*s*. bonus is given in column (9).

Table IV.

Whole Life Policies. (King's Model Office).

The new business of a Quinquennium followed through its history at intervals of 5 years.

O^{M} $2\frac{3}{4}$ and 3 per-cent.

Experience Rates $3\frac{3}{4}$ and 4 per-cent. (at least).

Commencement of Year	Released loading if Valuation at 3 % instead of $2\frac{3}{4}$ %	1 % on $2\frac{3}{4}$ % Reserves, less 1 % on 3 % Reserves	Difference (2)−(3)	Cost of 30*s*. Simple Reversionary Bonus			Increase in available surplus if Valuation Rate increased from $2\frac{3}{4}$ % to 3 % (4)+(7)		Increase in available surplus if Valuation Rate retained at $2\frac{3}{4}$ % (*i.e.*, $\frac{1}{4}$% on $2\frac{3}{4}$ % reserves)	
				At $2\frac{3}{4}$ %	At 3 %	Difference	Amount	Ratio % of (6)	Amount	Ratio % of (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6	338	12	326	3,925	3,705	220	546	14·7	89	2·3
11	267	20	247	3,385	3,217	168	415	12·9	160	4·7
16	225	25	200	3,081	2,946	135	335	11·4	210	6·8
21	190	26	164	2,804	2,695	109	273	10·1	241	8·6
26	158	25	133	2,507	2,421	86	219	9·0	253	10·1
31	125	22	103	2,160	2,096	64	167	8·0	243	11·2
36	98	16	82	1,772	1,727	45	127	7·4	216	12·2
41	73	12	61	1,367	1,337	30	91	6·8	177	12·9
46	49	7	42	983	964	19	61	6·3	132	13·4
51	29	5	24	634	624	10	34	5·4	90	14·2
..	1,552	170	1,382	22,618	21,732	886	2,268	10·4	1,811	8·0

TABLE V.

Endowment Assurances. (Buchanan's Model Office).

The new business of a Quinquennium followed through its history at intervals of 5 years.

On $2\frac{3}{4}$ and 3 per-cent.

Experience Rates $3\frac{3}{4}$ and 4 per cent. (at least).

Commencement of Year	Released loading if Valuation at 3 % instead of $2\frac{3}{4}$ %	1 % on $2\frac{3}{4}$ % Reserves, less 1 % on 3 % Reserves	Difference (2)−(3)	Cost of 30s. Simple Reversionary Bonus			Increase in available surplus if Valuation Rate increased from $2\frac{3}{4}$ % to 3 % (4)+(7)		Increase in available surplus if Valuation Rate retained at $2\frac{3}{4}$% (*i.e.*, $\frac{1}{4}$% on $2\frac{3}{4}$% reserves)	
				At $2\frac{3}{4}$ %	At 3 %	Difference	Amount	Ratio % of (6)	Amount	Ratio % of (5)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6	1,048	37	1,011	10,593	10,156	437	1,448	14·3	372	3·5
11	889	59	830	10,045	9,728	317	1,147	11·8	768	7·6
16	758	59	699	9,588	9,369	219	918	9·8	1,060	11·1
21	536	43	493	7,450	7,323	127	620	8·5	978	13·1
26	290	24	266	4,367	4,306	61	327	7·6	620	14·2
31	150	10	140	2,178	2,155	23	163	7·6	329	15·1
36	37	3	34	664	658	6	40	6·1	103	15·5
...	3,708	235	3,473	44,885	43,695	1,190	4,663	10·7	4,230	9·4

It will be seen that in the case of both whole life and endowment assurances the office would have on a change in the valuation rate, an increase of 14 per-cent on the cost of a 30*s*. bonus at the end of the quinquennium of the issue of the policies, but that to maintain that rate of increase there must be something more than a quarter per-cent increase in the experience rate. It may be noted that if the actual increase in the experience rate proved to be 7*s*. 6*d*. instead of 5*s*. per-cent, the ratio of 14 per-cent would be approximately maintained. The rates of column (9) would then be :

Year	Whole Life Per-cent	Endowment Assurance Per-cent
16	14·8	15·4
26	14·1	14·7
36	13·5	13·8
46	13·3	...

If the valuation rate is retained at $2\frac{3}{4}$ per-cent and the experience rate increases by a quarter per-cent, the effect on bonuses in respect of the business of a quinquennium of the model offices will be as shown in columns 10 and 11.

The summation of the figures of Tables IV and V gives the position of the offices when they have reached a stationary position, but the incidence of the allocation of surplus at the various durations of assurance must not be disregarded. It will be noticed that both classes of policies show similar results for offices with a level influx of business over a series of years.

It seems to be desirable to consider these alternative methods of apportionment of the increased profits from interest in relation to the existing methods of bonus distribution and the estimated contributions of policies of various durations to the scale of bonus adopted by the office.

VII.—Conclusion.

In bringing these notes to a conclusion the writer would briefly submit the following propositions for consideration.

(1) That at the periodical investigation a strict valuation of assets should be made, no security being taken at a price above that which, if a marketable security, it would realize according to market quotations at the date of the balance sheet; if not a marketable security the criterion of value should be taken as the price which the office would be willing to pay if the opportunity of making the investment occurred at the date of the balance sheet.

(2) That this strict valuation of assets having been made, the charge for depreciation so far as represented by a future increase in the interest surplus on the company's contracts will be treated properly by the actuary if, in his valuation, he provides for it by an increase in the valuation rate of interest or by an equivalent method, and that the balance of the depreciation (due to the increased rate of interest) may, in many cases, be similarly treated.

Abstract of the Discussion.

Mr. L. A. MOUAT JONES said that the paper came at a most opportune time. Mr. Tilt had shown that, notwithstanding depreciation, the average office was in much the same position financially as if its securities had stood at a higher value, because of the higher rate of interest it was able to earn upon its funds. It seemed, therefore, a little unreasonable that present policyholders should bear the whole of the depreciation. It was only necessary to consider the case of an office which made its valuation as at the 31 December last. What was the position of the ordinary with-profit policyholder? He had seen prices dwindle away to 31 December and then suddenly jump up again in January, just when it was too late to affect his bonus—at any rate his present bonus. And, if he happened to be the holder of an endowment assurance maturing that year, his position was all the more galling because he would not get any compensation out of higher bonuses in the future. No doubt he would be told that the very fact that prices were low meant that any bonus he might get was more valuable to him than if prices were high, because he was able to invest it to better advantage—but whether he would be satisfied with that explanation was another matter.

What was required was some system which would tend to equalize surpluses quinquennium by quinquennium, and Mr. Tilt's method would have precisely that effect. In the first place he took the assets and rigidly wrote them down to their present market prices. That seemed absolutely sound. Even in those cases where the full par value was secured by means of a sinking fund he quite agreed it would be wiser to write down the securities to their market values. In that connection he would like to point out that certain Colonial Government securities, bearing a fixed rate of interest and redeemable at par, partook very much of the nature of well secured mortgages. If mortgages were taken at their full value there was some case apparently for writing up these Colonial securities to their full value. That, however, he thought, would be unwise, because unlike well-secured mortgages they could not, of course, be called in at their full value. He did not think Mr. Tilt expressly said so in his paper, but he gathered he would be in favour of writing up to the market value securities which stood in the books below that value, and it seemed that was quite a reasonable course to pursue, although there were apparently many who objected to it. After all, what was wanted was the true value of the assets on the day when stock of the position was being taken, and the only way to get that was to take the actual values on the particular day.

The more important part of Mr. Tilt's method, however, was his treatment of the valuation of the liabilities. If the valuation of the assets depended—as it obviously must—on the current rate of interest, then there seemed no reason in theory why the value of the liabilities also should not be made to depend upon the current rate of interest; and Mr. Tilt had provided for that by making the

valuation rate of interest vary with the current rate of interest. He put up the valuation rate of interest when the current rate was high, and reduced it when the current rate dropped, so that the valuation of the liabilities corresponded to the valuation of the assets. There were, however, some practical objections to that. In the first place it was no doubt a temptation to weak offices to put up the rate of interest in the valuation in order to maintain their rate of bonus, but there was the more serious objection that the public and agents would not understand the course adopted. Agents of other companies would make the most of the fact that the office was using a higher valuation rate of interest, and that might have a serious effect on new business. The difficulty would be overcome to a large extent if several offices, especially the stronger ones, would agree to adopt the method. Unfortunately it was just the stronger offices that did not need to do it, and the weaker offices that would take advantage of it.

Mr. Tilt pointed out that the balance sheet strictly referred to existing contracts only and had nothing to do with new business, but later on he went on to trace the effect of a depreciation on perpetual and long-term debentures, and said that in those cases there would be an actual loss to the company, because the policies of the company were for comparatively short terms. He was afraid he could not follow Mr. Tilt in that. In considering the effect of depreciation one had to look upon the company as a going concern, and in the case especially of debentures redeemable at par there seemed to be no particular reason why there should ever be a loss.

With regard to policy loans, he did not know whether Mr. Tilt intended to include them in the mortgages. The question whether it was possible to obtain a higher rate of interest on policy loans was a difficult one. It was conceivable that a higher rate of interest might be obtained on new policy loans, but it would be almost impossible in practice to call in the old ones and increase the rate of interest.

Mr. ERNEST W. PHILLIPS said that he believed it was Dr. Moll, the Actuary of the Netherlands Fire and Life Insurance Company, who, in a letter to the *Journal* (xliii, p. 105), first put forward the suggestion that the valuation rate of interest could be increased when the experience rate had risen through depreciation in values. He pointed out the practical possibility of arriving at a valuation on, say, a $3\frac{1}{8}$ per-cent basis by means of interpolation. A few months subsequently, in his Presidential Address, Sir George Hardy referred to the proposal to increase the valuation rate of interest and touched upon the possibility that such a change might arouse the criticism of the "amateur expert." Mr. Tilt had pointed out that the valuation rate could not be increased to the full extent of the increase in the experience rate, so that perhaps there was the further danger that the system readily lent itself to abuse. It was also a question whether circumstances were likely to render the use of an increased valuation rate necessary. So far as he knew there had been no instance of such an increase, while at the same time bonus rates had, in the majority of cases, been

maintained or increased. The circumstances in which a higher valuation-rate would be required would be just those in which there would be a tendency to use the whole of the investment reserve fund in certifying, as required by Note 3 to the statutory balance sheet, that the assets "are in the aggregate fully of the value stated therein, less any investment reserve fund taken into account." If the whole of the reserve were utilized in that way the effect would be the same for practical purposes as if the fund were dispensed with and a sum equal to its amount written off the values of the securities, so that each stood at its market value.

When the difference between the valuation and net experience rates of interest was comparatively large, for example $1\frac{3}{4}$ per-cent, the fact that there was not at the moment a real investment reserve was not, perhaps, of great importance. The accumulation of such a reserve could be accomplished in the near future from interest profit. Moreover, when securities definitely redeemable above their book prices had depreciated in value, a hidden reserve was provided by the certainty that the securities must appreciate as the redemption date approached. If they were to calculate the yield on their securities allowing for redemption, and then value liabilities at a rate depending upon that yield, not only would they be less able to provide a reserve in the future, but also they would have anticipated future appreciation and have eliminated the hidden reserve such appreciation would otherwise have created. If the valuation rate, then, was to be increased they were bound to consider carefully, from the point of view of the investment expert, the necessity for a special reserve.

In a paper read before the Institute in 1912 (*J.I.A.*, xlvi, p. 134), Mr. G. E. May pointed out that high-yielding investments could be considered as analogous to under-average lives, a portion of the interest being treated as a risk premium towards securing the safety of the capital. An important point that arose for consideration was the question of the incidence of the extra risk. The difficulty in forming any definite conclusion on that point was caused by the lack of data—the absence of any cases where a supposed risk had materialized. One was forced to the conclusion that the risk incident to the higher-yielding good-class investments did not appear for some years, and that it was then of an increasing nature. Such a hypothesis would certainly seem reasonable, and since it had always been an established principle among actuaries to provide against a possible future risk, however remote the possibility might be, they must recognize the necessity for accumulating a reserve fund which should not be used in the manner described in Note 3 to the balance sheet as a set-off against depreciation. They must have that reserve as a provision against any possible future loss in the recently purchased high-yielding securities. Mr. May suggested that instead of carrying the risk premium to a reserve fund it could be used year by year in writing down securities, thus creating a hidden reserve. In the circumstances under consideration that method would not be allowable, because when securities were valued in bulk those standing

below their market values would automatically be set off against those standing above, and the hidden reserve would disappear. It was, therefore, essential if they were to increase the valuation rate of interest that they should have an investment reserve that would remain unused until the future, as otherwise the present policyholders might benefit unduly at the possible expense of the policyholders in years to come.

Coming, now, to the conclusions expressed by Mr. Tilt at the end of the paper, he thought that in circumstances such as those prevailing on 31 December last when prices of securities were at an extremely low level, it was not necessarily desirable to write redeemable securities down to their market prices, but, of course, they must hold a reserve against any depreciation which they did not write off. For example, during the early months of the present year a considerable recovery took place, and it might be that such low levels would never again be reached. If the rule of never writing up book values were enforced they would have securities standing at very low values which they chanced to fall to on a particular date, a position which seemed to be almost as illogical as the retention of the original purchase prices. If they were never to write up the values of securities he did not think that in the case of redeemable securities they should also bind themselves with a hard-and-fast rule to write down to the quotations at date of valuation.

Mr. H. J. RIETSCHEL said that in ascertaining the amount of the funds which could be released on account of the increase in the realized rate of interest, the author had dealt with the question on the basis of a net premium valuation. In the net premium method of valuation no specific provision was made for the whole of the liabilities of the office. For example, the provision for the expenses might be derived partly from the loadings reserved in the valuation and partly from the surplus interest. Without a careful analysis of the various liabilities provided for in a net premium valuation it could by no means be assumed that an increase in the experience rate of interest justified the employment of a corresponding increase in the valuation rate of interest. Personally, therefore, he preferred to approach the subject from the point of view of an office premium valuation in which specific provision was made for all the liabilities of the office and experience rates of interest were employed. On that basis, and assuming a depreciation of 12½ per-cent of the assets, corresponding to an increase in the rate of interest from 3½ per-cent to 4 per-cent, he had made certain valuations which showed that the greater part of the depreciation was met by the reduction in the reserves due to the increased valuation rate of interest; and that the balance of the depreciation which was not so met was recouped by the reduction in the liabilities in respect of a very few years' new business, consequent on the higher valuation rate of interest which could then be employed. Rates of bonus and rates of expenses differing from those which he himself employed would not, he thought, materially alter the general conclusion at which he arrived, that for a strong and progressive office a large depreciation

due to an increase in the general market rate of interest was not a weakness, but a source of ultimate profit.

Mr. E. B. NATHAN said that the principle now adopted by life assurance companies in valuing their securities differed widely from the principle adopted in valuing their liabilities, the former being analogous to that employed by a bank. But the position of a life assurance company was quite distinct from that of a bank. A bank had a very large liability which was liable at any time to be called in; on the other hand, apart from guaranteed surrender values, insurance companies' liabilities accrued slowly and could be foreseen a long time in advance. It therefore seemed to him that in adopting a method of valuation drawn to a great extent from banking practice, which involved the rigid writing down of securities from time to time to their market prices, insurance companies were rather hampering their field of investment and penalizing their policyholders. In that connection it was useful to consider the type of depreciation which might affect market securities. Depreciation might be divided into four classes. In the first place there was actual depreciation of the security itself, and as an example of that he might instance the conditions which prevailed when a Government had to suspend payment for a certain period. There was no doubt that in that case the security was depreciated, but frequently the market depreciation exaggerated the actual loss. Secondly, there was depreciation due to an increased demand for money. That demand did not really affect an insurance company at all. Thirdly, there was depreciation arising from what might be termed "fashion." And finally there was depreciation due to market manipulation. The majority of offices were called upon to value their securities at the 31 December in the valuation year, a time of year when the demand for money was at its highest, and the market was more or less susceptible to any "bearing" manipulation.

In giving weight to these varying influences an office converted depreciation which was frequently purely nominal and temporary into a permanent loss to its present policyholders. At the present time there was almost a parting of the ways, and it seemed to him that insurance companies must consider whether they intended to deal with their investments from a Stock Exchange point of view or independently of the Stock Exchange altogether. If they adopted the first course it seemed a fair suggestion that they should make an attempt, not perhaps to control the market, but to some extent to influence it. That brought him to the question of "jobbing in and out" of securities. He believed Mr. Burn had criticized that, but if insurance companies were prepared to write down their securities on every occasion to their market price it seemed only fair to the policyholders that when prices stood above the book values the company should avail themselves of the profit and take it in to the realized surplus. The other alternative was to stand outside the market altogether. Although that would be an ideal condition, he was afraid from a practical point of view it was impossible. If an office could follow up the valuation of its liabilities

by valuing its assets in a similar manner, *i.e.*, working over long periods instead of working over short periods, the assets could be valued so as to yield a definite margin over the rate assumed in the valuation. This interest margin might vary with the different classes of security. Any actual loss would, of course, be written off as it arose. While this suggestion would, with proper safeguards, give as sound a valuation as the method of following the market fluctuations, and would eliminate heavy writing down of securities at the end of a valuation period, it would be difficult to explain to the public and might be subject to a good deal of criticism. The alternative seemed to be to follow Mr. Tilt's suggestion, and vary the valuation rate to meet market depreciation, although this course could also be misrepresented by competitors.

Mr. S. G. WARNER said that the paper and the discussion upon it led him to reflect how very unlikely it would have seemed twenty years ago that such a paper should ever require to be written, or that members of the Institute should be occupying themselves with the questions and problems which it presented for their consideration. At that time the conditions were precisely opposite to those prevailing at present. The rate of interest had then fallen to a very low point, and there was a belief in the minds of many of the ablest financiers that that condition of things had come to stay. From that period might be dated the origin of some of the difficulties which were confronting life offices to-day. It was under the influence of those conditions that the step began to be taken—a step the full significance of which was only now beginning to be realised—of reducing valuation rates below 3 per-cent. There was, of course, a great deal to be said for such a measure under those conditions; but some actuaries were deterred from taking it by the reflection that once taken it would be very difficult to retrace; difficult not for any abstract reason but because of a practical objection to which reference had been made—the effect on public opinion. Public opinion might not always be reasonable, but it had a very important influence upon the conditions under which life assurance business was done and upon the success of the companies' operations.

With such very different financial conditions as now existed the natural course would be to return to a higher valuation rate, *i.e.*, to a valuation rate which would correspond to present conditions just as the lower valuation rates corresponded to the former conditions. It was unreasonable that such a step should be taken as an indication of weakness, but it certainly would be so taken, and therein lay the chief difficulty. The holding in hand of an extra reserve, without actually advancing the rate, as a precaution against varying financial conditions, would, he was afraid, be likely to be misinterpreted also. There were very few things about which the public were less reasonable than the matter of precautionary reserves. A reserve might be set up especially to meet "unforeseen contingencies", but if any unforeseen contingency happened, and an attempt was made to apply even a part of that reserve, it was sure

to be interpreted as a sign of weakness. Under those circumstances what was to be done? That led him to the third course, the departure from the strict method of writing securities down to the value of the day in the balance sheet. He was afraid, however, that was the least desirable of all the courses suggested. It seemed to him that at the date of valuation an office must regard the position as if it had immediately to realize. There were certain securities on the books: What could they be sold for in the market? That was all for which it ought properly to take credit.

The previous speaker had referred to the gradual tendency of offices to put a larger proportion of their funds into Stock Exchange securities. He sometimes wondered whether that had not gone quite far enough in view of the fluctuations of which sad experience had shown such securities to be capable. Fifteen or twenty years ago it was very customary to hear that mortgages were an undesirable form of investment, because, while it was never possible to get any profit on them there might be heavy losses; and on the other hand Stock Exchange securities offered golden opportunities to the prudent and judicious investor. Recent events might have taught them that there was something to be said on the other side, and that the despised mortgage might have its solid value when it was found that although no profit could be made on it, neither could a loss—provided, of course, a sufficient margin had been required.

Mr. H. J. P. OAKLEY observed that Mr. Tilt referred to the canons of investment laid down by Mr. Bailey, but there were two further canons which sometimes seemed to be lost sight of. The first was: When prices are high, place your money in non-fluctuating securities; and the second: When prices are low, realize non-fluctuating securities and invest in securities likely to rise in value. From an examination of the investments made in the past twenty years, it would appear either that those canons had not been acted on, or that those who had made the investments had been mistaken as to which were the times of high and low prices respectively. Taking the period 1895 to 1900, the quinquennium of high prices, it would be found that the sums invested in mortgages rose about 5 millions only, on a total of 120 millions, while the sums invested in Stock Exchange securities rose from 81 millions to 121 millions. He was aware that in applying the canons in question it was necessary to make a correct forecast of the future, and to have the courage to put one's theories into practice. At the same time the canons held good, and it was desirable as far as might be possible to put them into practice. On the other hand, it had to be admitted that had they been put into practice it was possible that to a certain extent they would have defeated themselves, because if the enormous sums which had been placed in Stock Exchange securities during those years had been invested in mortgages there would have been a glut of investment money available to mortgagors, while the fall in Stock Exchange prices which then began might have been even greater, and there would have been a tendency to throw the canons on one side in order to secure the higher rate of interest.

With regard to the writing down of securities, he thought most members would agree with the first part of Mr. Tilt's first conclusion, that a strict valuation of the assets should be made ; but he was not sure they would all agree with his conclusion that if the security was not a marketable security the criterion of value should be taken as the price which the office would be willing to pay if the opportunity of making the investment occurred at the date of the balance sheet. If the capital was secured it seemed to him that a good many would prefer to keep the price unaltered and be satisfied with a lower rate of interest. While most would agree that the valuation rate should not be increased to the full extent of the increase in the experience rate. Mr. Tilt's suggestion might perhaps lead some—where there was now a wide margin between experience and valuation rates—to value at a higher rate, and so arrive nearer the truth in regard to the respective margins of loading and interest.

The CHAIRMAN (Mr. Geoffrey Marks. Vice-President) said that the subject of the paper must have been very much in the minds of many there, not only at the end of last year but for the considerable time during which the continuous fall in securities had been so apparent. So far as the second part of Mr. Tilt's first conclusion was concerned, it was a counsel of perfection, but, in dealing with insurance interests, it was their desire as actuaries to place their companies in such a position that counsels of perfection could be adopted. The ideal situation in regard to investments was that in which all depreciation could be written off and no credit taken for any appreciation which had not been actually realized. It was unnecessary for him to point out the advantages of such a method : how the writing-down created a hidden reserve, and how the realization of an investment which had been dealt with in that way released automatically the particular proportion of the hidden reserve appropriate to that investment. Between this principle and that other principle which was very seldom but still occasionally followed, by which no precautions were taken to meet such conditions as now existed, there lay a wide possibility of choice of which the offices took advantage. He had recently looked into the statements made in the Board of Trade returns in reply to the question as to the valuation of Stock Exchange assets at an ordinary date, not a valuation date, and he found that, among the 66 offices making returns and statements, there were 9 different methods in use, and the largest number of offices adopting one particular method was 15.

In the consideration of such questions as that before them some confusion of thought was introduced by the fact that the term "balance sheet" was applied to both the accountant's and the actuary's statements of the position of their office. The accountant's balance-sheet, as had been pointed out, was a statement of the position of the office in regard to its assets and liabilities, on a particular date and at a given moment, and it was concerned not with the past or with the future, but only with the present. The actuary's balance-sheet on the other hand, was one to which that prepared by the accountant was a preliminary. The actuary

incorporated in his statement the accountant's results, but he also had to take into consideration rates of interest and mortality, and these involved him in a wide survey of the past and in an intelligent appreciation of the future. He had to apply to present conditions the lessons of the past, and to use an enlightened judgment in estimating the future. He had to deal with elements which were unstable in operation, except over long periods and in large masses, and to exercise a judgment and consider conditions which were outside the range of the accountant's training and duties. In fact the essential condition which had to be considered in actuarial work was that life assurance as an institution and life assurance offices as institutions were of a very permanent and lasting character, and in considering the problems which arose, whether financial or otherwise, it did not become actuaries to adopt temporary expedients, which were unfitted for application to the circumstances in which they worked. The period during which the value of money might be above or below the average was small as compared with the life history of an office, or even with the average duration of the policies in that office. As the author had pointed out, and Mr. Warner had emphasized, a long period of cheap money had led many offices to reduce their valuation rate, but he was inclined to think that they had taken this course not so much on account of the condition of the money market but because they appreciated the fact that having once got down to a low rate they placed themselves in a very strong position with regard to the future. However this might be, there was no doubt that those offices which did reduce their rate had adhered to a lower rate as a matter of financial policy, a policy which tended to safeguard existing interests and to provide a favourable outlook for the future. High prices made the change possible and the converse condition of things existing at the moment might justify a reversal of the process which was then adopted.

But there was a great difference between a change in policy which strengthened the position of an office and one which weakened it, and he could not help thinking that to raise the valuation rate of interest was a method which did weaken the position of the office —both its actual position and its standing in public estimation. He doubted whether the method which had been suggested was really justified from the point of view of the interests of the policyholders as a whole. Did it conform with the real object of life assurance, to provide the greatest good for the greatest number? Was it justifiable to release for bonus purposes money which would otherwise go to meet depreciation? It would take too long to enter fully into a discussion, which must necessarily have something of an ethical character, but the point was one which ought to be considered. In thinking for a moment who would suffer in the circumstances supposed, if the valuation rate were not raised, it would appear that old policyholders had probably derived advantage from a time when easier conditions prevailed, and new policyholders would get their advantage from the fact that the maintenance of the valuation rate on its lower basis provided them with

a larger margin of interest profit and therefore with better prospects of bonus. The only classes of policyholders who seemed likely to suffer to any appreciable extent were those who had short-term endowment assurances and those who died early, and he did not think either of those classes was entitled to very special consideration.

Life assurance in its modern manifestations was not a business for temporary expedients, and particularly so owing to the length of time necessary before the results of mistakes could be ascertained. It took many years to discover mistakes which might be the result of a judgment formed in perfect good faith and with a complete appreciation of the responsibility involved in making any change in familiar methods. Changes which looked at the moment desirable or even necessary might prove not only dangerous, but often disastrous.

The late Mr. Bailey, when he laid down the canons for investment referred to by Mr. Tilt, had in mind investments in Consols and similar irredeemable securities familiar to managers in his day, and commonly called "gilt-edged." Such securities could not possibly offer any guarantee for the safety of the capital, and the more one studied the history of prices in the last few years the more evident it was that the only security for capital, no matter how invested, lay in the exercise of sound judgment in the choice of investments, whatever the class to which they belong.

Mr. TILT, in reply, said he was amply repaid by the discussion for the trouble he had taken over the paper. He had intended to go through the criticisms of the various speakers, but many who had spoken appeared to agree with the conclusions in the paper, and it was not, therefore, necessary to reply at length. The Chairman seemed to suggest that the methods proposed had the effect of weakening the bases of valuation, but the object of the paper was to show that the reserves would remain as strong as if not stronger than before the depreciation arose. It had been suggested that a company should be treated as a going concern and that, therefore, the discounted losses described in the paper should not be taken into account. But, surely, if a perpetuity or a long-term debenture were written down and present policyholders were not charged with any of that depreciation they would be credited with too much benefit from the increased rate of interest. The present policyholders would not derive any benefit from the interest that the reduced value of the investments would earn after the policies had gone off the books, and if the adjustment for depreciation were not limited as suggested in the paper, they would be given the benefit of the whole increase. Another suggestion was that term debentures should not be written down. If the office was to be treated as a going concern and was not to write down the redeemable securities but to write down the irredeemable, where was the line to be drawn? Was a stop to be made in 1950, 1960 or when? He thought, for the reasons given in the paper, that it was illegitimate to keep debentures at above market values.

On a Short Method of Constructing an Abridged Mortality Table. *By* GEORGE KING, F.I.A., F.F.A., *Consulting Actuary.*

MEDICAL Officers of Health and others interested in the Public Health Service of the country, frequently wish to compare the mortality of a particular district with that of other districts or of the general community, without having to go through the labour of constructing a complete mortality table. Also it may be frequently useful to make similar comparisons in the case of the mortality experience of a Life Office at the stage when only the numbers at risk and the deaths are available. The following is a short and simple method of constructing an abridged mortality table for quinquennial points of age, including in the functions tabulated, in addition to the probability of living five years, the expectation of life and the annuity value, and it will be found to serve with ample accuracy the purposes in view.

In dealing with census returns and records of deaths, the population and the deaths are given in quinquennial age groups, whereas in the case of the experience of a Life Office the numbers exposed to risk at the beginning of each year of age are supplied, and the deaths. Hence, from Life Office statistics, we obtain directly q_x, whereas the national statistics give us first the central death rate, m_x, from which p_x can be derived by the usual formula. The same methods, however, after this point has been reached, apply to both sets of data.

The first step in the abridged process is to obtain graduated pivotal values at quinquennial age points of the exposed to risk, or population as the case may be, and of the deaths, and the formula given in my paper "A New Method of Constructing and of Graduating Mortality and other Tables", *J.I.A.*, vol. xliii, p. 115, is suitable. That formula has also been demonstrated by Mr. S. T. Shovelton, *J.I.A.*, vol. xlvii, p. 288. The following is a simplified way by which it can be derived:

Let there be a series of fifteen values of u, the function to be dealt with, from u_0 to u_{14}. To find the graduated value of u_7 the central term of the series.

Let y be the finite integral of the function u, so that $y_x = \Sigma_0^{x-1} u$; and let Δy, $\Delta^2 y$, etc., be the differences of y for quinquennial intervals, so that $\Delta y_0 = \Sigma_0^4 u$, $\Delta y_5 = \Sigma_5^9 u$, etc.

Then $$u_7 = y_8 - y_7$$

But $$y_8 = y_0 + \frac{8}{5}\Delta y_0 + \frac{24}{50}\Delta^2 y_0 - \frac{8}{125}\Delta^3 y_0$$

$$y_7 = y_0 + \frac{7}{5}\Delta y_0 + \frac{14}{50}\Delta^2 y_0 - \frac{7}{125}\Delta^3 y_0$$

$$u_7 = \frac{1}{5}\Delta y_0 + \frac{1}{5}\Delta^2 y_0 - \frac{1}{125}\Delta^3 y_0$$

$$= \cdot 2\Delta y_5 - \cdot 008\Delta^3 y_0 .$$

The differences, Δ, of y are the sums of five values of the function u, and may be represented by the symbol w, so that

$$w_x = u_x + u_{x+1} + \ . \ . \ . \ + u_{x+4} .$$

Making this change in notation, we have

$$u_7 = \cdot 2w_5 - \cdot 008\Delta^2 w_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{i})$$

We may use as an example the O^M Table, and, seeing that the construction and graduation of that table were effected by methods totally different from those employed in the abridged process, the test is severe. Taking the exposed to risk and the deaths, and starting from age 10, we group them quinquennially, 10–14, 15–19, &c. These groups are the w_x of formula (i), and, differencing twice, the formula is applied. The actual working, which is very short, need not be reproduced here, but the following are the results :

O^M Table.

Original Data			Graduated Pivotal Values			
Ages	At Risk	Deaths	Ages	At Risk	Deaths	q_x
10 to 14	3,056	10	17	4,150·9	14·424	·00347
15 ,, 19	25,844	97	22	33,719·7	152·400	·00452
20 ,, 24	175,866	806	27	101,854·7	516·776	·00507
25 ,, 29	507,580	2,615	32	160,622·2	1,042·256	·00649
30 ,, 34	796,958	5,202	37	187,404·6	1,512·848	·00807
35 ,, 39	932,507	7.557	42	191,596·0	1,949·232	·01017
40 ,, 44	955,150	9,731	47	181,838·2	2,302·408	·01266
45 ,, 49	907,042	11,526	52	161,322·1	2,735·760	·01696
50 ,, 54	805,206	13,670	57	133,759·9	3,122·200	·02334
55 ,, 59	668,260	15,594	62	103,599·0	3.425·920	·03307
60 ,, 64	517,825	17,093	67	72,514·4	3,552·288	·04899
65 ,, 69	363,146	17,677	72	44,311·8	3,249·408	·07333
70 ,, 74	222,818	16,150	77	22,467·4	2,446·816	·10891
75 ,, 79	113,960	12,197	82	8,850·9	1,459·976	·16495
80 ,, 84	45,674	7,317	87	2,396·3	554·768	·23151
85 ,, 89	12,871	2,865	92	394·14	125·864	·31934
90 ,, 94	2,309	692	97	25·760	13·016	·50528
95 ,, 99	205	86	...	...	...	...
100 ,, 104	6	3	...	...	...	...

Having thus the graduated pivotal values of q_x, we pass to $\log p_x$. In order to get the column l_x at quinquennial points we must have $\log_5 p_x$, the logarithm of the probability of living five years, so that, starting from the earliest age, in this case 17, with the logarithm of a suitable radix, and adding continuedly $\log_5 p_x$, we get $\log l_x$ at the quinquennial points. We must therefore find a formula to obtain from $\log p_x$ at quinquennial points the values of $\log_5 p_x$.

Employing the usual formula of finite differences

$$u_n = u_0 + n\Delta u_0 + \frac{n(n-1)}{2}\Delta^2 u_0 + \frac{n(n-1)(n-2)}{6}\Delta^3 u_0,$$

and remembering that the differences are for quinquennial periods, and taking n successively as

$$\frac{5}{5}, \frac{6}{5}, \frac{7}{5}, \frac{8}{5}, \frac{9}{5}, \text{ and } \frac{10}{5},$$

we have the following scheme—

$$\begin{aligned}
u_5 &= u_0 + 1{\cdot}0\Delta u_0 \\
u_6 &= u_0 + 1{\cdot}2\Delta u_0 + {\cdot}12\Delta^2 u_0 - {\cdot}032\Delta^3 u_0 \\
u_7 &= u_0 + 1{\cdot}4\Delta u_0 + {\cdot}28\Delta^2 u_0 - {\cdot}056\Delta^3 u_0 \\
u_8 &= u_0 + 1{\cdot}6\Delta u_0 + {\cdot}48\Delta^2 u_0 - {\cdot}064\Delta^3 u_0 \\
u_9 &= u_0 + 1{\cdot}8\Delta u_0 + {\cdot}72\Delta^2 u_0 - {\cdot}048\Delta^3 u_0 \\
u_{10} &= u_0 + 2{\cdot}0\Delta u_0 + 1{\cdot}00\Delta^2 u_0
\end{aligned}$$

If now we add the first five lines, we have the sum of five values of u from u_5 to u_9 inclusive, that is, we have the sum in what may be called "initial form," and if we add the last five lines we have, similarly, the sum in what may be called "terminal form." The sum of the five values being our function w, the following equations result:

$$w_5 = 5u_0 + 7\Delta u_0 + 1{\cdot}6\Delta^2 u_0 - {\cdot}2\Delta^3 u_0 \quad . \quad . \quad . \quad \text{(ii)}$$

$$w_6 = 5u_0 + 8\Delta u_0 + 2{\cdot}6\Delta^2 u_0 - {\cdot}2\Delta^3 u_0 \quad . \quad . \quad . \quad \text{(iii)}$$

These formulas are central, there being three groups of five, and the formulas giving the middle group. At the youngest age of the table, however, we cannot use the central method, but, as it happens, with very little loss of accuracy we may find the corresponding sum for the first group of five values in the set of fifteen. In the formula of finite differences we take n successively as

$$\frac{0}{5}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \text{ and } \frac{5}{5},$$

and we have the following scheme

$$u_0 = u_0$$
$$u_1 = u_0 + \cdot 2\Delta u_0 - \cdot 08\Delta^2 u_0 + \cdot 048\Delta^3 u_0$$
$$u_2 = u_0 + \cdot 4\Delta u_0 - \cdot 12\Delta^2 u_0 + \cdot 064\Delta^3 u_0$$
$$u_3 = u_0 + \cdot 6\Delta u_0 - \cdot 12\Delta^2 u_0 + \cdot 056\Delta^3 u_0$$
$$u_4 = u_0 + \cdot 8\Delta u_0 - \cdot 08\Delta^2 u_0 + \cdot 032\Delta^3 u_0$$
$$u_5 = u_0 + 1\cdot 0\Delta u_0$$

whence summing as before we have the initial and terminal forms

$$w_0 = 5u_0 + 2\Delta u_0 - \cdot 4\Delta^2 u_0 + \cdot 2\Delta^3 u_0 \quad . \quad . \quad . \quad \text{(iv)}$$
$$w_1 = 5u_0 + 3\Delta u_0 - \cdot 4\Delta^2 u_0 + \cdot 2\Delta^3 u_0 \quad . \quad . \quad . \quad . \text{(v)}$$

To obtain $\log {}_5p_x$ we use the initial forms, namely, equations (ii) and (iv), and it will be seen how short these are, and how simple are the coefficients. The formulas can be applied with great facilty on the arithmometer.

Table 1 annexed shows all the figures required to find the quinquennial values of l_x by the O^M Table, but with regard to them there is one further point to be mentioned. From the nature of the case, seeing that the last value of $\log p_x$ derivable from the data is that for age 97, the last difference of the third order is that for age 82, and hence the last value of $\log {}_5p_x$ given by the formula is that for age 87. To carry the Table on to the end, we take the fourth difference of $\log p_{77}$, and by continued addition carry $\log p_x$ down to age 107. Hence we get $\log {}_5p_x$ as far as age 97, and $\log l_x$ as far as age 102, where the Table comes practically to an end. For the radix at age 17 in the example, the value of l_{17} by the O^M Official Table is assumed.

In the Tables positive values are printed in ordinary type, and negative values in old style type.

Having now l_x at quinquennial points, the next step towards the construction of e_x is to obtain at each quinquennial point the sum of five values of l_x, and to do so we take the terminal form of the equations, and use equations (iii) and (v) in exactly the same way as we used equations (ii) and (iv) to obtain $\log {}_5p_x$. Table 2 annexed shows the working, the sum of the five values of l_x at age x being represented by the usual notation $N'_{x\overline{5}|}$. The last value in the Table must be filled in empirically as the formula gives $N'_{x\overline{5}|}$ only down to age 92. We already have l_{97} and p_{97}, from which we derive l_{98}, and we have also l_{102}, and by inspection we fill in the three missing values.

Now summing $N'_{x\overline{5}}$ from the bottom upwards we form the column N'_x, and dividing each value therein by the corresponding value of l_x we obtain the curtate expectation of life.

The expectation of life is that for the central *integral* age of the group, but in the Public Health Service it is more usually required for the central *point* of age of the group, that central expectation of life being the mean expectation for the five ages included in the group. We can by second differences very easily and accurately obtain this central expectation, $e_{x+\frac{1}{2}}$. Taking three values of e, which may be written e_0, e_5, and e_{10}, and applying the formula of finite differences,

$$e_{\frac{1}{2}} = e_0 + \cdot 1\Delta e_0 - \cdot 045\Delta^2 e_0 \quad . \quad . \quad . \quad . \quad . \quad \text{(vi)}$$

$$e_{5\frac{1}{2}} = e_0 + 1\cdot 1\Delta e_0 + \cdot 055\Delta^2 e_0 \quad . \quad . \quad . \quad . \quad . \quad \text{(vii)}$$

$$e_{10\frac{1}{2}} = e_0 + 2\cdot 1\Delta e_0 + 1\cdot 155\Delta^2 e_0 \quad . \quad . \quad . \quad . \quad . \quad \text{(viii)}$$

Formula (vi) applies to the first value of e in the table, and formula (viii) to the last; while formula (vii) applies to all the intervening values. For short distances the expectations of life do not differ much from an arithmetical progression, and the second difference employed above has very little effect. We might bring in a third difference, but that would not affect the third decimal place in the expectation. To Table 2 has been added the expectation of life for these central points of age obtained in this way.

Instead of obtaining the expectation of life, we may with equal ease obtain the value of the annuity. We have merely to multiply each quinquennial value of l_x by the corresponding value of v^x and obtain D_x, and then use D_x in the same way that l_x was used in Table 2. The values of annuities at 3 per-cent interest by the O^M Table are given in the annexed Table 3, with the full working.

In Table 4 is given a comparison of the values of the functions derived by the abridged method above explained with those of the official table graduated by Sir George F. Hardy, and it will be seen how remarkably accurate the approximate results are. Until age 82 the expectations of life never differ by more than 2 in the second decimal place, while from age 82 onwards the differences are unimportant. The approximate values of the annuities are even more accurate, there never being a difference of more than 1 in the second place of decimals until we reach age 82, and the largest difference, being that at age 87, amounts to 110 in the decimal places. We could scarcely desire anything better.

Table 1.

Abridged O^{M} *Table.*

Age	$\log p_x$	Δ	Δ_2	Δ_3	Δ_4	$\log {}_5p_x$	$\log {}_5p_x$	$\log l_x$	l_x	Age
17	·00151	46	22	60	...	·00868	$\bar{1}$·99132	4·98932	97.571	17
22	·00197	24	38	31	...	·01030	·98970	·98064	95,640	22
27	·00221	62	7	16	...	·01220	·98780	·97034	93,399	27
32	·00283	69	23	6	...	·01547	·98453	·95814	90,811	32
37	·00352	92	17	64	...	·01936	·98064	·94267	87,633	37
42	·00444	109	81	12	...	·02418	·97582	·92331	83,813	42
47	·00553	190	93	58	...	·03110	·96890	·89913	79,274	47
52	·00743	283	151	136	...	·04232	·95768	·86803	73,796	52
57	·01026	434	287	118	...	·05910	·94090	·82571	66,944	57
62	·01460	721	405	170	...	·08604	·91396	·76661	58,427	62
67	·02181	1,126	575	545	...	·12961	·87039	·68057	47,926	67
72	·03307	1,701	1,120	334	...	·19598	·80402	·55096	35,560	72
77	·05008	2,821	786	878	6,044	·30301	·69699	·35498	22,645	77
82	·07829	3,607	1,664	6,922	...	·45869	·54131	·05197	11,271	82
87	·11436	5,271	8,586	12,966	...	·65672	·34328	3·59328	3,920	87
92	·16707	13,857	21,552	19,010	...	1·05221	$\bar{2}$·94779	2·93656	864	92
97	·30564	35,409	40,562	...	...	2·11215	$\bar{3}$·88785	1·88435	77	97
102	·65973	75,971	...	...	...	...	...	$\bar{1}$·77220	1	102
107	1·41944	...	...	...	...	...	...	...	...	...

Table 2.

Abridged O^{M} *Table.*

Age	l_x	Δ	Δ^2	Δ^3	$N'_{x\overline{5}\vert}$	N'_x	e_x	$e_{x+\frac{1}{2}}$	Age
17	97,571	1,931	310	37	482,179	4,454,514	45·65	45·23	17
22	95,640	2,241	347	243	471,608	3,972.335	41·53	41·12	22
27	93,399	2,588	590	52	459,418	3,500,727	37·48	37·08	27
32	90,811	3,178	642	77	444,767	3,041,309	33·49	33·10	32
37	87,633	3,820	719	220	426,977	2,596,542	29·63	29·25	37
42	83,813	4,539	939	435	405,780	2,169,565	25·89	25·52	42
47	79,274	5,478	1,374	291	380,399	1,763,785	22·25	21·88	47
52	73.796	6,852	1,665	319	349,032	1,383,386	18·75	18·41	52
57	66,944	8,517	1,984	119	309,900	1,034,354	15·45	15·13	57
62	58,427	10,501	1,865	1,316	261,402	724,454	12·40	12·11	62
67	47,926	12,366	549	2,090	203,015	463,052	9·66	9·41	67
72	35,560	12,915	1,541	2,482	138,857	260,037	7·31	7·10	72
77	22,645	11,374	4,023	272	77.990	121,180	5·35	5·18	77
82	11,271	7,351	4,295	2,026	32,638	43,190	3·83	3·70	82
87	3,920	3,056	2.269	1,558	9,119	10,552	2·69	2·58	87
92	864	787	711	...	1,363	1,433	1·66	1·57	92
97	77	76	...	arbitrary	70	70	·91	·85	97
102	1	...	...	...	...	...	...	...	...

TABLE 3.

Abridged O[M] *Table—continued.* *Interest* 3 *per-cent.*

Age	D_x	Δ	Δ^2	Δ^3	Age	N_{x+5}	N_x	a_x
17	59,032·	9,118·	1,251·	166·	17	267,272	1,377,752	23·339
22	49,914·	7,867·	1,085·	212·	22	225,502	1,110,480	22·248
27	42,047·	6,782·	873·	101·	27	189,497	884,978	21·047
32	35,265·	5,909·	772·	94·	32	158,269	695,481	19·722
37	29,356·	5,137·	678·	112·	37	131,079	537,212	18·300
42	24,219·	4,459·	566·	124·	42	107,469	406,133	16·769
42	19,760·	3,893·	442·	59·	47	86,919	298,664	15·115
52	15,867·	3,451·	383·	49·	52	68,817	211,745	13·345
57	12,416·	3,068·	334·	19·	57	52,733	142,928	11·512
62	9,348·	2,734·	353·	120·	62	38,401	90,195	9·649
67	6,614·	2,381·	473·	108·	67	25,762	51,794	7·831
72	4,233·	1,908·	581·	47·	72	15,230	26,032	6·150
77	2,325·	1,327·	628·	172·	77	7,402	10,802	4·646
82	998·41	698·87	456·28	266·26	82	2,676	3,400	3·405
87	299·54	242·59	190·02	141·78	87	641	724	2·417
92	56·95	52·57	48·24	...	92	79	83	1·457
97	4·38	4·33	...	Arbitrary	97	4	4	·913
102	·05	...	...	...				

TABLE 4.

O[M] *Table. Comparison of the values of the functions derived by the abridged method with those of the Official Table.*

ABRIDGED METHOD					OFFICIAL TABLE				
Age	q_x	$_5p_x$	e_x	a_x 3 %	Age	q_x	$_5p_x$	e_x	a_x 3 %
17	·00347	·98021	45·65	23·339	17	·00375	·98044	45·67	23·347
22	·00452	·97656	41·53	22·248	22	·00431	·97701	41·54	22·252
27	·00507	·97230	37·48	21·047	27	·00523	·97181	37·46	21·041
32	·00649	·96501	33·49	19·722	32	·00648	·96509	33·49	19·725
37	·00807	·95640	29·63	18·300	37	·00804	·95684	29·63	18·303
42	·01017	·94585	25·89	16·769	42	·01001	·94606	25·87	16·764
47	·01266	·93089	22·25	15·115	47	·01277	·93057	22·23	15·104
52	·01696	·90715	18·75	13·345	52	·01693	·90726	18·73	13·336
57	·02334	·87277	15·45	11·512	57	·02338	·87187	15·43	11·498
62	·03307	·82028	12·40	9·649	62	·03344	·81907	12·39	9·644
67	·04899	·74198	9·66	7·831	67	·04900	·74273	9·67	7·840
72	·07333	·63682	7·31	6·150	72	·07281	·63750	7·31	6·154
77	·10891	·49773	5·35	4·646	77	·10882	·50208	5·34	4·645
82	·16495	·34778	3·83	3·405	82	·16240	·34546	3·75	3·356
87	·23151	·22044	2·69	2·417	87	·24001	·19242	2·51	2·307
92	·31934	·08867	1·66	1·457	92	·34788	·07672	1·60	1·497
97	·50528	·00772	·91	·913	97	·48276	·01724	·97	·917

[It will be seen that the process adopted by Mr. King for obtaining the pivotal values of q for the abridged table is identical with that followed in constructing a complete graduated table by his method, so that his abridged table would have the advantage of being in close agreement with the complete table if the same five-year groups were used for both.

It may, however, be of interest to indicate briefly an alternative method of procedure which would probably, in most cases, give sufficiently accurate results. The successive steps by this method would be as follows:

(1) Tabulate the ungraduated values of $\log {}_5p_x$ (*a*) in the case of an assurance experience by addition of the observed values of $\log p_x$; (*b*) in the case of a population experience by the approximate formula,

$$\log {}_5p_x = -\mathrm{M}m_x^{\left(\frac{1}{5}\right)}\left(1+\frac{m_{x+5}^{\left(\frac{1}{5}\right)}-m_{x-5}^{\left(\frac{1}{5}\right)}}{24}\right) = -\mathrm{M}m_x^{\left(\frac{1}{5}\right)}\left(1+\frac{a_0}{12}\right)^*$$

where $m^{\left(\frac{1}{5}\right)}$ is the central death rate per 5 years for a five-year age group, and a_0 is its first central difference.

(2) Graduate these values, if desired, or obtain intermediate graduated quinquennial values by interpolation.

3. Calculate $\frac{1}{5}e^{\left(\frac{1}{5}\right)}$ by the usual continued formula

$$\log \frac{1}{5}e^{\left(\frac{1}{5}\right)} = \log {}_5p + \log\left[1+\frac{1}{5}e_5^{\left(\frac{1}{5}\right)}\right]$$

and thence obtain e by the approximate formula

$$e = e^{\left(\frac{1}{5}\right)} + 2 - 2\mu;$$

---

* $$m^{\left(\frac{1}{5}\right)} = \frac{5(l-l_5)}{\int_0^5 l\,dt} = \frac{5(1-{}_5p)}{2\frac{1}{2}(1+{}_5p)-2\frac{1}{12}(\mu-{}_5p\mu_5)}.$$

Hence,

$${}_5p = \frac{1-\frac{1}{2}m^{\left(\frac{1}{5}\right)}+\frac{5}{12}\mu m^{\left(\frac{1}{5}\right)}}{1+\frac{1}{2}m^{\left(\frac{1}{5}\right)}+\frac{5}{12}\mu_5 m^{\left(\frac{1}{5}\right)}}$$

and

$$\log_e {}_5p = -m^{\left(\frac{1}{5}\right)}\left[1+\frac{5}{12}(\mu_5-\mu)\right] = -m^{\left(\frac{1}{5}\right)}\left[1+\frac{m_5^{\left(\frac{1}{5}\right)}-m_{-5}^{\left(\frac{1}{5}\right)}}{24}\right] \text{ approximately.}$$

the value of μ may be taken as

$$\frac{1}{10}\left({}_5p_{x-5}^{-1} - {}_5p_x\right).$$

Similarly for a.

With regard to (1) (a) it may be necessary at the beginning of the Table, or elsewhere if the data are few, to take an average value of p for the whole group instead of summing the five separate values.

The process is illustrated by the following applications to the O^M and English Life No. 6 (Males) Experiences. In the application to the O^M, for convenience in comparison with Mr. King's results, the values of ${}_5p_x$ have been tabulated from the data for ages 20, 25, 30, &c., and the values for ages 22, 27, 32, &c., have been interpolated by the central interval second difference formula $u_{\frac{2}{5}} = u_0 + \frac{2}{5}a_0 - \frac{3}{25}\beta_0$; the values of ${}_5p_{22}$, &c., consequently depend on four 5-year groups of the original data, whereas Mr. King's depend on six. In the application to the English Life No. 6 the values of $\log {}_5p$ have been used without graduation.

TABLE I.—O^M *Experience.*

x	$\log {}_5p_{-2}$	$-a$	$-\beta$	$\log {}_5p_x$	$\log \frac{e_x^{(\frac{1}{5})}}{5}$	$\frac{1}{5}e_x^{(\frac{1}{5})}$	${}_5p_{x-5}^{-1}$	${}_5p_x$	$\frac{{}_5p_{-5}^{-1} - {}_5p}{10}$	e_x
17	$\bar{1}$·99191	·00158	...	...	...	...	...	...	...	...
22	·99033	·00145	·00072	$\bar{1}$·98984	·89822	7·911	1·0198	·9769	·00429	41·55
27	·98888	·00302	·00102	·98779	·85117	7·098	1·0237	·9723	·00514	37·48
32	·98586	·00350	·00078	·98455	·79940	6·301	1·0285	·9651	·00634	33·49
37	·98236	·00459	·00105	·98065	·74265	5·529	1·0362	·9564	·00798	29·63
42	·97777	·00560	·00245	·97582	·67954	4·781	1·0456	·9458	·00998	25·88
47	·97217	·00949	·00430	·96889	·60798	4·055	1·0573	·9309	·01264	22·25
52	·96268	·01421	·00630	·95775	·52583	3·356	1·0743	·9073	·01670	18·75
57	·94847	·02208	·01104	·94096	·43120	2·699	1·1022	·8729	·02293	15·45
62	·92639	·03629	·01730	·91395	·32066	2·092	1·1456	·8203	·03253	12·40
67	·89010	·05667	·02482	·87041	·19056	1·551	1·2191	·7420	·04771	9·66
72	·83343	·08594	·04305	·80422	·03747	1·090	1·3477	·6371	·07106	7·31
77	·74749	·14276	·04684	·69601	$\bar{1}$·85165	·711	1·5696	·4966	·10730	5·34
82	·60473	·17963	·07006	·54129	·63401	·431	2·0137	·3478	·16659	3·82
87	·42510	·28289	·14216	·32899	·37704	·238	2·8754	·2133	·26621	2·66
92	·14221	·46396	...	·04804*	·06882	·117	4·6882	·1117	·45765	1·67
97	$\bar{2}$·67825	...	...	$\bar{2}$·68737*	$\bar{2}$·68737	·049	8·9528	·0487	·89041	·47

* By constant 3rd differences.

It will be seen that the values of e_x obtained by this method are almost identical (except at age 97) with those of Mr. King and the graduated O^{M} Table. The values of $\log p_x$, and thence of q_x, would be best obtained from col. (2) by Mr. King's formula $\log p_x = \cdot 2 \log {}_5p_{x-2} - \cdot 008 \Delta^2 \log {}_5p_{x-7}$.

TABLE II.—*English Life (No. 6) Experience. Males.*

x	$m^{(1)}_x$	$1+\frac{a_0}{12}$	col$_e$ ${}_5p_x$	col ${}_5p_x$	$\log \frac{e_x^{(1)}}{5}$	$\frac{1}{5}e_x^{(1)}$	${}_5p_{x-5}^{-1}$	${}_5p_x$	$\frac{{}_5p_{-5}^{-1} - {}_5p}{10}$	e_x
20	·02530	...	...	...	...	...	..	...	...	...
25	·02974	1·0005	·02975	·01292	...	...	...	...	...	...
30	·03834	1·0009	·03837	·01666	·78666	6·119	1·0302	·9624	·00678	32·58
35	·05061	1·0011	·05067	·02200	·72897	5·358	1·0391	·9506	·00885	28·77
40	·06536	1·0013	·06544	·02842	·66610	4·636	1·0520	·9367	·01153	25·16
45	·08185	1·0019	·08201	·03562	·59653	3·949	1·0676	·9213	·01463	21·72
50	·11009	1·0027	·11039	·04794	·51684	3·287	1·0854	·8955	·01899	18·40
55	·14632	1·0042	·14693	·06381	·42671	2·671	1·1167	·8634	·02533	15·30
60	·21005	1·0061	·21133	·09178	·32102	2·094	1·1582	·8095	·03487	12·40
65	·29155	1·0095	·29432	·12782	·20056	1·587	1·2353	·7450	·04903	9·84
70	·43832	1·0137	·44432	·19297	·05304	1·130	1·3423	·6413	·07010	7·51
75	·62032	1·0230	·63459	·27559	·88187	·762	1·5593	·5302	·10291	5·60
80	·98985	1·0347	1·02420	·44480	·64085	·437	1·8861	·3591	·15270	3·88
85	1·45429	...	...	·70060*	·33843	·218	2·7847	·1992	·25855	2·57
90	...	...	...	1·04299*	·97153	·094	5·0201	·0906	·49295	1·48
95	...	...	...	1·47197*	...	·034	11·0375	·0337	1·10038	...

* By constant 2nd differences.

Here again the values of e are practically the same as the official values—except at the oldest ages when they are in closer agreement with those given by Mr. King's graduation. The values of $\log p$ (and thence of q) could be conveniently obtained by the formula

$$\log p = \frac{3}{25} \log {}_5p + \frac{2}{25} \log {}_5p_{-5} - \frac{4}{125}\beta_0 .$$

EDS. *J.I.A.*]

LEGAL NOTES.

By WILLIAM CHARLES SHARMAN, F.I.A., *Barrister-at-Law.*

TWO cases dealing with the question of claims on the Statutory Deposit have recently come under the consideration of the Courts. The first of these is that of the liquidators of *Glasgow Assurance Corporation, Limited* v. *Welsh Insurance Corporation, Limited*, which came before the Scotch Courts, 51 S.L.R., 271.

Assurance Companies Act, 1909. Claim under Reinsurance treaty on Employers' Liability Fund, including Statutory Deposit.

The question in the case was whether an insurance company which has entered into a re-insurance treaty as regards its employers' liability insurance business with another insurance company is entitled in the liquidation of the latter company to a ranking on the employers' liability insurance fund (including therein the statutory deposit). This was argued before the Second Division of the Court of Session, Scotland, and decided by them in the affirmative. The facts are as follows :

The *Glasgow Assurance Corporation, Limited*, now in liquidation, carried on various kinds of insurance business, and, *inter alia*, that of employers' liability insurance. The respondent company, the *Welsh Insurance Corporation, Limited*, carries on, *inter alia*, an employers' liability insurance business. The two companies entered into a re-insurance treaty, dated 30 November 1909, whereby the Welsh company agreed to pay to the Glasgow Company $21\frac{1}{2}$ per-cent of its workmen's compensation premiums, and the Glasgow Company, in consideration thereof, agreed to indemnify the Welsh Company under its policies in respect of fatal accidents (with certain exceptions), and also to a certain extent against claims in respect of disablement.

The re-insurance treaty was terminated on or about 31 August 1911, subject to the risks still current, for which the Glasgow Company had previously become liable under its provisions. During the period while the treaty was in full force the Welsh Company paid over the stipulated proportion of its premiums. The Glasgow Company carried the amounts so received by it into the account of its "separate assurance fund", applicable to employers' liability insurance business, kept in pursuance of the Assurance Companies Act, 1909, and similarly entered on the other side of that account the payments of claims made under the treaty. It thus treated the re-insurance transaction as part of its employers' liability insurance business under the Act.

The Welsh Company had, admittedly, a claim in the liquidation for sums due under the treaty, and the main question raised was as to the mode of ranking the due amount of this claim. On 18 December 1913, the Lord Ordinary (Cullen) pronounced the following interlocutor :

". . . that the respondents are entitled to a ranking on "the employers' liability fund of the company (including therein "the statutory deposit) in respect of their claim, in so far as the

"same arises out of claims paid under policies issued by the "respondents of which the company was under obligation to "relieve the respondents in respect of the agreement or treaty."

The liquidators of the *Glasgow Assurance Corporation, Limited*, reclaimed, and argued that Section 3 of the Assurance Companies Act, 1909, gave the policyholders in any one of the particular classes of insurance business dealt with in the Act a security over the whole funds applicable to the particular class of business, and conferred on such policyholders a preference over ordinary creditors of the company in that particular class of business. The respondents were not policyholders, nor were they creditors, by virtue of the contract of re-insurance, of the Glasgow Company so far as its employers' liability business was concerned. Re-insurance differed from direct insurance in respect (1) that the risks covered were different, (2) that re-insurance depended not upon the risks of the original policies, but upon the careful management of the re-insured company. The legislation contained in the Assurance Act was directed to the security of policyholders, and did not apply to re-insurance.

Lord Hunter, after reviewing the provisions of the Life Assurance Acts 1870 and 1872, said: "The Act of 1909 was "passed to consolidate the provisions of the Life Assurance "Acts and to extend them to certain other classes of insurance. "As in the preceding Acts, so in that Act there is nothing to "indicate that the policyholders in the case of a company "carrying on only one class of insurance business should be "preferred on the assurance fund to other creditors of the "company. In the case of a company carrying on several "businesses, the policyholders in one class of business were to "be put in the same position as regards the insurance fund "applicable to that business as if the company carried on no "other business.

"The object and effect of Section 3 is in my opinion to "effect separation of the funds of different businesses carried "on by the same company, but not to grant preferences over the "separated funds to some as contrasted with other creditors "having claims in connection with the same class of business. "If the view which I have just expressed be sound, the re-"spondents are entitled to rank on the fund, provided that "they are creditors of the Scots Company with reference to "their employers' liability business. The Act of 1909 defines

" that business as ' the issue of or the undertaking of liability " ' under policies insuring employers against liability to pay " ' compensation or damages to workmen in their employment.' " The contention of the liquidators is that the issuing of direct " policies to employers is in its nature distinct from re-insuring " risks undertaken by other companies. This may be true. " At the same time re-insurance for the spreading of risks is an " ordinary feature of various kinds of insurance business both " in this country and elsewhere. In the case of a company " carrying on only one of the enumerated classes of insurance " businesses, say employers' liability insurance, and therefore " having only one insurance fund, and in the course of its " business re-insuring other companies in connection with " employers' liability risks undertaken by the latter, either " in virtue of power to do so in their memorandum or as an in- " cident of their business. I think that it would be only natural " to treat the premiums received from re-insurance as receipts " from their employers' liability business and the re-insured " companies as creditors upon the insurance fund. Direct " insurance and re-insurance appear to be two methods of " carrying on the same class of insurance business rather than " two separate classes of such business. I think the issue of " policies apt to cover cases of direct insurance, and the words " ' undertaking of liability under policies ' apt to cover cases " of indirect insurance."

Lord Guthrie in expressing his agreement, said: " I read " the statute as making a clear distinction in Section 1 between " two classes of documents—a distinction which excludes these " re-insurance contracts from the category of ' policies insuring " ' employers against liability to pay compensation or damages " ' to workmen in their employment', and includes them within " the category of documents involving ' the undertaking of " ' liability under policies insuring employers against liability " ' to pay compensation or damages to workmen in their " 'employment ' . . . it seems to me clear that contracts of " re-insurance cannot be reasonably excluded from the scope " of the alternative in Section 1 of the Act. Cases of amalga- " mation of companies may be provided for under this alternative. " but I read it as primarily applicable to contracts of re-insurance " which have been familiar features of insurance business in " Great Britain, at all events since 1880, and the existence of " which was recognized both in the Act of 1870 and in the

"Act of 1909, the former referring to re-insurance of both kinds, "and the latter to re-insurances of company's risks."

Claim on Statutory Deposit in respect of Annuity granted for other than cash consideration.

The second case is that of *In re The British Union and National Insurance Company (Limited)*, previously reported in these Notes (*J.I.A.*, vol. xlviii, p. 197), which has since come before the Court of Appeal, 30 T.L.R. 520, on a point other than that to which reference was made. The question raised on appeal was whether, in the winding up of the insurance company, a certain annuity ought to be satisfied out of the deposit of £20,000 made on behalf of the company in pursuance of the Life Assurance Companies Act, 1870, in priority to the claims of the general creditors. The facts are as follows:

The *British Union and National Insurance Company (Limited)*, which carried on life assurance and annuity business, had, immediately after its incorporation, engaged Mr. Frank Urch as general manager at a salary of £1,500 a year and commission. In August 1910, a new agreement of service was entered into, under which Mr. Urch accepted the post of manager of the life department only, at a salary of £600 a year and a commission on all new business introduced by him. In the same year the company discontinued its business of industrial life assurance, and negotiations ensued with a view to Mr. Urch's retirement, which culminated in his bringing an action for damages against the company. The action was compromised on terms set out in an agreement dated 30 December 1910, by which the company covenanted during Mr. Urch's life to pay his wife £240 a year, and in case Mrs. Urch died in his lifetime to pay the same to him for the remainder of his life.

In June 1912, the company went into voluntary liquidation, and a liquidator was appointed under supervision of the Court. The claims of all life assurance policyholders had been satisfied or provided for, and the only annuity for which the company was liable was that of Mr. and Mrs. Urch. At the original hearing of the action, Mr. Justice Astbury said he thought this was merely a claim for general damages compromised, and was not an annuity within Section 30, subsection (*b*) of the Assurance Companies Act, 1909, and ought not therefore to be satisfied out of the deposit in Court in priority to the claims of general creditors.

The Court, consisting of Cozens-Hardy, M.R., and Swinfen Eady and Pickford, L.JJ., allowed the appeal.

The Master of the Rolls in his judgment said that the result of Section 3 (2) of the Act of 1909 was that the deposit fund which was part of the life assurance fund, was absolutely the security of the life policyholders, and could not be dealt with as free general assets of the company until after all such claims had been satisfied. The only question was whether Mr. Urch was a life policyholder within the meaning of the Act. By Section 30, where a company granted annuities upon human life, "policy" included the instrument evidencing the contract to pay such an annuity and "policyholder" included annuitant. Mr. Urch held a deed dated 30 December 1910 under the Company's seal by which the company covenanted to pay during his life an annuity of £240. The validity of the deed was not challenged, but it was contended that as the annuity was not granted for a cash consideration it was not such an annuity as was contemplated by, or was entitled to the benefit of, the Act of 1909. There was abundant consideration for the grant of the annuity, and it was not *ultra vires*. The consideration was truly recited in the deed. Mr. Justice Astbury considered that the annuity was not granted in the ordinary course of business, *i.e.*, in consideration of a payment to the life assurance fund, and, therefore, was not within the Act, but with great respect, he (the Master of the Rolls) was unable to agree with that view. It involved the insertion in Section 30 (*b*) of the Act of the words "for a cash consideration only" and for this his lordship thought that there was no justification. The annuity was granted for valuable consideration, and the nature of that consideration was not important.

Notice to policyholders upon transfer of Assurance Company.

Section 13 of the Assurance Companies Act, 1909, provides that before the application for the transfer of one assurance company to another is made, a statement of the nature of the transfer, together with an abstract of the facts and copies of the reports, must be sent to each policyholder of each company unless the Court otherwise directs.

In the case of *In re Hearts of Oak Life and General Assurance* Company, 30 T.L.R. 436, an application was made for the sanction of the Court to a proposed transfer of an assurance company under the above Act, and the Court was asked to

dispense with service of the statement on all holders of policies under £50.

Mr. Justice Eve in giving judgment, said that the statutory obligation to send a statement of the transfer to every policyholder was imperative, unless the Court otherwise directed. Generally, industrial policies were of very small amount, and it was said that if the section were to be strictly complied with, the statement would have to be sent to thousands of persons in a humble position of life, and would cause unreasonable expense to both companies. In these circumstances, the Court was asked to dispense with service on all policyholders under £50, which would be to dispense with service on all holders of industrial policies.

His Lordship thought that he ought to dispense with such service as regarded the transferee company, the small policyholders of which were not materially affected by the transfer. He therefore dispensed with such notice. He felt, however, more difficulty in taking the same course with regard to the transferor company. The policyholders in that company were probably quite unaware of the nature of the contract into which they were entering, and therefore they required protection. On the other hand, the notices would convey nothing to their minds, and they were not in a position to take steps to be heard in the matter.

Having regard, also, to the fact that the cost of giving the notices would be very heavy, and would ultimately fall on the policyholders, his Lordship thought he ought to take the same course with regard to the transferor company.

The Court would, therefore, dispense with notice on holders of policies in that company under £50, but it would only do so provided other steps were taken by advertisement to inform the policyholders of the proposed transfer, so as to give them an opportunity of objecting to it.

Income Tax in respect of Interest not remitted to this country.

Many life assurance companies have hitherto enjoyed exemption from Income Tax in respect of interest on foreign investments, when such interest is not remitted to this country.

This exemption has now been abolished by a financial resolution of the House of Commons, passed on 11 May 1914, to the effect that income tax shall be charged on the income

arising from securities or possessions in any place out of the United Kingdom, whether that income is received in the United Kingdom or not. This resolution will doubtless be incorporated in the forthcoming Finance Act, but under the provisions of the Provisional Collection of Taxes Act, 1913, it has immediate statutory effect.

REVIEWS.

The Construction of Mortality and Sickness Tables. A Primer by W. PALIN ELDERTON *and* RICHARD C. FIPPARD. London: A. & C. Black.

THE general plan of this little book seems to us excellent. Probably most students of the Construction of Mortality Tables are, as the authors suggest, repelled by the complicated notation and formulas which confront them almost on the threshold of the subject as it is ordinarily presented, and it is therefore most useful to have an elementary text book in which the main principles and methods are explained verbally and by numerical examples practically without the help—or hindrance—of symbols or formulas.

Chapters I, V, VI, and VIII, dealing respectively with the nature of the problem and the extraction of data from life-office records, rates of withdrawal and sickness, the construction of mortality tables from censuses and registered deaths, and the comparison of different mortality tables, are perhaps the best examples of the authors' method. The treatment of all these subjects is admirably simple and effective, although in Chapter I the "Age at Exit used in calculations," which is not inserted in the specimen card, might with advantage have been omitted also from Table I; its introduction at this stage, without the explanation given in Chapter III, brings in by inference a purely technical relation—age at exit = age at entry + duration—and seems to present an unnecessary difficulty, since (as pointed out in a footnote to Chapter III) the assumed age at exit may be as much as a year out either way.

In Chapters II-IV, dealing with the construction of select and aggregate mortality tables from life-office data, we think it would have been more in accordance with the authors' plan if they had used, for purposes of illustration, tables with self-explanatory headings instead of actual extracts from Experiences which presume some technical knowledge. The headings of Table II, for example —Duration, Withdrawals, Deaths, etc.—are practically meaningless apart from the formulas given in "Principles and Methods," and it would have been clearer to insert some such headings as Year of Duration, Withdrawals within 6 months before or after the End of Year, Deaths *during* the Year, etc. No doubt the explanations are given in the text, but this does not have quite the same effect in keeping before the student's mind the precise ideas represented by the Table. Besides, from the explanation given in

regard to the deaths the student might not unnaturally be led to suppose that the duration of a policy terminated by death in the 31st year ought to be 31—not 30, since it is under observation 31 years for the purpose of calculating the Exposed to Risk. It is, of course, taken as 30 because it is convenient to make $q_x = \frac{\theta_x}{E_x}$ instead of $\frac{\theta_{x+1}}{E_x}$, *i.e.*, on purely technical grounds. It may be mentioned incidentally that the explanation given in regard to withdrawals, namely, that "we balance the understatement of some cases with the overstatement of others", may not seem to the student to justify the entire omission from the Exposed to Risk at duration 0, in Table II, of the 30 withdrawals within the first half year. This is another slight disadvantage resulting from the use of an extract from the British Offices' Experience, because in that Experience the withdrawals were not in fact dealt with in the exact manner described by the authors, but were adjusted by a certain transference from the beginning to the end of the policy year. These, however, are minor points. The book as a whole, by giving in a simple form an account of the main principles and methods of Mortality Table construction, will certainly tend to put the student on good terms with himself and the subject, and should therefore be of service both to students and to teachers.

A word may be added in regard to Chapter VII, which deals with the application of the Census Method to Insurance data. The chapter practically follows the lines of the paper read by the authors before the Institute, but at the beginning an answer is supplied incidentally to the question raised in the discussion on that paper as to why the method should be called a census method. That question, however, was probably suggested by a doubt as to whether the virtues of the method really consisted in its being a census method or in its being continuous, and it would be interesting to have the authors' views on this point. As regards new cases for which individual cards have to be written it would apparently involve no more labour to compile a continuous experience on policy year than on census lines.

With a view to subsequent editions it may be mentioned that "they" should be substituted for "it" in line 24, p. 10, and that a "not" should be deleted in line 7, p. 60.

Théorie et Pratique des Opérations Financières. Par ALFRED BARRIOL.

Paris: Octave Doin et Fils.

BY the courtesy of the author a copy of the second edition of this work has recently been added to the Library of the Institute. As its name implies, M. Barriol's treatise is of wider scope than the ordinary interest text-book. International exchange and Stock Exchange transactions are dealt with in considerable detail, and a short section at the end of the book is devoted to the operations of the great

financial houses. In connection with speculative Stock Exchange transactions, M. Barriol includes an investigation—based on the assumption that the distribution of prices would be in accordance with the normal law of error—of the theoretical relation of the *prime* to the *écart*, that is, as we understand it, of the price of a call-option to the difference between the call-quotation and the ordinary quotation for next settlement. The operator in options of this nature would probably be as much surprised to learn that he is applying the law of error to finance as M. Jourdain was to find that he had been talking prose all his life without knowing it; but apart from doubts whether in such a case theory really exercises any influence over practice, and whether variations in prices could be regarded as following even approximately the law of error, it would seem to be difficult, in applying the formula practically, to determine the modulus of the particular curve to be employed for a specified security. At times of active speculation—when options are most in demand—the average deviation in the price of the security for the period covered by the option might be a very unreliable measure of the range of fluctuation.

The subjects of compound interest, annuities and loans repayable by instalments, are dealt with on very much the usual lines, except that the distinction between nominal and effective rates hardly appears. Presumably the nominal rate, payable half-yearly or quarterly, has not the practical importance in France that it has in our own country and America. In the chapters relating to loans repayable by instalments, the various classes of securities dealt in on the Paris Stock Exchange are discussed in detail, and some useful schedules are given—particularly one showing the practical method of adjusting annual capital repayments to allow for the fact that an integral number of bonds must be repaid at each drawing. For the purpose of calculating the yield on an annuity or security, M. Barriol recommends, in general, one or more applications of the first difference interpolation formula. This method, however, when it is necessary to obtain a more accurate result than that given by a single interpolation, entails a good deal of calculation, and a second central difference formula would appear preferable. We observe that M. Barriol does not employ any symbol either for the present value of 1 or for that of an annuity, although, we believe, that the use of both v and a was recommended by the International Congress; this makes a good many of his formulas more complicated in appearance than they are in fact.

CORRESPONDENCE.

SELECT NET PREMIUMS FOR ENDOWMENT ASSURANCES.

To the Editors of the Journal of the Institute of Actuaries.

SIRS,—I have recently had occasion to obtain a working equivalent to the select rates of mortality shown by the Institute Experience

of Endowment Assurances (Old and New combined). In default of a table of graduated rates, I have adopted one founded on simple percentages of the corresponding $O^{[M]}$ rates, which roughly reproduces the experience.

The premiums obtained from this table cannot differ materially from those which would be obtained from a "scientific" graduation, and so may be of interest. I therefore give specimens, comparing them with the corresponding $O^{[M]}$ and $\mathbf{O}^{[NM]}$ values. It will be seen that the rates are, generally speaking, as much below those of the $O^{[M]}$ table as the latter are below those of the $\mathbf{O}^{[NM]}$.

Pure Annual Premiums for Endowment Assurances			3½ per-cent.				$E^{[M]}$ approx. $O^{[M]}$ $\mathbf{O}^{[NM]}$		
Age at Entry	Term								
	10	15	20	25	30	35	40	45	50
20	8·42	5·24	3·68	2·78	2·21	1·83	1·57	1·39	1·27
	8·47	5·30	3·76	2·86	2·30	1·93	1·67	1·50	1·39
	8·55	**5·38**	**3·83**	**2·94**	**2·38**	**2·01**	**1·76**	**1·59**	**1·48**
30	8·45	5·27	3·74	2·86	2·32	1·97	1·75	...	...
	8·52	5·36	3·84	2·98	2·44	2·11	1·90	...	...
	8·61	**5·45**	**3·93**	**3·07**	**2·54**	**2·21**	**2·00**	...	...
40	8·53	5·39	3·90	3·07	2·59	...	...	..	...
	8·63	5·52	4·05	3·25	2·79	...	...	...	...
	8·74	**5·64**	**4·18**	**3·38**	**2·93**	...	...	...	...
50	8·73	5·68	4·29	..	...	...	...	...	...
	8·90	5·90	4·57	...	...	...	...	...	...
	9·08	**6·10**	**4·77**	...	...	...	...	...	...

Of the rates of mortality used it will be sufficient to say that the percentage for the first eleven years of assurance for ages attained 28 and over is 75 of the corresponding $O^{[M]}$ rate, that the ultimate percentage after twenty years of assurance is 85 of the $O^{[M]}$ ultimate rate, that the expected deaths for the more solid part of the Experience are 5,550 as against 5,517 actual, and for the whole Experience, excluding the first twenty years of assurance, 626 as against 617.

I am, Sirs,

Yours faithfully,

W. E. NORTON.

48, *Gracechurch Street, E.C.,*
3rd June 1914.

INTERNATIONAL CONGRESS OF ACTUARIES, 1915.

The Eighth International Congress of Actuaries will be held in St. Petersburg from the 30th May to the 5th June, 1915. We give below the programme of the subjects to be dealt with.

Subjects for Discussion.

1. The determination of probabilities and other quantities connected with social assurance. The consideration of a universal system of notation.
2. The scope and limits of compulsory clauses in assurance contracts.
3. The practical value of select tables.
4. The calculation of mathematical reserves based on office premiums.
5. The mortality among persons who have been disabled by accident.
6. Policy Reserves from the legal, technical, and economical points of view. The rights of the policy-holders to the reserves against their policies.

Subjects for Papers without Discussion.

1. The possibility of the application of the theory of stability of statistical series, of frequency curves and the theory of correlation to actuarial calculations.
2. Comparison between the mortality among the general population of a country and the mortality among assured lives.
3. The evolution of assurance law.
4. New tendencies in the region of the ordinary and industrial life insurance.
5. The Moratorium, as affecting insurance business.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES.

Section 72 of the National Insurance Act. Some other Features of Friendly Societies and National Insurance, including a Note on the Proposed Belgian National Insurance Act. By Eric B. Nathan, F.I.A., F.S.S., *a Public Valuer under the Friendly Societies Act.*

[Read before the Institute, 27 April 1914.]

PART I.

Section 72 of the National Insurance Act.

Introductory. In the discussion on Mr. Simmonds's Paper (*J.I.A.*, vol. xlvii) allusion was made by several speakers to the omission of any reference in that Paper to Section 72 of the National Insurance Act. The omission was, I understand, intentional. Mr. Watson devoted his sixth Lecture on Friendly Society Finance to this Section and to Section 73. He dealt very fully with the position as it then existed. Any present reference to the Section can only follow the lines he laid down, and may therefore appear superfluous. As, however, there was no opportunity of discussing this subject on the Lecture, and as in the discussion following Mr. Simmonds's Paper there appeared a certain interest in this Section, it seems probable that a short demonstration of this side of the National Insurance Act may lead to an interesting and useful discussion, notwithstanding the failings in the Paper itself.

Section 55 of the Original Bill. Section 72 of the Act corresponds with Section 55 of the Bill as originally drafted; the latter Section has, however, been considerably modified and extended. The modifications will be seen from a comparison of the two sections, which are set out in the Appendix to this

Paper side by side. A consideration of the earlier Section suggests that it was a skeleton which it was intended to amplify in Committee.

There are, however, fundamental differences between the two Sections with which I propose dealing. Section 55 opens with the words "Any Registered Friendly Society . . . shall submit a Scheme." In the Act the word "any" has been changed to "every." This amendment was introduced by the Attorney-General, and was made in conjunction with the amendment in the section which followed. The idea seems to have been that as Registered Societies were to be given an additional option in the scheme to be introduced, namely: that of "continuing" benefits, the submission of a scheme should be compulsory on all Societies.

Section 55 as originally drafted provided for quite a different type of Scheme from Section 72. Under the original Section when a Society decided to submit a scheme that scheme would have been compulsory on all its insured members. The first step would have been to ascertain for these members the share of the Funds of a Society reserved for Benefits similar to those under the National Insurance Act. This would have involved the separation of a Society's contribution for Sickness and Funeral Benefits, and the taking into account only of that part of the contribution applicable to Sickness Benefits ceasing at age 70. In many societies this would have been extremely difficult, as their custom is to keep a single benefit fund. Having ascertained the amount of funds held by a society against the State benefits or that part of them the society undertook, the original scheme would have cancelled these benefits, and utilized the funds set free in the provision for *the insured members* of one or other of the following options:

(*a*) In the payment of other benefits,

(*b*) In the reduction of the contributions of members for the other benefits paid by the Society,

(*c*) In the payment in whole or in part of the contributions paid by a member for National Insurance.

It is worth noting that the scheme would have been compulsory on all members of a society, and that the funds set free would have been applied for the benefit of those who set them free, after first getting rid of any deficiencies in the societies.

Under some such scheme it seems likely that some of the millions of reserves (£10,000,000 was mentioned) would have been released for the benefit of the societies, while the reduction of duplicate insurance would have had an effect on the rates of sickness under National Insurance.

The amendments to Section 55. It was, I believe, represented to the Government by the Friendly Societies that Section 55 as originally drafted would require an individual accounting for each insured person, and that such an individual accounting was not possible for Friendly Societies with their limitations both as regards the capacity of their staffs and the time and funds available. It was, in addition, pressed upon the Government that the compulsory retirement of the insured and younger members of a society would leave a small number of members in advanced age or impaired health, too small in numbers to benefit by the law of average, and in a state of health which would involve greatly increased rates of sickness. In view of these representations an amendment substantially in the form of the Section in the Act was introduced by the Government. In Committee the objects of the amendment were set out by the Attorney-General (Sir Rufus Isaacs) as follows:

"Under the Government Amendment . . . once a scheme "is submitted either for altering the benefits or for *continuing* "or altering the contributions if the scheme shows on a valuation "there are existing funds set free, and which it is not necessary "to employ for the purpose of meeting existing liabilities of the "Society those existing funds can be applied in three different "ways. The first would be for providing other benefit for "existing members of the Society whether they become insured "members or not. . . . The second would be in the reduction "of the contributions payable by the members, also in respect "of benefits independent of this Act. . . . The third way . . . "is towards the payment of contributions by such persons who "are already members under this part of the Act—that is, "insured persons—if they elect to receive their benefit under the "approved society of which they are already members. . . . "*The Amendment does not make any great change.* The principal "change is one found necessary in consequence of the enormous "difficulty of carrying out what at first was thought to be the "more practicable and just method—that is to say, to apply to "each person individually an increase of benefit or reduction "of contribution . . . the money set free in any particular

" society to which that member belonged before the State " Scheme came into existence."

It will be noticed from the above extract that while there is a suggestion of continuing contributions under the amended clause, there is no mention of continuing benefits. Throughout the whole discussion in Committee no mention was made as to the option of continuing benefits. The word "continuing" [" for continuing benefits "] was inserted in an amendment on which there was no discussion whatever. Presumably it was a Government Amendment, but it was thought of so little importance that no member on either side of the House considered it worth discussing. Except for the reference by the Attorney-General no mention was made as to inserting the provision for continuing contributions. Members on both sides of the House of Commons do not appear to have had the slightest doubt that members of friendly societies would be only too anxious to reduce their contributions and benefits. Who originally introduced the word " continuing " into the Section I have been unable to discover. Far from the amendment not making any great change as was asserted by the Attorney-General, the introduction of this single word completely changed the clause, greatly to the prejudice of the friendly societies, while it has in my opinion contributed not a little to the increased cost of sickness under the National Insurance Act which is so much under discussion at the present time.

Section 72.

In the sixth Lecture on Friendly Society Finance, Mr. Watson explained fully Sections 72 and 73, as at the date the Lecture was delivered. Following on a legal ruling of the Law Officers of the Crown, which has up to the present been uncontested, the Section has been modified in practice, and options which Mr. Watson suggests might be given voluntarily if funds were available, have now to be incorporated as a compulsory part of the Scheme. To save cross reference I propose giving a short explanation of the Section as at present interpreted.

Societies affected.

The Section aims at every Registered Friendly Society (or branch of such society), whether it became an approved society, or whether it stood outside the Act, the only provision as to the liability to submit a scheme being that the society grants similar benefits to those provided under the Act.

It may be said to affect all Registered Friendly Societies granting some or all of the following benefits :

(1) Sick Pay.

(2) Medical and Sanatorium Benefit.

(3) Maternity Benefit.

Registered Societies restricting their transactions to the following classes of benefit are not required to submit schemes.

(1) Death Benefits.

(2) Superannuation Benefits and Deferred Annuities.

(3) Widows and Orphans Pensions.

(4) Endowments.

(5) Indemnities in the case of Fire, Shipwreck or Fraud by Officers.

(6) Unemployment Benefits. (Specially authorized Societies.)

It does not affect Trade Unions or Unregistered Societies, a suggestion made by Mr. Bathurst, M.P., to include the latter not being accepted by the Government. Section 73 makes provision for employers' superannuation or provident funds which grant benefits similar to those paid under the Act, whether registered or unregistered.

Operation of the Scheme.

The scheme applies to the "insured" members of friendly societies who were members of their respective societies on 16 December 1911 ; it must be available for such members whether they decide to make their friendly society, or some other body, their approved society. It grants these members the option of *continuing*, abolishing, reducing or altering the benefits similar to those granted under the Act, and the option of continuing, abolishing, reducing or altering the contributions required for these benefits. It has been held that some members may select one option, while others may select another. So that, while some members may decide to continue their full contributions and benefits, other members can reduce benefits and contributions up to the full extent allowed by the National Insurance Act. In this manner much of the simplicity which was claimed as one of the advantages of the amendments to Section 55 has been lost.

Only the following benefits can be effected as they are the only benefits "similar to those conferred by this Act."

(1) Medical Benefit throughout life.

(2) Sickness Benefits ceasing at age 70 at a rate not exceeding—

10*s*. first 26 weeks and 5*s*. per week for remainder of sickness for Men.

7*s*. 6*d*. first 26 weeks and 5*s*. per week for remainder of sickness for Women.

(3) Sickness Benefit starting at the earliest on 13 January 1913. Disablement Benefit (after first 26 weeks) starting at the earliest on 15 July 1914.

(3*a*) Under the original Act persons aged over 50 last birthday on the 15 July 1912 received lower rates of sick pay, but as a result of the Amending Act (1913 iii and iv George V. Ch. 37) these insured persons now receive the full rates of sick pay.

(4) Maternity Benefit of 30*s*. for the wife of an insured member or for a female member.

Effect of a Scheme.

It is provided that the scheme must not prejudice the solvency of the Society existing prior to its introduction.

Friendly societies are accustomed to measure their solvency as so much assets per £1 of gross liability.

The assets consist of accumulated funds, present value of contributions, value of reassured benefits, and the amount of arrears, accrued interest, &c.

The liabilities consist of the value of the benefits, sundry debts, deficiencies in the reassured benefits.

The solvency is measured by the ratio $\frac{\text{Assets}}{\text{Liabilities}}$ expressed in terms of £1. If in any particular case the value of this ratio were equal to ·8, the society would be said to be worth 16*s*. per £1 of liability. This system is understood by members of friendly societies and from circulars issued by the Registrar of Friendly Societies in connection with Section 72 it appears to have received official sanction. It is required that the degree of solvency after the scheme shall not be less than before its introduction. A point arises in connection with societies holding

a surplus prior to the introduction of the scheme, thus being "worth" more than 20*s*. per £1 of liability. I submit that the scheme must not be used as a method of distributing this surplus, and as a result after the scheme the surplus measured per £1 of liability must be intact.

The case of a society paying a small sickness benefit, such as 6*s*. per week for the first 26 weeks and 3*s*. per week for the remainder, may be taken as an illustration. Under ordinary circumstances it would not be possible to grant the full abatement of 3*d*. per week to the insured member of such a society, without increasing or creating a deficiency. If, however, the society had a large surplus it might be possible to grant the insured members under the scheme the full abatement and still leave the society solvent.

The effect of such a scheme would be that the surplus created by all the members of the society would be distributed in part or in whole among the insured members under the scheme.

This cannot, I think, be the intention of the Act, and a scheme drawn up on these lines would not I think fulfil the conditions of the Section.

If the society is worth 25*s*. in the £1 prior to the scheme it must retain the same degree of solvency after the scheme has been put in operation, and any distribution of surplus should be dealt with subsequently by an amendment of rules.

If any funds are set free as a result of the scheme, then they must be applied first of all towards eliminating the deficiency of a Society or Branch. If after eliminating the deficiency there remains a surplus, that is to say, if the degree of solvency is as a result of the scheme increased (providing it exceeds 20*s*. per £1 of liability), the surplus can be distributed by means of a supplementary scheme, either in paying the contributions of insured members making the society their approved society, or in providing extra benefits to all the members of the society, namely:

> Insured members who, having selected the scheme to reduce, have created the surplus.
>
> Insured members who do not select the scheme to reduce but continue full benefits and contributions.
>
> Uninsured members of the society.

The last two classes have not contributed in any way to the surplus by setting free reserves against Sickness Benefits.

The scheme and any supplementary scheme must be passed by the members of the society as an amendment of rules, and must then be submitted to the Registrar of Friendly Societies for his confirmation. All confirmed schemes are automatically incorporated in the society's rules. A society with branches may submit a scheme for all its branches, but as the scheme, in practice, has to give effect to the scale of benefits paid, the age distribution and the condition of solvency of any individual branch, I think that, for the purposes of a scheme, the branches of an Affiliated Order must be considered as separate societies.

Effect of the Amendments in Section 55 of the Bill.

Before dealing with the actual preparation of a scheme it may be as well to mention that the result of giving the members of friendly societies the option of continuing or reducing their benefits and contributions has been that in most societies the great majority of members have decided to continue all their contributions and benefits with the exception of the medical contribution.

I do not think that 10 per-cent of the eligible members of friendly societies have accepted the scheme. Such a result was unexpected and was not even suggested when the Bill was before the House of Commons in Committee.

Preparation of a Scheme.

For the preparation of a scheme under Section 72 we require enough data to enable us to make a valuation of a friendly society as at 31 December 1911. If a reliable valuation had been made somewhere near to this date, it might be used as a basis for a preliminary scheme. As, however, a large percentage of the valuations of friendly societies are prepared by unqualified valuers and are quite unreliable, I think that a valuation of the Society immediately prior to the scheme is a sound preliminary. In addition to the ordinary data for a friendly society valuation we need the following information :

(*a*) Separate indication of all insured members with their ages.

(*b*) Insured members who have advised the society that they desire an abatement in their contributions and benefits to be separately scheduled with their ages on 31 December 1911. These members must, according to the Section, have been in the society on 16 December 1911, but probably if they are in the society on 31 December 1911 this will be near enough.

(*c*) Insured members under (*b*) should be further subdivided into (1) ordinary insured persons; (2) special classes of insured members. These latter include members who are aliens for the purposes of National Insurance, members engaged in the military, naval and marine services, members engaged in the Mercantile Marine, and members under age 21 without dependants.

Having ascertained the position of the society prior to the adoption of the scheme we next prepare a valuation of the reduced benefits for members coming under paragraph (*b*) above.

I suggest that this valuation can be best dealt with by treating all members as entitled to the full reduction of benefits, and subsequently making adjustments for the extra reserves in the case of the special classes and for the additional liabilities remaining to a society after the release of the State Benefits.

The tables to be employed for the scheme should be on the same basis as the tables employed in the valuation (but see a suggestion on page 329). We are, of course, now dealing with Term Benefits ceasing at age 70 instead of Whole Life Benefits which arise in the ordinary valuation. If the tables in the valuation have been adjusted to meet light or heavy sickness rates experienced by the society, then, provided these adjustments are required for ages below age 70, a similar adjustment should be used for the scheme.

The principal liability released by the scheme is that for sick pay and disablement benefit, both ceasing at age 70. Sick pay will be at the rate of 10*s*. per week for 26 weeks, and disablement benefit at 5*s*. per week for the remainder of sickness. If, however, a society pays smaller benefits than these, the smaller benefit must be taken; while if the society's sick pay ceases under its rules at an earlier age than 70 that age should be taken.

Thus, if a society pays under its rules 8*s*. for three months, 4*s*. for three months, and 2*s*. for the remainder of sickness, sick benefits ceasing at 65, these are the benefits which should be dealt with under the scheme.

The release of liability for sickness benefits can be put in the following form:

$$X_{1},\, {}_{70}s_x^{t} + X_{2},\, {}_{70}s_x^{t/t} + X_{3},\, {}_{70}s_x^{2t/all},$$

where X_1, X_2, X_3, are weekly rates of sick pay, and ${}_{70}s^t_x$ is the present value at age x of a sickness benefit of 1 per week for the first t weeks of attack ceasing at age 70.

Sickness Benefits excluded by the Act.

For the ordinary insured members of Friendly Societies, the following benefits usually provided for by these societies are not paid under the National Insurance Act, and as a result of the Scheme insured members should not be deprived of them:

(1) Sick Pay in excess of 10*s.* per week (7*s.* 6*d.* for women) for the first 26 weeks of sickness.

(2) Disablement pay in excess of 5*s.* per week after the first 26 weeks of sickness.

(3) All sickness and disablement benefits after age 70 is attained.

(4) Sickness and disablement benefit where a member is in receipt of compensation from his employer under the Workmen's Compensation Acts, or at Common Law, from either his employer or third parties.

(5) Sick pay during the first three days of sickness, except where less than 12 months separates the two attacks.

(6) Sick pay between 15 July 1912 and 13 January 1913.

(7) Disablement benefit between 15 July 1912 and 15 July 1914.

(8) For the special classes of insured persons, referred to later, the difference between the ordinary scales of benefit and the reduced scales of benefit which they receive under the Act is not a benefit provided under the Act for these members, and extra reserves are required for these members to make up the deficiencies.

Extra Reserves.

In his sixth lecture, on page 102, Mr. Watson seems to suggest that it is optional for a society to grant its insured members under the scheme the additional benefits excluded by the Act and set out above under headings (4) to (7). Since these lectures were delivered I understand that the Registrar of Friendly Societies maintains that as the above benefits are not benefits similar to those granted under the Act, a society cannot, by its scheme, relieve itself of its liability to its insured members for these benefits; hence for

each of these members extra reserves for the items (4) to (7) must be provided, the extra reserves under headings (1) to (3) having been made by the use of term functions and fixed rates of abatement.

First 3 *days' Sickness.* The Act does not provide sick pay during the first three days of sickness. By Section 8, sub-section 5, it is, however, provided that a second attack occurring within a year of a previous illness shall be considered as a continuation of the previous illness. The independent side of a society has to provide, therefore, for attacks lasting less than three days, and for not more than the first three days of sickness in any one year. Mr. Watson suggests that in default of the society's data providing reliable information as to the cost of this benefit it should be taken as an annual sum of 1*s*. to 1*s*. 3*d*. In the illustrative tables I have assumed an annuity ceasing at age 70 ; 1*s*. per annum for Table C, and 1*s*. 3*d*. per annum for Tables D and E.

Waiting Periods.—The scheme must also provide (for insured members under it) sick pay at the rate of 10*s*. per week until the 13 January 1913, and disablement pay up to 14 July 1914 at the rate of 5*s*. per week. (If the present benefits are less than the National Insurance benefits the lower rates are taken.) As a reserve for this, the expected sickness during the first six months at age (x), and after the first sick months at ages (x) or $(x+1)$, have been taken.

Claims under the Workmen's Compensation Acts, &c.—Various authorities have recommended friendly societies either to discontinue the payment of sick pay while a member is in receipt of compensation from his employer, or, if they must pay this indemnity, then to meet it from a separate fund, supported by extra contributions or levies. While the submission of a scheme presents an opportunity of again recommending such a course, the exclusion of claims for Workmen's Compensation cannot, I think, be dealt with as part of the scheme. The National Insurance Act does not provide for the payment of sick pay when an insured person is in receipt of compensation under the Workman's Compensation Acts or at Common Law. If a society pays sick benefit during incapacity arising from industrial accidents, then, as this is not a benefit "similar to those granted under the Act", a reserve must be provided by the scheme for the continuing of the payment of sick pay under these conditions from the voluntary funds of a Society.

The rate of claim for industrial accidents is a function of the employment, and of the age of the workman, but the cost of claims arising from the duration of incapacity largely depends upon the age of the workman. In the absence of satisfactory data it is not possible to make an accurate reserve for this additional liability. Messrs. Hardy and Wyatt estimated that the exclusion of Workmen's Compensation claims reduced the sickness rates for the whole insured population by 10 per-cent. While it might be possible to make a reserve for the additional liability on these lines, the extra liability of 10 per-cent only applies to a particular table for the whole insured population. It would be unsuitable for the valuation of an individual society, where the "accident" risks might be far in excess of this figure, or where the valuation basis is different.

It is extremely difficult to come to any conclusion as to the cost of the "Accident Benefit" which will be left over to a society for their insured members under the scheme.

In *J.I.A.*, vol. xliv, Mr. Watson has investigated the cost of accident claims to friendly societies. His remarks deal chiefly with two classes of employment where the heavy cost of accident claims is well known.

Cases A and B deal with Railway Societies. Case C deals with a society containing 50 per-cent of miners in its membership.

In the following table I reproduce the average number of weeks' accident claims for all members.

Table A.

Age	Case A	Case B	Case C — 50 % Mining Members	
	Weeks—all Members	Weeks—all Members	Weeks—all Members	Average Cost
				£
15–19	·508	·218	·409	·253
20–24	·384	·269	·542	·299
25–29	·441	·273		
30–34	·450	·292	·574	·323
35–39	·462	·279		
40–44	·374	·309	·840	·426
45–49	·406	·312		
50–54	·532	·434	·745	·315
55–59	·707	·590		
60–64	...	·689	·396	·123

I have had recently placed before me some figures relating to the experience of the British Steel Smelters, Mill, Iron, Tinplate and Kindred Trades Association. This is a Trade Union with a large membership engaged in steel smelting, tin-plate working, and similar occupations.

It is known that these employments involve a considerable accident risk which is excluded by the National Insurance Act. This Union became an approved society, and set up a separate fund to supplement the benefits granted under the Act. The chief benefit granted from this separate fund was the payment of sick pay when a member is in receipt of compensation from his employer under the Workmen's Compensation Act or at Common Law. The rate of benefit granted from the Fund is 10*s.* first 13 weeks, 5*s.* second 13 weeks, and 2*s.* 6*d.* for the second six months, after which time the indemnity for any one accident ceases.

The rates of benefit are considerably lower than are provided under the National Insurance Act and no permanent sick pay is available. The results of the working of this separate fund during 1913 are now available, and thanks to the courtesy of Messrs. John Hodge, M.P., John Baker, and D. Jenkins, Officers of the Association, I am in a position to include these results in Table B.

A single year's working cannot give data of much weight, but from the fact that the type of compensation paid is similar to that remaining over to a friendly society as a result of Section 72 of the Act it may be of some interest.

The following points may be noticed from the experience set out on page 328.

(*a*) The year's working does not include a full year's claims. Cases that were on the funds on the 13 January 1914 have not contributed a full year's claims to the experience while there are no compensating claims brought forward for a previous year. The number of cases on the funds was 33.

(*b*) The large numbers of lapses which took place after compensation had been paid, 12·65 % of the claimants having lapsed. It will be seen that the proportion of these lapses increases with the age of the members. I suggest that a part of these lapses may be due to permanent or continued incapacity. The experience is that of a Trade Union and in the

TABLE B.

Experience of the British Steel Smelters for the year 13 January 1913–12 January 1914.

Incapacity due to Accident (W.C.A.) Claims only.

Age	Number Exposed to Risk E_x	Number of Claims	Number of Claims outstanding end of year	Number of Lapses after Benefit	Number of Weeks "Accident" Claims			Percentage of Claims to Members	Percentage of Claimants who lapsed after Benefit	Number of Weeks "Accident" Claim per Member per Annum		
					First Three Months	Second Three Months	Second Six Months			First Three Months	Second Three Months	Second Six Months
16–20	374	49	1	5	196	14	21	13·10	10·20	·523	·038	·056
21–25	777	138	4	16	578	42	13	17·76	11·59	·744	·054	·017
26–30	927	129	5	20	618	24	0	13·92	15·50	·667	·026	·000
31–35	980	138	1	11	701	64	26	14·08	7·97	·715	·066	·027
36–40	958	130	1	15	605	36	4	13·57	11·54	·631	·037	·004
41–45	779	98	4	7	503	81	38	12·58	7·14	·645	·104	·049
46–50	632	79	2	14	391	40	0	12·50	17·72	·618	·064	·000
51–55	418	62	7	11	291	38	38	14·83	17·73	·696	·091	·091
56–60	236	45	5	8	275	44	31	19·07	17·77	1·164	·186	·131
61–65	111	14	2	4	106	34	35	12·61	28·57	·955	·306	·315
66–70	36	3	1	1	39	27	22	8·33	33·33	1·083	·741	·602
Totals	6,228	885	33	112	4,303	44[illegible]	228	14·21	12·65	·691	·071	·037

case of a member leaving the employment as a result of a serious injury he would discontinue his membership of the Trade Union or of this Fund at least.

(*c*) The actual cost of benefits during the year's working amounts to £·368 per member for all ages.

So far the classes referred to above deal only with employments where the existence of a heavy accident risk is well known. There are many employments where there is some risk, although it may not be heavy.

It therefore seems that the additional risk for accident claims remaining over to a friendly society as a result of the scheme involves a more or less heavy liability for all insured members.

To provide the additional reserves I suggest one of the two following alternatives.

If the valuation of the society has been made by tables loaded for excessive sickness or by tables involving heavy rates of sickness, then the supplementary valuation for the release of liability might be made by tables involving the same mortality, but the normal rate of sickness.

Thus, if the principal valuation had been made by the Manchester Unity, 1893–97, Class B.C.D., the valuation of the released liability might be made by a table deduced from the group A.H.J., the same mortality being involved in each case.

The assumption is, that the whole of the excess sickness in the B.C.D. group is due to claims for compensated accidents. Mr. Watson's investigations would seem to suggest that for many employments this is a fair approximation. It must be borne in mind, however, that some heavy employments require such a high state of physical condition that sickness which would involve only short claims in a light occupation may keep a member on the funds for a lengthy period.

A difficulty in making allowance for accident claims in the above manner is caused in the case of those societies where the valuation has been made on the normal basis without adjustment for heavy sickness. Although the risk in these cases may not be heavy there is undoubtedly an appreciable risk, and some reserve should be made.

Another alternative method of making an allowance for the accident risk is to assume that it can be reserved in the form of an annuity ceasing at age 70, the amount of the annuity

increasing with the attained age of the member at the time of the valuation.

The advantage of making an allowance in the form of an annuity is that effect can be given to the various classes of employments to be found in a friendly society, a reserve being made for those cases where the risk, although light, is still appreciable, and should not be ignored. In the illustrative tables I have made use of this method, assuming an annuity increasing in amount with the rates of sickness and with the attained age.

Contributions.

So far I have only considered the release of liability for sick pay and the extra reserves required. As against the release of liability we have on the other side of the valuation balance sheet a reduction in contributions for these members under the scheme. This reduction in contribution ceases at age 70. So far as the scheme is concerned, after age 70 the member resumes the full rate of contribution, and receives the full rate of sick pay. Mr. Watson has suggested in his lectures, that in view of the allowance for administration under the National Insurance Act part of this abatement in contributions should come from the management fund. I have reason to believe that the Committees of Management of many approved societies find the sum allowed for administration for State Insurance inadequate. This is, I understand, especially the case with the approved sides of many affiliated orders where branches contain from 100–200 insured members, but the accounts and records that have to be kept are nearly as full as for a centralized society with many thousands of members. Under these circumstances I do not think that the scheme can rely on an abatement in contributions from the Management Fund.

The actual abatement under the scheme which a society can make in a member's contributions depends on various factors. The abatement can be set out symbolically as follows :

Release of liability =

$$X_1 \sum_{x=16}^{x=69} {}_{70}s_x^{t} + X_2 \sum_{x=16}^{x=69} {}_{70}s_x^{t,t} + X_3 \sum_{x=16}^{x=69} {}_{70}s_x^{2t/all} - \text{(extra reserves)}.$$

$$\text{Release of contributions} = c \sum_{x=16}^{x=69} \bar{a}_{x\overline{70-x}|}$$

Where X_1, X_2, X_3 are weekly scales of sick pay:

$_{70}s_x^t$ = value of sick pay ceasing at 70 for first t months of attack.

c = annual abatement allowed on contribution.

Except in the case of societies with a surplus the value of c must be such that the value of the release of contributions does not exceed the total value of the release of liabilities.

Tables C, D, and E give the reserves at quinquennial ages, allowing a full abatement of 3*d.* per week.

For the purpose of setting up these illustrative reserves, I have taken sick pay on two bases, namely:

(1) 10*s.* per week 26 weeks,
5*s.* per week remainder.

(2) 10*s.* per week 26 weeks,
5*s.* per week 26 weeks,
2*s.* 6*d.* per week remainder.

Reserves have been made for the "waiting periods", and the first three days on the lines suggested above.

The extra reserves for compensated accidents have been taken as an annuity. The amount of the annuity varies with the table used for the reserves and is shown in each table.

The reserves have been calculated throughout at 3 per-cent interest.

TABLE C.

Sample reserves based on the Manchester Unity, 1893–97, Group A.H.J., Area (1) Rural and Urban. Interest 3 per-cent. A contribution of £·65 per annum (3*d.* per week) ceasing at age 70 is assumed.

Deduction is made for extra reserves for the following contingencies:

(*a*) First 3 days of sickness.

(*b*) Compensated accident claims, taken as an annuity ceasing at age 70, at the following rates:

Age attained	Annuity *s.*	*d.*
16–25	0	6
30–40	1	0
45–50	1	6
55–65	2	0

(*c*) Sick pay during the waiting periods.

The amount of the deductions under *a*, *b* and *c*, is given in column III.

Column I. Reserves on the basis of sick pay at rate of 10*s*. first 26 weeks, 5*s*. remainder.

Column II. Reserves on the basis of sick pay at rate of 10*s*. first 26 weeks, 5*s*. second 26 weeks, 2*s*. 6*d*. remainder.

Age	Sample Reserve I	Sample Reserve II	Extra Reserves III
16	− 2·36	−3·88	+2·25
20	− 1·23	−2·95	2·13
25	+ ·43	−1·57	2·06
30	+ 1·85	− ·46	2·48
35	+ 4·07	+1·40	2·39
40	+ 6·13	+3·09	2·34
45	+ 7·90	+4·44	2·68
50	+ 9·71	+5·89	2·70
55	+10·54	+6·43	3·19
60	+10·06	+5·96	3·62
65	+ 5·69	+2·41	4·92

Table D.

Sample reserves based on the Manchester Unity, 1893–97, Whole Society. Interest 3 per-cent.

A contribution of £·65 per annum (3*d*. per week) ceasing at age 70 is assumed. Deduction is made for extra reserves for the following contingencies :

(*a*) First 3 days of sickness.

(*b*) Compensated accident claims, taken as an annuity ceasing at age 70, at the following rates :

Age attained		Annuity *s*.	*d*.
16–20	... ...	1	0
25–30	... ...	1	6
35–40	... ...	2	0
45–50	... ...	2	6
55–60	... ...	3	0
65	... ...	4	0

(*c*) Sick pay during the waiting periods.

The amount of the deductions under *a*, *b* and *c*, is given in column III.

Column I. Reserves on the basis of sick pay at rate of 10*s*. first 26 weeks, 5*s*. remainder.

Column II. Reserves on the basis of sick pay at rate of 10*s.* first 26 weeks, 5*s.* second 26 weeks, 2*s.* 6*d.* remainder.

Age	Sample Reserve I	Sample Reserve II	Extra Reserves III
16	− 2·08	−3·65	+3·18
20	− 1·03	−2·81	3·02
25	+ ·16	−1·91	3·41
30	+ 2·14	− ·25	3·25
35	+ 3·81	+1·06	3·57
40	+ 6·03	+2·89	3·41
45	+ 7·78	+4·24	3·65
50	+ 9·70	+5·75	3·53
55	+10·60	+6·33	3·89
6	+10·10	+5·80	4·27
6	+ 5·43	+1·97	5 66

Table E.

Sample reserves based on the Manchester Unity, 1893–97, Group E.F., Area (2) Rural and Urban. Interest 3 per-cent. A contribution of £·65 per annum (3*d.* per week) ceasing at age 70 is assumed.

Deduction is made for extra reserves for the following contingencies :

(*a*) First 3 days' sickness.

(*b*) Compensated accident claims, taken as an annuity ceasing at age 70, at the following rates :

Age attained	Annuity *s.*	*d.*
16–20	3	0
25–30	4	0
35–40	4	6
45–50	5	0
55–	6	0
60–65	7	0

(*c*) Sick pay during the waiting periods.

The amount of the deductions under *a*, *b* and *c*, is given in column III.

Column I. Reserves on the basis of sick pay at rate of 10*s.* first 6 months, 5*s.* remainder of sickness.

Column II. Reserves on the basis of sick pay at rate of 10*s.* first 6 months, 5*s.* second 6 months, 2*s.* 6*d.* remainder.

Age	Sample Reserves I	Sample Reserves II	Extra Reserves III
16	− ·01	−1·88	+5·65
20	+ 1·05	−1·07	5·38
25	+ 1·88	− ·57	6·11
30	+ 4·07	+1·27	5·77
35	+ 5·90	+2·69	5·86
40	+ 8·18	+4·50	5·54
45	+10·07	+5·90	5·52
50	+12·39	+7·68	5·10
55	+13·41	+8·18	5·56
60	+12·61	+7·14	6·34
65	+ 7·02	+2·41	7·71

Scales of Sick Pay.—A consideration of the Tables C, D, and E, and the formula set out above shows that the rates of sick pay granted by a society have a considerable influence on the amount of abatement that can be granted. This is only to be expected, and is only mentioned because there is a "popular" idea that the abatement must always be 4*d.* per week (including 1*d.* per week for the medical benefit).

Age Distribution.—It will be seen from the tables that the reserves, being the difference between the value of the benefits released and the value of the reduction in contributions, are negative at the younger ages, rise to a maximum about age 55, and then fall somewhat to the end of the table.

In Tables C and D, up to age 30, and in Table E up to age 25, the liability from which a society is relieved, under the lower scale of benefits, is less than the value of the abatements of 3*d.* per week granted to the members under the scheme. Hence there will be no funds set free for the younger members of societies if they are to receive the full abatement. If the age distribution of members coming under the scheme is towards the younger ages in one society, the abatement which can be granted will be less than in another society paying similar benefits and under like conditions of solvency, but where the ages of the members under the scheme are older.

Mortality and Sickness.—If it has been necessary to adjust the standard sickness tables for excessive sickness then the release of liability is increased and the abatement is corres-

pondingly increased. [Unless the release for excessive sickness is reserved against accident claims as suggested above.] If an abatement has been made in the standard sickness rates to allow for light sickness, the release of liability will be reduced and the abatements may also be less. When the mortality is heavy and is given effect to in the valuation, the abatement in contributions can be greater than in an exactly similar society valued on a table giving a light rate of mortality, the heavy mortality tending to decrease $\sum \bar{a}_{x\overline{70-x}|}$ more than the values of the sickness benefits.

Accident Reserve.—The society retains the full liability for sick pay arising from accidents. The extent of this liability depends on the occupations of those members who come under the scheme. Societies with a large proportion of their members engaged in mining and heavy industrial occupations will find that the scheme leaves behind a considerable liability for which it is particularly desirable that adequate reserves should be made. In all societies, even if the employments are light, substantial reserves should be provided against accident claims, for this is a risk the society undertakes without any direct contribution.

Condition of Solvency.—An important factor in the amount of abatement that can be allowed under the scheme is the condition of solvency of the society prior to its introduction. If a society holds assets to the extent of 85 per-cent of its liabilities it cannot release, against part of its liability, assets of an amount equal to the liability without prejudicing its degree of solvency. It can only relieve its members of a percentage of the full amount of abatement. As a result of sickness rates, age distribution, and the other factors referred to above, it is possible that this proportionate abatement may amount to the full 3*d*. per week. Where, however, the deficiency is very considerable, the members cannot under any circumstances receive the full abatement in their contributions.

Special classes. Insured persons.

Under the National Insurance Act insured persons whose ages exceeded 50 last birthday on 15 July 1912 received reduced benefits; under the amending Act, however, these members now all receive full rates of benefits, and thus for the purposes of a scheme, framed after the amending Act came into force, must be considered as ordinary insured persons. There are still various classes of insured persons receiving reduced benefits who require special treatment under the scheme. I suggest that the best method is to consider

these members as ordinary insured persons, and subsequently set up extra reserves for the additional liability remaining to a society owing to the reduced benefits they receive under the Act.

(i) *Unmarried Minors.*—Insured persons under age 21 who have no dependants receive reduced rates of sick pay until age 21. For males these rates are 6*s.* for 13 weeks, 5*s.* for second 13 weeks, and 5*s.* for the remainder of sickness, the additional liability remaining to the Society being sick pay at the rate of 4*s.* for the first 13 weeks, and 5*s.* for the second 13 weeks, ceasing at age 21. For this small liability provision can easily be made on an exact basis.

(ii) *Aliens.*—For the purposes of the National Insurance Act, Alien has a particular interpretation. This is set out in Section 45 of the principal Act, and attention is also called to sub-section (4) of this section. Shortly, those foreigners are not treated as "aliens", who on 4 May 1911 were members of a friendly society (which ultimately became an approved society) and had then been resident in the United Kingdom five years or over. These foreigners are treated for National Insurance as British subjects. All other foreigners are aliens for the purposes of the scheme. Aliens receive reduced rates of sick pay. These reduced rates arise from the fact that no reserves are credited to approved societies for these members. As the sick pay under the Act is reduced the society retains in its voluntary side a larger share of the sick pay originally granted. The abatement in contributions should therefore be lower than for ordinary members. The alien members' sickness benefits vary with age, and if an exact account were to be taken of the rates received in fixing the abatement it would involve an individual valuation. This is against the spirit of the Section and would cause an amount of work out of all proportion to the results obtainable.

The Insurance Commissioners have issued a table (Table G), showing the rates of benefit receivable by aliens under the National Insurance Act. The contributions being constant, these rates of benefit vary with the age. The adoption of the table was not compulsory, but as any other table would have required to be actuarially certified, I believe most if not all approved societies have adopted Table G for their alien members. The table is in a convenient form for the purpose of Section 72 and should be taken into account in fixing the extra reserves required for this class of member under the scheme.

An illustration of the extra reserves required may make this point clearer.

For aliens between the ages of 35 and 40, the rate of benefit provided is 6*s.* first 6 months and 3*s.* the remainder.

The society will only be relieved of liability to this extent. I suggest that the alien should in the first instance be treated as insured for the full rates under the Act, and, assuming that the society pays at present from its voluntary funds the full State rates of benefits, extra reserves for sick pay of 4*s.* for first 26 weeks, and 2*s.* per week remainder of sickness should be set up.

In dealing with the reduction in contributions this should be proportionate to the value of the reduced benefits taken over by the State side.

As an approximation to the reduction in contributions for alien members I suggest that the abatement for ordinary insured persons should be reduced in the ratio that the total full sick pay for aliens bears to the total full sick pay that would have been received had the members been A1 members eligible for full benefits.

Thus, in the above example, if all the alien members were between 35-40 the reduction would be $\frac{\Sigma 6s. \times n}{\Sigma 10s. \times n} (=\cdot 6)$ of the reduction for ordinary members under the scheme.

(iii) *Naval, Military and Marine Members.*—Until retirement from the services these members do not receive sick benefit, the only benefit is maternity benefit. The contribution paid by them individually is $1\frac{1}{2}d.$ per week. After retirement full rates of benefit are received under National Insurance, and the full contribution of 4*d.* per week is paid. For these members the scheme should, I think, be arranged so that after retirement they receive the full abatement, and are treated as ordinary members under the scheme.

Until retirement the society pays from its voluntary funds the full benefits to which the member is entitled under the rules. A difficulty arises in fixing the age at which retirement takes place. This varies with the different services. For the purpose of the scheme I suggest that 35 might be taken as the age at retirement. Assuming that the society pays the State rates of sick pay, the release of reserves can be expressed as follows:

Release of liability

$$= \cdot 5 \sum_{x=16}^{x=34} \frac{K_{35}^{6} - K_{70}^{6}}{D_x} + \cdot 25 \sum_{x=16}^{x=34} \frac{K_{35}^{6/\text{all}} - K_{70}^{6/\text{all}}}{D_x} - (\text{extra reserves})$$

Release of contributions

$$= c \sum_{x=16}^{x=34} \frac{\overline{N}_{35} - \overline{N}_{70}}{D_x} + c' \sum_{x=16}^{x=34} \bar{a}_{x:\overline{35-x}|}$$

Where c is the full annual abatement for ordinary insured members, and c' is the reduced scale to be found, the value of c' should be such that the release in the value of the contributions does not exceed the release in the value of the benefits. I would point out that the extra reserves for the first 3 days' sickness and accident claims are deferred liabilities, arising after retirement from the services. The reserve for the waiting periods is not required.

(iv) *Mercantile Marine Members.*—It is doubtful whether friendly societies having members engaged in the foreign going sea-service pay them any sickness benefits while outside the United Kingdom. I think they usually restrict the membership to the payment of death benefits. Assuming, however, that there are such members who come under the scheme, their position under the National Insurance Act is that while on a foreign voyage or in a foreign port they do not receive sick pay or medical benefit. They only receive this benefit on their return to the United Kingdom. Probably the National Insurance Act will provide disablement pay in all cases. For the purpose of fixing a scheme, I suggest that we can assume for this class that the society is only relieved of liability for sick pay at reduced rates (Disablement Benefit). It remains liable to pay full pay from its voluntary funds. The abatement in contributions should be reduced for these members, and I suggest that the abatement for ordinary members might be reduced in the ratio

$$\frac{\cdot 25 \Sigma_{70} s_x^{6/\text{all}}}{\cdot 25 \Sigma_{70} s_x^{6/\text{all}} + \cdot 5 \Sigma_{70} s_x^{6}}$$

On retirement from sea service the member receives the full abatement in contributions as provided for other members of the Society.

(v) The National Insurance Act provides that where an employer undertakes to pay the full wages for the first six weeks of sickness, no sick pay is receivable from the approved society during this period. A reduced contribution is paid. It is not possible to give actuarial effect to this abatement. The arrangement may be varied from time to time and may vary with different employers. I suggest, therefore, that these

members should be treated as ordinary members under the scheme.

Medical benefit.

The National Insurance Act provides medical attendance for all insured persons. In the past friendly societies used a certain portion of the member's contribution as a fee to the doctor. Frequently this charge was 4*s*. per annum, although in some societies it was more, and in others it was less. Whatever the sum taken for medical attendance may have been, a refund of the medical contribution should be made for all insured persons in a society, whether they desire to continue benefits or come under the scheme proper.

Completion of scheme.

The results of the scheme are shown in a valuation balance, and they are embodied in a series of resolutions. These resolutions must be passed by the members of the society at a general meeting called as for an amendment of rules. The resolutions when passed are submitted to the Registrar for confirmation and registration, and on registration they form part of the rules of the society.

Provisional schemes.

To meet the delay in preparing a scheme the Registry got out a system for provisional schemes based on the last valuation of a society. These schemes were adopted by many societies. When, however, the final scheme was issued, some members who had adopted the provisional scheme decided to resume full contributions and benefits. During a period of twelve months or more they had had their contributions reduced. As on a resumption of full benefits increased reserves are needed for the full benefits, a certain proportion of the reduction made in contributions should be refunded by these members.

Supplementary schemes.

If the scheme brings out a surplus or an increased surplus a supplementary scheme can be submitted for its distribution (but not by a cash payment to the members). This distribution can be by means of increased benefits or reduced contributions for all members of the society. If increased benefits are granted, I suggest that they should take the form of an addition to the death benefits granted, instead of an increase in sick pay. In the case of a branch of an affiliated order, part of the surplus set free in one branch can be utilized to make good the deficiencies in other branches.

The Act further provides that where funds are set free they can be used for the payment of part of the contributions of those insured members who were members of the society on 16 December 1911, and who made the society their approved

society for National Insurance. This certainly seems one of the best methods of distributing a surplus arising under the scheme. The chief objection to it is that insured persons, deciding to continue benefits, participate equally in the surplus set free by those who reduce benefits.

Consideration of the section.

(i) It will be seen from the Sample Reserves that at the young ages the abatement under the scheme results in negative reserves, while after age 30 the reserves are positive, and continue to increase up to age 55. This is inevitable in view of the system of a uniform abatement for all members who come under the scheme. As a result the younger members receive too large an abatement, and the older members too small an abatement in contributions. Except on the ground of expediency, there is, I would submit, nothing to be said for such a system.

As the old members of friendly societies have the idea that they have largely contributed to the success of their society, and consider that large proportions of the funds are standing to their individual credit, the system of a uniform abatement for all members has contributed to the failure of Section 72 to attract members to come under its operation.

(ii) The Section provides, as we have seen, for the distribution of any surplus among all the members of the society. This surplus is not the result of the usual operations of a society, but is created by the reduction in the contracts of some of its members under the operation of an Act of Parliament. When the section was discussed in the House of Commons particular attention was given to the old members of friendly societies, and the reason for this provision in the section was that they should receive some compensation for the loss of the young members' contributions and association. I would submit that better provision could be made for these older members by grouping branches of societies for purposes of permanent sick pay, and by extra valuation reserves, than by giving them a share in a surplus created by some of the insured members.

In addition to the old members who could not be insured under the Act, the Section enables insured members of a friendly society, who decide to continue contributions and benefits, to share in any surplus set free by those who reduce.

By their action this class have decided to stand outside the operation of the scheme, and it does not seem equitable that

they should share in any funds set free under it by the, probably, poorer members.

The fact that funds set free were to be used for the benefit of all members instead of being reserved for the insured members under the scheme weighed with these latter in deciding not to come under the scheme. Moreover, the benefit of 10*s.* for 26 weeks and 5*s.* thereafter is cheap for a contribution of $2\frac{1}{2}d.$ or 3*d.* per week which is the maximum abatement insured members could anticipate. In very insolvent societies the abatement would be less.

Failure of the section. I have above suggested reasons why Section 72 has failed to appeal to the large mass of members of friendly societies who are insured persons under the Act.

This failure is unfortunate for the societies. The financial position of many societies was weak before the passing of the National Insurance Act. Very large deficiencies were shown in the valuation reports, and owing to the inadequate manner in which many of the valuations had been made, it was thought that much larger deficiencies existed than were disclosed. It was asserted when the Section was discussed, that one result of the Section would be to eliminate these deficiencies and to place the societies in a solvent condition. As but a small proportion of members have come under the scheme, the societies in the aggregate have only benefited to a slight extent. They probably remain to-day with deficiencies nearly as large as before the introduction of the National Insurance Act. Conditions have changed, however, since the introduction of National Insurance.

Large numbers of the members of friendly societies receive sick pay from the voluntary side of their society, and from the State side in addition. Some of them are insured in two or more friendly societies or trade unions, and under the National Insurance Act in addition, the result being that members of friendly societies receive in sick pay a sum nearly equal to, and sometimes in excess of, their wages when at work. This is a condition which, experience shows, not only increases the number of claims, but also prolongs the duration of claims, and renders it increasingly difficult to get members off the funds once they are in receipt of sick pay.

Moreover, the medical administration under the National Insurance Act does not appear to be very efficient. This is the only medical attendance available for insured persons, and some of

the doctors on the panels grant certificates of incapacity with a freedom that enables insured persons to remain on the funds after they are able to resume their occupation.

The administration of approved societies is distinguished by a certain irresponsibility. This is partly due to the feeling that the Government must in a last resort make up any deficiencies shown on valuation. It is also in part due to outside pressure on approved societies to meet all claims, caused by an anxiety that the administration of National Insurance should prove to be popular.

The effect of the administration of National Insurance on the friendly societies is obvious. An insured member of a friendly society cannot claim against the State Section without also claiming against the voluntary funds. As a result of these influences, the sickness experience of friendly societies will increase in the future. Moreover, the attitude of members of friendly societies is different towards National Insurance. There is now no restraint on claiming benefit which was a feature in many friendly societies.

The year 1913 which has just closed was characterized by high rates of employment, a mild winter, and an absence of any extensive epidemics—all features which should contribute to light sickness claims. I believe, however, it is already well known that many friendly societies have had an extremely bad year, the amount of sick pay being heavily in excess of that required in previous years. I have myself seen the figures of a society where the amount required for sick pay in 1913 was nearly equal to the total of the amount required per head in the three years 1910, 1911, and 1912, and this notwithstanding the introduction of a number of young lives.

Amendments to Section 72.

Unless we are prepared to see many friendly societies collapse under the combined weight of their increased liabilities and old deficiencies, some early steps are required to enable them to get rid of the duplicate insurance resulting from National Insurance. I propose suggesting briefly the lines upon which an amended Section 72 might be drawn.

(1) All schemes should, I think, be compulsory. The experience of Section 72 proves that, unless a scheme is compulsory, only a small number of members will avail themselves of it.

(2) If the scheme is compulsory it should so far as practicable be strictly equitable as between members of

different ages and classes. This will involve a scheme where the abatement would vary with the attained age of the member. In fact, I suggest so far as this part of the scheme is concerned, a return to conditions provided for under the original section.

(3) The scheme should provide not only for a release of benefits similar to those granted under the Act, but also for extinguishing the remaining benefits for insured members paid by the friendly societies, namely:

(*a*) Sick pay after age 70.

(*b*) Sick pay during incapacity where compensation is payable under the Workmen's Compensation Act, &c.

(*c*) Sick pay during the first three days of sickness, when not received under National Insurance.

(4) The scheme might provide for additional benefits, in lieu of the reduction in contributions, providing these did not take the form of sick pay or come under headings (*a*), (*b*), (*c*) above.

Increased death benefits or pensions might be granted under this heading.

(5) The scheme might provide for strong reserves being made in the valuation, for the old members of friendly societies who remain uninsured. Provision might also be made for pooling permanent sick pay in the Affiliated Orders.

(6) After the above provisions, if any surplus is brought out it should be dealt with for the benefit of those insured members under the scheme who had created it.

In view of the fact that National Insurance has entered the area of party politics, it seems very doubtful whether any steps will be taken, unless the initiative comes from friendly societies themselves. Some scheme which will reduce benefits and contributions for all insured members is very important for these bodies. It could be provided for individual societies, I think, by amendment of Rules, but I am afraid that the members in many societies would reject any voluntary attempt to reduce sickness benefits, even if an adequate compensation were forthcoming in the form of abatements in contributions or alternative benefits.

PART II.

The Proposed Belgian National Insurance Act.

Introduction.

At the present time there is a certain agitation for the conversion of our National Insurance Act into a voluntary system. It may, therefore, be of some interest if we examine the proposed Belgian National Insurance Act. While this proposed Act cannot be said to be voluntary, it allows such a considerable freedom to all parties, that it approximates to a State Endowed Voluntary System. As the Act will be applied to a large extent through the friendly societies, I propose, in the first place, explaining their organization and position.

Belgian friendly societies.

Until quite recent years the operations of Belgian Societies were on a very small scale. As recently as 1891, there were only 412 registered societies, with a total membership of 54,870. In the last twenty years, however, the number of societies and their membership have increased in a surprising manner, as will be seen from the following :

Year	Number of Independent Societies	Membership
1895	734	87,312
1900	1,790	196,487
1911	3,299	419,979

The progress outlined by these figures is largely due to the encouragement the societies have received from both the central and the local Governments.

At the present time Belgian Friendly Societies are of three types :

(1) Independent Societies.

(2) Federations of Independent Societies.

(3) Unions of Federations.

(1) *Independent Societies.*—These are the original body and are organized frequently on a religious, political, or industrial basis. They are often an offshoot of some other movement, and restrict their membership to participants in that movement. There are, for example, a number of societies connected more or less closely with the Labour and Socialist Parties in Belgium.

There are, however, societies that are outside any political, religious or professional movement, and are known as "Neutral

Societies." These societies approximate more closely to the average English friendly society.

The benefits granted by all societies are as follows:

- (*a*) Medical attendance and drugs for a member and his family.
- (*b*) Sick pay during the first three or six months of illness.
- (*c*) Funeral benefit.
- (*d*) Maternity benefit for women members.
- (*f*) Agency for the National Pension Office, and some subsidy towards the cost of pensions purchased through the society.

(*b*) The sick pay does not begin from the first day of illness. A delay of from three days upwards is required before a member is entitled to benefit for any attack of illness. The initial interval varies in the different societies. I have seen a case where no sick pay was paid during the first month of illness.

The rate of sick pay varies between 50 centimes per day (5*d.*) and frcs. 3.33 per day (2*s.* 8*d.*).

Some societies pay for Sundays, other societies only pay six days to the week. As employment on Sundays is usual for half a day, there seems little reason why sick pay should not be paid for Sundays. Sick pay generally ceases at age 65.

(*f*) These societies act as agents for the National Pension Fund (Caisse de Retrait). This Fund was started as far back as 1850, but progress was extremely slow—in the year 1887 there were only 5,400 voluntary members. The co-operation of the friendly societies was then introduced, and, as a result, by the year 1899, 168,080 members of friendly societies were contributing to the National Pension Fund.

The pensions granted by the National Pension Fund start at age 65, and they are purchased by means of single premiums. The amount of pension is supplemented by means of Government grants. The pensions can be purchased either with or without return in the event of death. The contributions for pensions are on a graduated scale.

For the benefits directly administered by the friendly societies it is usual to charge a uniform contribution at all ages of entry. Members are not admitted after age 40. After age 25 a small entry fee is required, increasing with the age. This is supposed to compensate the societies for the extra risk arising among old members.

No provisions are included in the Rules, nor by Act of Parliament, for periodical valuations, nor for the certification by an actuary of the Tables employed.

(2) *Federations of Societies.*—Up to quite recent years the independent society was the only type of friendly society existing in Belgium. As the majority of these societies only granted sick pay during the first three or six months of attack, a demand was created for sick pay for a longer duration. The societies wisely decided not to undertake, themselves, the payment of permanent sick pay. Federations of friendly societies were formed to undertake the administration of permanent sick pay, These federations operate locally and follow the policy of the original societies. We thus have Religious, Political, and Neutral Federations. When a society joins a federation, all its members are not required to join for permanent sick pay. For present members of a society, it is only compulsory for those under age 40 at the date the society is admitted. The older members of a society can join on terms. After, however. a society has been admitted, all new members joining the society must also join the federation. Federations do not generally admit members over 35 or 40.

The members of societies pay an extra contribution for permanent sick pay, uniform at all ages. The federations grant permanent sick pay after the first six, or three months. This permanent sick pay ceases at age 65. The rate of sick pay granted is Frc. 1.00 per day. Some federations pay for Sundays, others only pay for six days to the week.

The contributions paid vary with the federation, but a fairly favourite rate appears to be 20 centimes (2*d.*) per month, for all ages at entry.

This rate of 20 centimes per month might be compared with the following contributions deduced from the Manchester Unity 1893–97 Whole Society Experience, at 3 per-cent, loaded with 20 per-cent for contingencies.

Age	Yearly Contribution for Frcs. 6.00 payable after first six months' incapacity, ceasing at age 65
	Frcs.
20	4.37
25	5.33
30	6.53
35	8.09
40	10.11

The payment of management expenses is met from the contribution. There are no statutory provisions by which Belgian friendly societies charge a separate contribution for management, and meet the expenses from a separate account.

From the report submitted to the Belgian Parliament, the expenses for these federations in the year 1911 worked out at 11·64 per-cent of the members' contributions. In addition to the members' contributions, the federations have since 1906 received considerable subsidies from the Central and Provincial Governments. The question of subsidy which is closely connected with the proposed Act, is dealt with in the next paragraph.

The progress made by these federations in the last seven years is shown by the following table. It is, I think, quite remarkable.

Year	Number of Federations	Membership
1906	42	62,921
1907	50	85,693
1908	58	108,622
1909	67	137,942
1910	74	197,670
1911	83	236,605
1912	88	280,414

(3) *Unions of Federations.*—As their name suggests, these societies consist of combinations of the federations. They are organized on the same lines as the other two classes of society. The only benefit granted is the negotiation of transfers of members between federations in the same union, but a different locality. Some type of reserve value estimated on empirical lines is granted to the society accepting the transferred member.

General points. Belgian friendly societies of all descriptions do not appear to be much given to the pageantry and semi-masonic ceremonies which is such a picturesque feature of British friendly societies, but which adds so considerably to the cost of working, and distracts members' attention from the efficient management of the Insurance features of the societies.

However, to encourage the formation and administration of friendly societies, a decoration has been instituted by a Royal Decree (Order in Council).

Another point of difference arising from the separation of

temporary sick pay and permanent sick pay, is the absence of any relation between the two rates of indemnity. Permanent sick pay is sometimes equal to, or greater than, the temporary indemnity, a condition which would have pleased the late Mr. Ansell, but appears curious when compared with the custom of our friendly societies.

It is not usual for Belgian societies to grant sick pay when a member is in receipt of compensation from his employer.

Some societies require a declaration from a member that he is not insured in another society. In this way duplicate insurance is avoided. How far this custom is general I am unable to say.

Subsidies and legislation.

The Act of 1894, granting powers of registration for friendly societies, corresponds closely to the Friendly Societies Act, 1875.

No provisions are included, however, for annual audit, nor for valuations.

A few other points of difference may be mentioned :

(1) Friendly societies receive free postal facilities.

(2) The benefits of friendly societies cannot be alienated by the members nor attached by creditors.

(3) Friendly societies are forbidden to divide their funds.

It is provided, by later Acts, that only registered friendly societies can receive subsidies from the Central or Provincial Governments. In this connection the societies receive an annual subsidy of Frcs. 2.00 for each of their members affiliated to the National Pension Fund.

In the year 1906, by an Order in Council (or the Belgian equivalent) the State undertook to subsidize federations of friendly societies. These subsidies were continued for six years and were then incorporated in the Act of 5 May 1912. This Act provides for subsidizing the federations by the State to the extent of 60 per-cent of the contributions received from the members for permanent sick pay. Certain conditions are imposed on a federation as a means of qualifying for the subsidy. Briefly they are as follows :

(1) Admission of new members up to age 40 at least.

(2) The payment of a minimum rate of sick pay of Frc. 1.00 per day (6 days to the week).

(3) Sick pay by a federation must begin from the expiration of the third month, at the earliest, and from the expiration of the sixth month, at the latest.

(4) Sick pay must be continued until age 65.

(5) "So as to create reserves and establish a condition of financial strength" to qualify for a subsidy, the federation's income from contributions and interest must at least equal 75 per-cent of the amount paid in claims to the members. This percentage is based on the previous year's working, or it may be based on the results of the previous five years' working.

The federations continue to receive grants from the Provincial Governments. These grants vary in amount. For the year 1911 the provincial and communal grants amounted to Frcs. 157,100, and were equal to 32·37 per-cent of the contributions paid by the members. The total amount received by the federations, in State and Provincial subsidies, amounted to 76·91 per-cent of the members' contributions.

The conditions for the Provincial subsidies vary considerably. Some provinces pay a percentage of the contributions, other provinces pay a share of the claims; the latter is the system adopted by the Province of Hainault (Coal Mining and Metal Workers).

The extraordinary method set out above under (5) by which a solvent federation might be deprived of its Government grant, while any number of insolvent societies received subsidies, reappears in the National Insurance Bill, and I propose examining it when dealing with the Bill.

Belgian sickness and mortality.

There is at the present time no sickness experience referring to Belgian friendly societies. Efforts are being made by the Friendly Society Department to gather the necessary data for such an investigation, and tables similar in origin to Sutton's Registered Societies Experience will eventually result. The absence of data is given as one of the reasons why the proposed National Insurance Act should not be established on actuarial lines. It is suggested that the experience deduced from English friendly societies is not applicable, as these societies restrict full and half pay to the first 100 weeks of sickness. This is not correct, as the large majority of our societies have no restriction as to the payments of full and half pay.

It is not desirable to employ the sickness experience of one country for the operations of another, conditions of employment, housing, mortality, and legislation varying considerably. I think it will be found, however, that conditions in Belgium

are not so different from those existing in this country, that the experience of British friendly societies would, in the absence of other data, provide a fair base for the operations of similar societies in Belgium.

Although conditions of employment vary in the two countries, the staple employments themselves are very similar. For its population, Belgium has large coal, metal, and textile industries. Probably a greater proportion of the workers are engaged in hazardous occupations than is the case in this country. The Belgian Workmen's Compensation Acts do not provide such a complete indemnity. The mortality of the general population is, however, higher than in England.

Through the courtesy of Monsieur Ver Hees, of the Ministère de l'Industrie et du Travail, certain data, referring to the operations of federations of friendly societies, have been placed before me. These data refer to the years 1908–1911. Unfortunately they are not of sufficient age to permit of an extensive investigation, nor are they in a form which lends itself to comparison. They give at the end of each of the four years the number of members and the number of days' sickness experienced at each age. No particulars as to deaths, withdrawals, and new entrants are available.

A comparison, however, of the membership at one age, in any year, with the membership at an age one year younger, in the previous year, gives the increase, or net movement due to the above causes.

Another point of difficulty is, that the rules of the federations are not uniform in the amount of waiting period required before a member is free to benefit. While the general rule is to pay benefit after the first six months of attack, some federations pay after three months' sickness.

It must be remembered that the federations do not accept members directly, but only through an affiliated friendly society. This involves a further difficulty in dealing with aggregate figures deduced from the whole body of federations as the increased membership might be due to the admission of new members in old affiliated societies, or to the admission of new societies to affiliation with the federations. The latter event takes place by negotiation between the federation and the society, and the conditions vary as to age and waiting periods.

The increase in the membership of federations has been so large during the period, that the terms as to new entrants will

have an important bearing on the experience. On 31 December 1907, the membership was 85,693, while on 31 December 1911, it was 236,605.

I desired to compare the Experience in these federations with some Standard Table, and for this purpose selected the Manchester Unity 1893–97, Whole Society.

So as to give the utmost weight to the waiting periods I adopted the following approximation for arriving at the exposed to risk :

Exposed to Risk 1911 age $(x) = \frac{1}{2}$ [membership end of 1910 aged $(x-1)+$ membership end of 1909 aged $(x-2)$].

This approximation to the Exposed to Risk while easy to apply, gives, I think, weight to the fact that a member joining a friendly society will not be at risk in the federation with which it is affiliated for a year, or even longer.

The results of the comparison are shown in the following table.

TABLE F.

Experience of Belgian Federations of Friendly Societies. Years 1910-11. *(Males).*

Sickness after the first 6 *months compared with the Expected by the Manchester Unity, Whole Society,* 1893-97.

ge.	YEAR 1910				YEAR 1911				YEARS 1910-11		
	E_x	Expected Weeks	Actual Weeks	Per-cent	E_x	Expected Weeks	Actual Weeks	Per-cent	Expected Weeks	Actual Weeks	Per-cent
–20	8,898	342	754	220·47	12,030	462	1,343	290·69	804	2,097	260·82
–25	13,433	1,585	1,612	101·70	18,360	2,166	2,035	93·95	3,751	3,647	97·23
–30	17,066	2,885	3,569	123·71	23,406	3,956	3,510	88·73	6,841	7,079	103·48
–35	18,296	4,420	5,370	121·49	24,588	5,940	4,681	78·80	10,360	10,051	97·02
–40	16,565	6,248	7,190	115·07	22,792	8,597	8,096	94·17	14,845	15,286	102·98
–40	...	15,480	18,495	119·47	...	21,121	19,665	93·10	36,601	38,160	104·11
–45	14,660	8,388	7,365	87·80	19,674	11,250	9,387	83·44	19,638	16,752	85·30
–50	11,210	9,661	8,225	85·13	15,163	13,068	10,133	77·54	22,729	18,358	80·77
–55	7,912	11,645	10,171	87·34	10,804	15,900	12,337	77·59	27,545	22,508	81·71
–60	5,187	13,148	11,395	86·66	6,901	17,492	13,490	77·12	30,640	24,885	81·22
–65	2,740	13,133	10,305	78·46	3,534	16,942	15,306	90·34	30,075	25,611	85·16
–65	...	55,975	47,461	84·79	...	74,652	60,653	81·23	130,627	108,114	82·75
–65	115,967	71,455	65,956	92·30	157,252	95,773	80,318	83·86	167,228	146,274	87·47

A few points may be noted from the above experience:

(1) The actual sickness in the aggregate appears to follow closely the Manchester Unity Whole Society Experience, but to be more favourable than that Experience.

(2) The examination by age group shows, however, that at the youngest age group, 16–20, it is very heavily in excess. Various trials were made, but this feature remained, whatever system was adopted in getting out the exposed to risk.

(3) When the ages are further grouped, it appears that the sickness for age group 21–40 is well up to the

Table G.

Occupations of Male Members of Belgian Federations.

Occupations	Year 1908		Year 1909		Year 1910		Year 1911	
	Membership	Percentage of Total	Membership	Percentage of Total	Membership	Percentage of Total	Membership	Percentage of Total
1. Mining (mostly Coal and Iron)	11,665	11·15	19,824	14·89	36,168	19·08	43,570	19·19
2. Quarry Work	3,176	3·03	3,585	2·69	4,487	2·37	6,677	2·94
3. Metal Work (including Iron and Steel Foundry)	19,683	18·82	23,614	17·74	32,995	17·41	39,364	17·34
4. Glass, China and Chemical Works	7,215	6·90	8,932	6·71	10,943	5·77	12,558	5·53
5. Building	6,115	5·84	7,681	5·77	10,807	5·70	12,986	5·76
6. Woodworkers	5,341	5·11	6,819	5·12	9,383	4·95	11,279	4·96
7. Textile	6,070	5·80	7,566	5·68	10,761	5·67	12,637	5·56
8. Leather	2,071	1·98	2,873	2·17	3,693	1·95	4,161	1·83
9. Railway and Transport Workers	8,139	7·78	9,698	7·28	13,290	7·01	14,345	6·31
10. Agriculture	7,277	6·96	9,037	6·78	10,441	5·51	12,317	5·42
11. Clerks, Shop Assistants, Travellers, etc., and Professions	9,383	8·97	11,110	8·35	15,113	7·97	20,803	9·17
12. Various Employments (mostly light)	18,453	17·64	22,380	16·82	31,445	16·59	36,304	15·99
—	104,588	100·00	133,119	100·00	189,526	100·00	227,001	100·00

Manchester Unity Experience, being 104 per-cent of the expected, while the sickness from 41–65 is only 82·75 per-cent of that anticipated. When a further examination is made into the data there seems a reason for the apparent break at age 40 in the sickness experienced.

Belgian federations restrict their membership to persons under age 40 at entry. A table of occupations for members of these federations for each year was also furnished. An extract may prove of interest, and throw some light on the experience, although it is rather too recent to have its full effect on the sickness experience during 1910 and 1911.

The table opposite gives the percentage the membership in any particular occupation bears to the whole membership of these societies.

It will be noted that during the past four years most of the occupations numbered 2 to 12, have shown slight decreases in the percentage of the whole membership, and items 10 and 12 (Agricultural Labourers and Light Occupations) decreases of 1·54 per-cent and 1·65 per-cent. On the other hand the percentage of members engaged in coal mining has shown a remarkable increase of 8·04 per-cent. This is due, I believe, largely to legislative encouragement. The Province of Hainault, where one of the staple industries is coal mining, has during recent years heavily subsidized the federations working within its borders. There are now nearly 80,000 members of these societies in this Province.

The large increase in the membership engaged in coal mining, which has been going on since 1906 or 1907, accounts, I think, for the excessive sickness in the first age group, and it is a feature which will, I think, eventually affect the whole experience of these societies.

With regard to the light sickness after age 40, a somewhat different influence is, I suggest, at work. The membership of these federations was up to 1906 comparatively small. Probably these federations then contained a large proportion of men engaged in professional occupations, and who were interested in the political, social, or religious movements to which the friendly society formed an adjunct. These members are now chiefly to be found at the old ages where they tend to keep down the rates of sickness. Many of them, I would suggest, are in fact

honorary members, although they are included among the ordinary members of the federations.

Under these circumstances I do not think that the light rate of sickness after age 40 could be looked on as a feature of the experience that can be anticipated in the future.

The proportion of coal miners under a scheme of National Insurance, compulsory on all workers, would, I think, decline.

When the abnormal features suggested are taken into account the experience appears, from the limited data in Table F, to approach that anticipated by the Manchester Unity Whole Society. In the absence of any specific data referring to Belgian Sickness Experience, the divergencies shown above are not, I think, so large that the Manchester Unity Whole Society data, or some other Standard Table, would be inapplicable. Under any system of National Insurance there are bound to be wide divergencies from the present existing tables and experience, and the contributions and reserves should be heavily loaded to allow for contingencies.

Proposed Belgian National Insurance Act.

I have dealt at some length with the position of Belgian friendly societies, because the projected Act is to be operated to a large extent through these bodies; moreover, it follows closely the Act of 5 May 1912 in its methods, while this Act is directly incorporated in the scheme of National Insurance.

Scope of Act.

The Act provides that insurance against sickness, invalidity, and the provision of a pension at age 65 is compulsory on all workers, of both sexes, over age 16, engaged in manufactories, mines, agriculture, or commerce, providing they work for an employer, private or public, unless, in the latter case, the public authority makes an equivalent provision. (Invalidity is sickness lasting more than 3 months.) It is estimated that the insured population will amount to 1,800,000 persons.

Exemptions.—The following classes are exempted by the Act:

(1) Workers aged 65 and over.

(2) Those whose salary or earnings are more than Frcs. 2,400.00 [£95] per annum.

(3) Outworkers, working for more than one employer.

The following classes can obtain exemption by making application :

(4) Workers who receive board and lodging from their employer. This class would include domestic servants, and shop assistants living in.

(5) Miners who are in receipt of a pension under the Act of June 1911.

In this connection it is as well to point out that the employer is to pay contributions in every case of exemption, except where exemption is granted on the salary exceeding Frcs. 2,400 per annum. If the income exceeds this figure, but the salary or earnings are less than Frcs. 2,400, the member has to be insured and the employer has to pay.

The above exempted classes, and also persons working on their own account, can enter into insurance voluntarily. Aliens may be admitted into National Insurance if they have lived 10 years in Belgium, and if their native country offers Belgians equivalent advantages. Otherwise, there is no compulsory insurance of aliens, but the employer pays his contribution for any of them he may employ.

Administration of benefits.

For members of friendly societies and federations the administration of National Insurance is through their societies, providing they have become approved.

The primary society (the friendly society) provides:

(1) Medical and chemists' benefit.

(2) Maternity benefit, for female members only.

(3) Sickness benefit for the first 3 months of sickness.

The federations of friendly societies provide:

(4) Invalidity benefit for sickness of a longer duration than the first three months.

For those who do not join friendly societies and for those whose health or age prevents admission to these societies, the above benefits are administered through Local Councils, somewhat similar to our Insurance Committees. These Councils can arrange with the friendly societies in their area for the transfer of the Council's members to the society, or they can arrange with the societies to administer the benefits on behalf of the Local Council.

Old Age Pensions are provided directly from the State Pension Fund (Caisse de Retrait), the friendly societies continuing to act as agents.

Benefits granted.

Sick and invalidity benefits cease at age 65 so far as National Insurance is concerned.

In addition to medical benefit the minimum benefits which can be granted by approved societies are :

(1) Sick pay, first 3 months, Frc. 1.00 per day.

(2) Invalidity pay, after first 3 months. Frc. 1.00 per day.

(3) Maternity benefit, Frcs. 45.00 for female members.

(4) The payment during sickness of the minimum contribution for pension to the Caisse de Retrait, namely, 50 centimes per month.

(5) Sick pay to start at the latest from the fifth day.

The above benefits will be those granted by the Local Councils. The friendly societies may, however, grant any benefits in excess of these rates that they desire.

The payment of sick pay for Sundays is not compulsory.

Both sexes receive the same rates of sick pay.

The Old Age Pension is normally Frcs. 365.00 (£14. 12*s.*) per annum, starting at age 65 ; it is procured by means of single premiums.

Members' contributions.

So far as the members of friendly societies are concerned, the contribution payable is fixed by the rules of the society and federation through which they take their benefits.

For insured person who are not members of friendly societies the contributions are :

(1) Sickness maternity and medical benefits, Frcs. 12.00 (9*s.* 6*d.*) per annum.

(2) Invalidity (Sickness after first 3 months), Frcs. 6.00 (5*s.*) per annum.

For low wage workers, the contribution for sickness under (1) is reduced by half. The sick pay is also reduced by half. A low wage worker is one whose earnings are less than Frcs. 15.00 (12*s.*) per week.

Local Councils, however, have the right to increase contributions for their members. These supplementary contributions can be fixed for some or all of the members. In this way the heavier risks can be charged increased contributions.

The contribution for all insured persons towards Old Age Pensions is a minimum of Frcs. 6.00 per annum. This contribution must be employed in the purchase of a deferred annuity,

without return in the case of death. Insured persons can, however, pay additional contributions to increase their pension, or to compensate for their advanced age at entry into insurance. These additional contributions can be employed for the purchase of a deferred annuity with or without return in the case of death.

Low wage workers can pay a reduced contribution of Frcs. 3.00 per annum.

Employers' contributions.

The employers are required under the Act to pay a contribution of Frcs. 6.00 (5*s.*) per annum for each employee whose salary or remuneration is less than Frcs. 2,400 (£95) per annum, whether they are or are not insured persons.

This contribution is divided as follows:

(1) For sickness insurance (first 3 months), Frcs. 2.00 (1*s.* 8*d.*).

(2) For invalidity insurance (after first 3 months)—

Up to the year 1938, nothing.
After the year 1938, Frcs. 2.00 per annum at least.

(3) For Old Age Pensions—

Up to the year 1938, Frcs. 4.00 per annum.
After the year 1938, Frcs. 2.00 per annum or less.

The medical benefit is a first charge on the employers' contribution.

National subsidies.

The Government subsidy is divided under four headings:

(*a*) Sickness insurance. The subsidy is 0.25 centimes for each franc of contribution. The maximum amount of subsidy is Frcs. 3.00 per annum. The subsidy for insured persons born prior to 1865 is 0.50 centimes per franc. Presumably the maximum will be Frcs. 6.00 per annum.

The subsidy for those obtaining exemption is Frc. 1.50 per head per annum. This together with the employer's subsidy is used for medical benefit.

(*b*) Medical Benefit. A supplementary subsidy of from Frc. 1.00 to Frcs. 3.00 per head per annum, will be granted for this benefit in the case of those insured persons residing at a great distance from the doctor.

(*c*) The State subsidies for invalidity sickness are based on the Law of 5 May 1912. They are, however, slightly increased—

(i) Insured persons born in 1865 and afterwards, 0.60 centimes for each franc of contribution up to the first six francs of contribution, or a maximum of Frcs. 3.60 per member per annum. Insured persons born prior to 1865 receive a subsidy of one franc per franc of contribution up to a total of Frcs. 6.00 per annum.

(*d*) The State subsidies of Old Age Pensions follow the Laws of 1900 and 1911—

(i) Born after 1870, 0.60 centimes per franc for the first 15 francs of contribution.

(ii) Born between 1866–1870, Frc. 1.00 per franc for the first six francs, and 0.60 centimes per franc for the following eighteen francs.

(iii) Born between 1861–65, Frc. 1.50 for the first six francs, and 0.60 centimes for the next eighteen francs.

(iv) Born before 1861, Frcs. 2.00 for the first six francs, and 0.60 centimes for the next eighteen francs.

(v) The first six francs contribution per annum must be employed in the purchase of a deferred annuity without return.

In addition to the above subsidies granted as a proportion of the contribution the following old age pensions are granted by the Government without corresponding contribution from the insured.

(vi) In case of necessity a pension of Frcs. 120.00 per annum is granted to all Belgians living in Belgium and who were born prior to 1 January 1843.

(vii) Providing they have paid a minimum of Frcs. 18.00 into the State Pension Fund, all Belgians born between 1843 and 1872 receive in case of need a pension of Frcs. 120.00 per annum.

(viii) All Belgians born after 1873 receive an addition to their pensions. These additions are on a graduated scale, starting at Frcs. 115.00 for those born in 1873, and falling by Frcs. 5.00 for each year for those born in subsequent years.

(*e*) An immediate grant of Frcs. 5,000,000 for building Sanatoria is provided. In addition an annual payment will be made by the State towards their support.

Provincial subsidies.

It is probable that the subsidies to the federations by the Provinces will be continued. These vary in amount, but it is provided by the Bill that subsidies cannot be granted as a percentage on the disbursements for sick pay, they must in future be in the form of additional contributions either *per capita* or as a percentage of the contributions.

Payment of contributions.

The insured person's contribution is paid directly to his friendly society, or to the local council where he resides, and through whom he takes his insurance. The society, or local council, gives him a receipt book with a counterfoil. This counterfoil contains no indication as to the society or local council by whom it is issued. It is given to the employer and remains in the latter's hands so long as the workman is in his employment.

The employers' contributions are paid, in the first case, to the local councils who distribute them in accordance with the membership among the societies and the Pension Fund.

From the fact that the employer pays for all exempted persons, the amount of his contribution works out at more than Frcs. 6.00 per insured person.

Some of the exempted persons receive medical benefit : it is, I believe, the intention to ear-mark the employer's contribution for medical benefit, and thus the surplus arising from uninsured employees would go towards meeting any excessive cost in this benefit. The State subsidies are paid directly to the local councils or to the friendly societies (primary society).

It will be seen that the suggested methods of payment avoid any deduction by the employer from the wages and eliminate the whole system of stamping cards. So far as the employer is concerned he is not directly involved in the payment of contributions, unless the workman falls in arrears in his society. In this case the friendly society notifies the employer, through the local council, to deduct the amount of the arrears from the workman's wages. If the employer does not deduct these arrears he becomes liable for them to the society.

Local Councils.

The provisions made for these bodies are fairly extensive. They are administered by a Committee consisting of a majority of members appointed by the federa-

tions of friendly societies in their district and those who are directly insured through these councils. The other places on the councils are divided between the nominees of the Government and the doctors and chemists, the Government having the majority of these latter seats.

In addition to administering benefits for insured persons outside the friendly societies, the local councils organize the medical benefit, and fix the rates of remuneration for doctors and chemists in their locality. They may administer medical benefits for the members of friendly societies, provided the cost of management for these latter does not fall on the members insured through the local councils. The expenses of the local councils are met half by the State and half by the provinces.

Investment of funds.

Friendly societies, federations and local councils are restricted in the class of investment in which their funds can be placed.

The following classes of investment are authorized:

(1) The shares of workmen's dwellings societies, provided they are without uncalled liability.

(2) Loans to the above societies.

(3) Belgian and Congo securities authorized for the savings bank.

(4) First mortgages on freehold property, provided that the mortgage must not exceed fifteen times the registered annual rental value.

(5) In the provision of sanatoria, hospitals, and similar institutions required for the purpose of National Insurance. Not more than 20 per-cent of the total funds may be invested in this manner.

Criticism of the proposed Act.

Comparison with English rates.

The Bill in its present form bears a very close resemblance to our National Insurance Act, leaving out the provisions of Old Age Pensions. The benefits are similar, although the minimum rates granted are lower than under the English Act. Table H shows the two benefits compared.

Administration through friendly societies.

One of the outstanding features of the Bill in its present form is the prominent position accorded to the friendly societies. Not only are they now placed in a favourable position for the administration of the various benefits, but they have a controlling voice in the local councils.

TABLE H.

Sickness Insurance.

	British			Belgian
			7 9ths Shillings	Minimum Benefits
Benefits	Sick Pay (1st 26 weeks) ...	10/-	7·78	5/- (Frcs. 6.0) [13 weeks]*
	Disablement (after 26 weeks)	5/-	3·89	5/- (Frcs. 6.0) [after 13 weeks]*
	Maternity Benefit (Male and Female)	30/-	23·22	32/-(Frcs. 45.00)[Female members only]
	Medical and Sanatorium Benefits...	Yes		Yes
	Benefits cease	Age 70		Age 65
	Accident Claims (W.C.A.) ...	Not paid		Not paid
Contributions	Insured	16/- per annum†		14/3 (Frcs. 18.00)
	Employer	12/- per annum†		1/8 (Frcs. 2.00) 1/8 (Frcs. 2.00) after 1938
	Government	$\frac{2}{9}$ths cost of all Benefits		5/6 (Frcs. 6.60)
	Local and Municipal Governments	Nil		Varies
	Contributions payable during Sickness	No		Yes (except contribution for pensions)
Initial Reserves	Set up and repaid by means of an annuity-certain			No initial reserves set up Extra subsidies provided for old members

* Does not include the contribution for Pension of 50 Centimes per month.

† These rates of contribution allow four weeks arrears per annum.

a majority of whose members are nominated by the societies. These local councils in turn control the medical benefit, and fix the rates of remuneration for the doctors.

It is difficult to estimate the cost of the medical benefit. An inspection of the Returns for 2,948 societies, out of which 1,749 pay directly for medical benefit, seems to suggest that the present cost is Frcs. 5.35 per member. This includes, however, a provision in some cases for the member's family. Under National Insurance the medical profession will receive better remuneration; in fact, it is the declared intention of the Bill that they should do so.

So far as the present members of friendly societies are concerned their present insurance through their society becomes

their National Insurance. As the rules of Belgian societies frequently restrict membership to a single society or group of societies, duplicate insurance under the Act and through independent societies will not be possible.

The rates of benefits paid by the societies are left to the option of the societies, the only reservation being that the rates must not be less than the minimum benefits granted by the Act. Generally speaking the rate of sickness benefit paid during the first three months is in excess of the minimum rates.

The rates on invalidity pay after the first three months, paid by the federations, do not vary greatly from the minimum rates, the most frequent point of difference being the payment for seven in lieu of six days in each week.

The present scales of invalidity pay per diem, and the question whether they are paid for Sundays, are shown by the following table :

Table J.

Amount of Invalidity Pay.

Year	50-74 Centimes		75-99 Centimes		1 Franc	
	Number of Federations		Number of Federations		Number of Federations	
	Sick Pay Sundays	No Sick Pay Sundays	Sick Pay Sundays	No Sick Pay Sundays	Sick Pay Sundays	No Sick Pay Sundays
1907	5	2	0	1	19	23
1908	6	1	2	0	20	29
1909	7	2	1	0	22	35
1910	9	2	2	0	27	34
1911	7	3	1	0	33	39
1912	6	3	1	0	33	44

Year	Number of Federations		Total number of Societies
	Sick Pay Sundays	No Sick Pay Sundays	
1907	24	20	50
1908	28	30	58
1909	30	37	67
1910	38	36	74
1911	41	42	83
1912	40	47	87

From the above table it would seem that, assuming a uniform membership in all federations, those paying for seven days a

week pay on the average Frcs. 6.58 per week, while those paying for six days pay an average of Frcs. 5.85 per week.

Looked at from another point, however, we see that 33 federations paid seven francs per week, and the balance of 54 federations paid six francs, or less, per week. Under National Insurance these 54 federations will have to bring up their sick pay to the full six francs per week.

Not only does the Act leave the scales of benefits to the choice of the society, but it also gives the society entire liberty in the rates of contribution payable by its members. It is suggested that the principle of subsidy, proportionate to the contribution for both sickness and invalidity, will induce societies to fix adequate contributions for the benefits granted. While the societies may fix their contributions sufficiently high to procure the full subsidy, this is no guarantee that the contributions will be adequate for the benefits granted. The history of the friendly society movement in this country shows that the competition between societies encourages the promise of excessive benefits out of proportion to the contributions charged, nor is there any reason to anticipate that the position will be different in Belgium. The fact that the friendly society movement in Belgium is subordinate to some other movement, political or religious, would seem to make it probable that the competition in granting the greatest benefits for the contribution will be excessive. It seems possible that the antipathies between the principle movements will be reflected in the competition among the friendly societies affiliated with these movements. At present the societies charge uniform contributions to all entrants irrespective of age. Practically no society charges graduated contributions according to the age at entry. Under the National Insurance Act this system can continue and will encourage societies to restrict their membership to young members, notwithstanding the increased subsidy for old members.

Expenses of Management.—These will be met out of the contribution, no specific proportion being set out on one side for expenses. At the present time the expenses of the societies, including the federations, are low. Most of the work is done by voluntary effort and without any expense to the societies. I do not think that such a condition will continue under National Insurance, nor do I think it is desirable that it should continue.

The administration of sickness benefit under a scheme of National Insurance requires skilled workers, who can only

be obtained by the payment of sufficient remuneration to enable a suitable class to devote their whole time to the work. The organization of Belgian societies into federations and primary societies approximates to the organization of our affiliated orders, and while this organization enables a close check to be placed on the claimants to benefit, the expenses of management must be higher than in centralized societies. The contributions will, I think, require a very substantial loading for expenses.

Government subsidies.

The conditions laid down to entitle both a society and a local council to the subsidies are as follows:

Sickness Insurance.—The contributions from the benefit members and the receipts from interest must be at least equal to 85 per-cent of the amount paid for sick pay and medical benefit. Where, however, a society has increased its reserves during the period by a sum equal to 25 per-cent of the expenditure they are exempt from the provision.

Invalidity Insurance.—The contributions from the benefit members and the receipts from interest must at least equal 75 per-cent of the expenditure on benefits.

In both classes of benefit the comparison between the expenditure and income can be made either on the past year's experience, or on the mean of any number of past years up to five.

In the early years of a friendly society there is an accumulation of funds, notwithstanding the fact that the contributions may be quite inadequate. This is particularly the case in a society granting permanent sick pay like the federations. On the other hand, where a society consists of members advanced in age, the claims exceed the contributions, even in a society in a strong and solvent position. It is for this reason that reserves are created.

In the case of the Belgian societies the subsidies are so large that they form a substantial part of the contribution, hence at a period earlier than would normally occur the contributions paid by the members alone and the interest will be unable to fulfil the conditions laid down for the receipt of a subsidy. The subsidies are by these provisions made to depend not on the solvency of the societies, but on the power of the societies to attract young members. They will, I think, encourage competition among societies and introduce a principle similar to the dangerous working of divisional societies and slate clubs, in which the existence of the society and the payment of benefits to old members depends on the introduction of young lives.

Actuarial Basis. The scheme of the proposed Act does not proceed on actuarial lines, the reasons given being as follows :

(1) Absence of any Belgian data and the inapplicability of the tables deduced from foreign data. From a consideration of the rough data in Table F I think that it will be seen that the present experience of Belgian societies is not so divergent from that shown by the Manchester Unity Whole Society Experience 1893–97, as to make the use of such a standard inapplicable when account is taken of the varying conditions of mortality and the tables are loaded to allow for the proportion of the population engaged in the various occupations. As conditions of National Insurance vary from the conditions under voluntary insurance through friendly societies a substantial loading would be required under any circumstances to allow for fluctuations. The Belgian system of applying National Insurance through friendly societies, and on individual lines, makes the employment of Standard British Tables deduced from friendly societies Experience in this country more suitable than were the whole insured population to be dealt with *en masse* as under our own Act.

(2) The second objection raised to the foundation of the scheme on actuarial lines is that the contributions might be "excessive" and beyond the capacity of members of friendly societies. The idea is to gradually lead the members by means of the subsidies towards an actuarial basis. In answer to this it might be pointed out that while the members are being "led" towards actuarial solvency deficiencies are increasing, and what would be an adequate contribution to-day will be inadequate in the future, when large deficiencies have accrued. Many societies in this country have been founded on insolvent lines with the idea of gradually teaching the members the lesson of solvency ; by the time the lesson has been learnt the deficiencies have grown so large that the knowledge has come too late to be of use in preserving to old members the insurance for which they have paid for many years.

By founding the scheme without an actuarial basis inequities will arise between the various classes of members, and as a uniform contribution is charged by all societies, either the old members will pay inadequate contributions, or the young members pay excessive contributions, and in either case there is no security for the benefits for which the members contribute.

Conclusion. A few features of the Bill are, I think, an advance on our own Act :

(*a*) The method by which a member of a friendly society uses his present insurance as his State Insurance, avoids double insurance, and eliminates all need for a section such as Section 72 of the National Insurance Act. It reduces the "profit" from malingering to a minimum.

(*b*) The liberty granted to friendly societies and federations in fixing their contributions and benefits might prove a feature of great value. Better effect can be given to local needs and conditions of employment than under a uniform system applicable to all insured persons.

(*c*) The system of local councils is an improvement on our Deposit Insurance. Even if the benefits are small, compared with the contributions, some amount of insurance can be granted, whereas so far as deposit contributors in this country are concerned, there is no insurance at all.

Defects.—The chief defects of the Bill seem to me to be actuarial, and the following points are suggested :

(1) The creation of actuarial reserves for members entering into insurance at an advanced age. These reserves should be based on the minimum benefits granted at the youngest age of entry. They should, I think, take into account the occupation of the members. So that while the total reserves for all societies might follow a uniform basis, the reserves for heavy occupations, such as miners, would be in excess of the reserves for light occupations, such as clerks. The reserves credited to the societies would be repaid by means of an annuity-certain as in the British method. The creation of these reserves would do away with the need of additional subsidies in the case of old members. If the State does not set

up these reserves, the unions of federations might undertake this work for the societies in their unions.

(2) Societies and federations while being permitted to grant their members the rates of benefit they desire. should be required to show, by means of actuarial certificate, that these contributions are adequate for the benefits granted.
These tables would take into account the suggested standard reserves so far as the minimum benefits are concerned. They would also take account of the State and provincial subsidies.

(3) A periodical valuation of the liabilities and assets for societies and federations. Care would have to be taken that any surplus brought out was due to the transactions of the society in the period, and was not due to the creation of initial reserves in excess of the requirements for that society.

(4) A revision of the principle of subsidy provided under the Act 5 May 1912, and by the present Act. The method by which a relation to contributions and the claims is set up as a standard for the receipt of a subsidy is quite unsound and should be abandoned. Subsidies should be payable to those societies charging adequate contributions for the risks undertaken. The subsidies should be based on the minimum benefits, and additional subsidies might be granted for hazardous occupations.

In conclusion, I desire to thank Monsieur Ver Hees, of the Ministry of Industry, for all the information and data placed at my disposal with reference to the Belgian Insurance Act.

Appendix.

National Health Insurance Act.

Section 55 of Bill.	Section 72 of Act.
55.—(1) Any registered friendly society or branch which provides benefits similar to those conferred by this Act shall submit to the Registrar of Friendly Societies a scheme for abolishing, reducing, or altering such benefits as respects members who become insured persons under this Part of this Act, and for the application of the	72.—(1) Every registered friendly society which provides benefits similar to any of those conferred by this Part of this Act shall submit to the Registrar of Friendly Societies a scheme for continuing, abolishing, reducing, or altering such benefits as respects members who become insured persons and for con-

funds accumulated in respect of such members for the purposes of such benefits in any one or more of the following ways :

(*a*) in the payment of other benefits ;

(*b*) in the reduction of the contributions of members in respect of benefits payable by the society independently of this Part of this Act ;

(*c*) in the payment of contributions under this Part of this Act payable by its members ;

and any such scheme when confirmed by the registrar shall have effect as if enacted in this Act.

(2) This section shall come into operation on the passing of this Act.

tinuing, abolishing, or reducing the contributions of such members, so, however, that the combined effect of the alteration of the benefits and contributions shall not prejudicially affect the solvency of the society, and, if the scheme or a supplementary scheme shows on an actuarial valuation that, owing to the alterations in the benefits and contributions effected by the scheme any part of the existing funds of the society is set free as not being required to meet the liabilities of the society, the scheme or the supplementary scheme shall provide for the application of the part of the funds so set free in any or more of the following ways :

(*a*) towards the cost of the provision of other or increased benefits payable by the society independently of this Part of this Act to existing members whether insured persons or not ;

(*b*) in reduction of the contributions payable by such members in respect of the benefits payable by the society independently of this Part of this Act ;

(*c*) towards the payment or repayment of contributions payable under this Part of this Act by such of its existing members as are entitled and elect to receive benefits under this Part of this Act through the society.

(2) This section shall apply to branches of registered societies in like manner as to societies : Provided that a society with branches may, if it so desires (subject always to the exercise of any right of a branch, expressly conferred by the rules of the society, to dispose of any of its funds for the benefit solely of the members of the branch), submit a scheme applicable to all its branches, and it shall be competent for the society to provide by its scheme or supplementary scheme for the application of the whole or any part of any sums so set free towards the discharge of any deficiencies in any of its branches which may be found to exist on such actuarial valuation as aforesaid.

(3) Any scheme adopted by a society or branch of a society in accordance with its rules when confirmed by the Registrar of Friendly Societies shall be deemed to be incorporated in the registered rules of the society or branch and may be amended accordingly, so, however, that no amendment shall be inconsistent with the provisions of this section.

(4) This section shall apply to seamen,

marines, and soldiers, from whose pay deductions are made under this Part of this Act as if they were insured persons, and for the purposes of this section "existing" means existing at the passing of this Act.

(5) This section shall come into operation on the passing of this Act.

Abstract of the Discussion.

Mr. R. G. MAUDLING, after referring to the differences between Section 55 of the original Bill and Section 72 of the Act, said that competent authorities had expressed their inability to give any meaning whatever to certain portions of Section 72, and no doubt the proper interpretation could only be arrived at in the Courts. The Section gave the option of continuing, abolishing, reducing, or altering the benefits, and altering or reducing the contributions, subject to the combined effect of the alterations leaving the solvency of the society unimpaired; and then it went on to deal with the moneys which might be set free as a result of that operation. Unless the Section were read as a whole it seemed to him it would be open to any society to submit a scheme which would defeat the latter portion; it could give deductions dependent on the age to such an extent that no sum would be set free. It had been suggested that the object of the Section was to enable provision to be made for persons who were already members of friendly societies, and who therefore might be assumed not to be able to pay the additional contribution without hardship. Another suggestion was that it was the intention that the Section should form a means of creating "released reserves." Taking those two views into account some idea would be obtained as to how the present form of scheme had been evolved. A rather peculiar point with regard to the Section was that if the scheme provided for "continuing" the benefits the latter portion of the Section became entirely meaningless. But perhaps the most awkward word in the whole Section was the word "altering." It was possible to reduce a benefit, or abolish a benefit, but how a benefit could be altered in any way without being either reduced or abolished he could not conceive. He believed that with regard to this word the view was held that, as it was impossible to assign any meaning to it, it should be disregarded.

Section 72 in its present form was an improvement upon the original clause in the Bill, because it allowed rather more equitable treatment of the different classes of members. It had, however, now been decided that it was possible to have a scheme allowing individual members the option of reducing or continuing as they might think fit. A scheme, either in the "continuing" or "reducing" form, provided it applied to all insured members, would not raise any serious question of inequity, but in an optional reducing scheme, if any money were set free, grave injustice might be done, as amongst insured members themselves. He thought it was in some

degree due to the recognition of this fact that members had decided as far as they were able to obtain the advantages of the released reserves for themselves individually and had made sure of getting those advantages by taking "double" benefits. This had no doubt led to a certain amount of excessive insurance, but he was not inclined to attribute too much weight to the apparently excessive sickness shown by comparisons of present and past experience. Allowance had to be made for the fact that no matter with what period of sickness the illness of a member happened to coincide he was entitled to benefits under the Act for the first six months of sickness, *i.e.*, at full pay, so that by adding the payments on the State side and the voluntary side together, as he believed had been done in many cases, an error was introduced which would entirely vitiate the comparison. The preparation of a scheme involved an estimate of the value of the benefits of which the society was to be relieved, and he was in general agreement with the methods of valuation adopted. These methods, however, were distinctly conservative, and he should be inclined in some instances to say that the relief might be increased with safety. There were one or two points in the suggestions in regard to special classes to which he wished to refer. As regards soldiers, for instance, the author seemed to adopt a high retiring age, and against that took no account of the fact that the soldier, after a comparatively short period in hospital, was discharged, and therefore the State had to assume liability for a portion of the benefits during the first six months of sickness. Similar remarks applied more or less to naval seamen, and *mutatis mutandis* to the mercantile marine.

He was somewhat surprised to find that no reference had been made to the subject of the reductions in respect of female members—a very difficult question, and one, too, which was really a more difficult problem than any which had been referred to in the special classes. As regards the reserves for compensated accidents, he did not consider the method of valuation adopted by the author was so good as the one referred to in the Paper, namely, the utilization of tables based upon varying occupational risk. He was inclined to think that as the Manchester Unity Tables could be taken as showing approximately the effect of accident risk upon sickness experience, they provided a better method of measuring the accident liability than the adoption of what could but be called empirical "costs" varying with the age. He quite agreed that a valuation was absolutely necessary to start with, and that upon that valuation there should be imposed the effect of the scheme. His own general practice was to use the non-hazardous experience (which admittedly included a small amount of accident claims), and then to load that experience in accordance with circumstances—keeping in view the members' occupations, the Home Office Returns, and the tariff of the insurance companies transacting Workmen's Compensation business. The author had given the accident experience of a trades union for one year. He did not think the author intended much stress to be laid upon the comparison, but he should like to ask him whether a

member was entitle to claim without reference to previous claims in respect of each separate accident.

Mr. NATHAN—No.

Mr. MAUDLING said it would of course make a very substantial difference in the cost if he came back to full pay for every fresh accident; it would vitiate any value the comparison might have.

Another point in connection with this scheme is the necessity of making reserves in cases of chronic sickness. The problem had been dealt with very fully by the Actuarial Advisory Committee in connection with the computation of reserve values under the National Insurance Act.

If the valuation preparatory to the formulation of a scheme showed a surplus, he thought that that surplus should remain intact. The author apparently agreed in principle, but he stated that the surplus should be maintained at the same amount per pound of liabilities. The effect of this would be to reduce the surplus, since the liabilities would be reduced by the scheme. He considered, however, that a society which had a surplus before the Act came into operation should not have that surplus reduced by the Act. It was earned surplus and members should be entitled to their shares; therefore he did not think any of it should be distributed preferentially in order to grant a larger reduction of contribution to insured members.

The author had suggested certain ways in which Section 72 might be amended, and the first one was that all schemes should be compulsory. It was all very well within the precincts of Staple Inn to talk about making schemes compulsory, but any attempt to convert the various members of friendly societies would, he was afraid, prove wasted energy. Members had got the idea into their heads that by coming under the scheme they were giving other members something and they were not going to avail themselves of it if they could help it. The second suggestion of the author was that if the scheme were made compulsory it should be made as far as possible equitable as between different members. If that meant that the deduction was to be varied with the age of the member there would be no reduction of deficiency, because there would be a demand for the largest deduction it was possible to allow without actually affecting the rate of solvency adversely, and thus one of the principal objects which it was hoped would be attained would be defeated.

His own impression was that it would have been better if, instead of providing for so complicated a procedure, which had proved very confusing and was very imperfectly understood by the persons concerned, power had been given to members of friendly societies to reduce their existing benefits and receive in return an actuarially-certified reduction of contribution. Under that method more improvement would probably have been effected in the finances of friendly societies, and there would not have been so much excessive insurance. A large number of members would have been inclined to agree to retain, say, one-half of their existing voluntary benefits, whereas as it was they had either to retain the whole or none at all.

Another consideration which had a bearing upon double benefits was the fact that trade had been very good during the past few years, and consequently the extra contribution required under the Act did not form so serious a burden as was anticipated.

Though it was impossible to abolish benefits in compensated accident cases under the scheme, the desirability of that course would doubtless be recognized by members of the Institute. It was a reform that could not perhaps be effected at once, but societies might by degrees be convinced of its necessity. It was not so important in connection with the affiliated orders as with centralized societies. A lodge was generally composed of men subject to similar occupation-risks, consequently reduction of benefit owing to accident claims did not involve unfairness, but with a centralized society, admitting members in hazardous and non-hazardous occupations on the same conditions, there was an obvious injustice. The strange thing about it was that the non-hazardous risks did not appreciate the fact. As far as possible, in preparing new tables, he definitely excluded the accident liability, and advised that if it was desired to cover that liability it should be dealt with on a levy system, with a broad classification according to occupations.

With regard to Part II of the Paper, dealing with the Belgian Insurance Act, the friendly society movement in Belgium was of comparatively recent growth, though it was showing signs of expansion. The existing system was, no doubt, entirely unsound, and it might be due to that fact that the legislature preferred not to base the Act on the Manchester Unity Tables. A fair approximation could have been made by use of the Manchester Unity Tables, adjusted so as to allow for the different occupational distribution.

In the experience of the Federation of Friendly Societies during the year 1910-11 there were obviously very peculiar circumstances with regard to the rate of sickness experienced from ages 16 to 20, and one would be inclined to think there was some fallacy in the determination of the exposed to risk, though that would affect other ages. The fact that the percentage of actual to expected claims was no less than 220 might be explained to a large extent by the expected claims after six months having been taken, whereas he understood that a large number of societies paid benefits after three months. At ages of 16 to 20, there was practically no expected sickness after 12 months, and the second three months' sickness represented about double the expected sickness in the second six months. That might explain much of the peculiarity, otherwise he thought it was more or less impossible to account for it, but in any event no reliance could be placed upon the figures. The Belgian Insurance Act seemed in important features to resemble the German rather than the English scheme, in that there was the separate treatment of sickness, invalidity and pension. The principle of subsidy was absolutely empirical and was based upon erroneous principles. Apparently, the subsidy might be granted when contributions were wholly inadequate and might be refused where adequate provision was being made for the risk. However, actuarial criticism of the Act

was hardly possible, because it was not upon an actuarial basis, though it could hardly be urged that an "accumulation" basis was indispensable in a National Insurance Scheme.

Sir GERALD RYAN congratulated the author on having prepared an essay of a very able and useful character. It was not his intention to refer to the details of the Paper, but he desired to call attention to the general effect of Section 72 of the Insurance Act. In November last he was led to look into the subject somewhat closely owing to the fact that the sickness claims of a County Friendly Society, of which he was a President, had increased considerably, and further investigation seemed to show that that was a result of the new conditions under the National Insurance Act. On considering the matter it appeared to him that an increase in the rate of sickness was to be expected for three reasons. In the first place, there was the fact that under the National Insurance Act as it eventually passed into law duplicated sickness benefits were allowed (*i.e.*, members were allowed to continue their old friendly society sickness benefit, side by side with the new State benefits); in the second place, there was allowed a free choice of doctor instead of the compulsory consultation of the old club doctor; and in the third place there was the substitution of a State obligation with regard to sickness benefits for that mutual and fraternal obligation which had been the great feature of the friendly society system. For these three reasons it seemed to him only natural to expect an increase in the rate of sickness in future. When, however, he expressed an opinion to that effect at the annual meeting of the society in question he was met with the criticisms that any opinion of that kind was premature and not justified, and that it was a calumny upon the working classes of the country. It seemed hardly necessary to discuss the second criticism. Public opinion rightly held that it was not in the interest either of the individual or of the community that over-insurance in any form or shape—against sickness equally with other contingencies—should be permitted, and it was in no way a calumny upon the working classes to say that precautions should be taken.

It was interesting to consider for a moment how it was that the National Insurance scheme took the form in which it became law. There was not the slightest doubt that, as introduced in the first instance, it had other motives and followed very different lines. One of the objects of the Insurance Bill was avowedly to strengthen the position of friendly societies in the country. It was pointed out that perhaps 80 per-cent of the friendly societies were in a state of insolvency, and that by the operation of the Bill a large amount of sickness liability would be released, the effect of which, roughly speaking, would be to produce a state of solvency in the friendly society system. That was confessedly one of the underlying motives of the Bill, but during the discussion in Parliament, and while the Bill was passing through the Committee stage, changes were introduced, the effect of which was to enable the old benefits of the friendly societies to be continued side by side with the State benefit; an

as a matter of fact, in the society he had mentioned he found that about two-thirds of the members—and he afterwards heard that about the same proportion would hold good in the case of other county societies—continued their old sickness benefits in addition to the State benefits. That meant that an agricultural labourer in receipt of, say, 15s. a week wages, might be insured for 12s. a week in his old friendly society and 10s. a week under the State section, and so draw in case of full-pay sickness 22s. a week. It appeared to him that that was a dangerous thing. Why such a vital change was made in the Bill during its passage through Committee he had never been able to understand, and it was very curious that there did not appear to have been any discussion in Parliament on the subject. He could not say that none of the clauses dealing with continuing benefits came under discussion, because he thought some of them did, but certainly the effect of so altering the Bill as to substitute for a limit of 80 per-cent of a man's wages the unlimited powers of insurance which members now possessed was not discussed. It was suggested by the author that probably the friendly societies brought pressure to bear upon the Chancellor of the Exchequer to induce him to alter the Bill and permit the continuation of the old benefit. and therefore legalize the double insurance ; but if so the effect of the change could hardly have been understood at the time, nor was it clear that the friendly societies had any right to exclusive consideration in the matter, because the effect of duplicated sickness must be felt just as much on the State side as on the friendly society side, and therefore if it weakened the position of the voluntary section of friendly societies it would also weaken the position of the State section. It seemed very desirable to try and find out why the change took place, as it would then be easier to see how to remedy it. if a remedy proved to be necessary.

Bearing in mind how the cost of claims under the Workmen's Compensation Act had largely increased as soon as the obligation of the State was substituted for the obligation of the employer, they would expect an increase of sickness claims under the National Insurance Act. And when, in addition, they took into account the extraordinary temptation to sick people to remain on the claim list so long as they drew when sick considerably more than they did when performing a full day's work, they would, he thought. be led to the conclusion that there must be a marked increase in the sickness rate certainly in the case of agricultural societies. If that were so, the position of friendly societies would be weakened, and they would be weakened not as compared with the position which it was assumed they would be put in under the operation of the Bill as introduced into Parliament, but weakened in comparison with the position which they occupied before the introduction of the Bill. And it was by no means unlikely that the State sections of the societies would be adversely affected by the same causes ; that was to say, that the effect of the duplication of sickness insurance might be largely to increase the rate of sickness which had been assumed to occur in future by those distinguished

members of the Institute, Sir George Hardy and Mr. Wyatt, in framing the basis upon which the contributions were calculated. In that way both the Government authorities and the friendly societies might come to the conclusion that some change should be brought about. But what should that change be, and how could it be effected? He suggested that efforts should be made to get back as closely as possible to the scheme of the Bill as originally introduced. The original Bill was a sounder piece of work and better calculated to carry out the Government's object than the Act as it was eventually placed on the Statute Book. In the first place, the Government should hold an enquiry as to the effect of the changes in the fabric of the Bill, and a conference should be arranged with representatives of the friendly societies in order to see whether the joint interests of friendly societies and the State sections under the Insurance Act could not be promoted by both sides agreeing upon some means by which the Act could be restored to its original form, and the danger of extra or duplicated insurance avoided. It would be quite impossible for the Government to deal with that question unless they had the assent of a large number of the friendly societies, and an enquiry coupled with a conference seemed the most promising course.

With regard to the actuarial aspects of the Paper, there were two points that actuaries who were called upon to undertake the valuation and investigation of friendly societies in future would be interested in. First, the old division of sickness into city and rural, *i.e.*, into districts where population was dense and districts where population was sparse, would have to be extended, and it would be desirable to have an investigation into the sickness rates where duplicate benefits existed and where they did not. In that way it would be discovered whether there was any justification for the assumption which he, amongst others, had made that the various causes at work would produce an increased rate of sickness. The other point had reference to the basis of valuation. He could not but think that actuaries who had to report upon the position of friendly societies might find themselves in considerable difficulty. In many instances the sickness claims would no doubt be unduly high, but he did not think it very much mattered at the first valuation, occurring, say, three years after the Act had come into force, whether the rate of sickness had been excessive or not, because the valuation did not deal with the rate of sickness in the past three years but with the sickness that was likely to prevail during the continuance of the benefits which came under review. It would be a matter of considerable importance and perhaps some anxiety to members, when reporting on the position of some of the friendly societies, to consider what would be the future rates of sickness. In the old days it was a very fair thing to take the last five or ten years and assume continuing conditions, and then to vary the standard tables so as to give effect to any feature in the particular sickness of the society. But he was not at all sure that that would be the case in future. Certainly, in dealing with agricultural

societies (or light-occupation societies as the author called them) the actuary might have to assume, even where no very tangible evidence at present existed of increased sickness, that the important conditions at work would tend to send up the rate of sickness.

Mr. J. BACON said that over 30,000 annual returns were received from friendly societies during the period covered by the Registrar's last report, and yet only about 9,000 provisional schemes were submitted and only 1,050 final schemes were approved. Although the time fixed had long expired, the Registrar had apparently no power to enforce the submission of a satisfactory final scheme and Section 72 therefore appeared to have become practically a dead letter, and the centre of interest had shifted to the financial results likely to accrue owing to the practical adoption of "continuing" schemes by the great majority of friendly society members and the higher scale of insurance thus introduced. So far as his experience went it confirmed what had been already said as to the increased cost of sickness, but he thought it could not be definitely stated that that was due entirely or to any very great extent to double insurance. It was known that there were about 85 or 90 per-cent of the old friendly society members who had elected to take the State benefit as well as their own friendly society benefit, and undoubtedly there had been a higher sickness rate; in fact, he was told by the secretary of one of the affiliated orders that on the voluntary side the sickness insurance had gone up about 75 per-cent since the Act came into force, and he had heard that the experience of members who had taken both State and voluntary insurance, compared with the experience of members taking only the State insurance, had been considerably in excess. But there were the other causes mentioned by Sir Gerald Ryan, and in addition it had to be remembered that people coming under the State scheme were very different in character from those who came under the old friendly society schemes. There had been probably brought in a more thriftless class, a class who, either from personal character or other circumstances, were not insured previously. It would be somewhat dangerous to single out any one cause, such as double insurance, and say that that was the cause of the excess. The question of an increase in the rate of sickness paid for could be looked at from two points of view—from the point of view of the friendly society official, who was interested in anything that would bring down the rate of sickness, and from that of the politician who was engaged in launching a scheme of national insurance or a national health scheme, and it was quite possible that from the latter point of view the higher rate of sickness which was paid for, even if due to the higher level of sick pay now prevailing, might be a matter of congratulation rather than a thing to be deprecated, if it represented better care and more adequate treatment of the insured and resulted in the future in a more healthy nation and possibly in an actual reduction of the future rate of sickness.

With reference to points of detail, the speaker said that it was stated early in the Paper that the scheme under the original Section 55

would have applied to all members of a society, but he himself thought it was obvious that it would have applied to insured members only. Again, throughout the Paper the author appeared to speak of the maximum abatement of contributions as 3*d*. per week, and the 3*d*. was assumed to come from the benefit funds. The abatement might be as much as 3*d*. plus the amount paid to the doctor, and it was quite usual for some part of the 3*d*. to come out of the management funds. The author also stated that in addition to the ordinary data for a friendly society valuation there was needed, amongst other things, some separate particulars of insured members and their ages. He had not been able to find out why the author required this information in addition to that relating to members who were reducing their benefits and contributions, and he should be glad if he would enlighten them in his reply. A little later on the author suggested the adoption of a fixed rate of 1*s*. 3*d*. as the liability of the society in respect of the first three days' illness. The liability would obviously vary with the amount of weekly full sick pay for which the member was insured, and he supposed that the author intended the 1*s*. 3*d*. to apply only to the case of the statutory 10*s*.

With regard to the waiting periods, the author's method of obtaining the reserve, as stated in the Paper, appeared to be incorrect. The reserve should be for sick pay for the first six months of illness in respect of half-a-year's exposure only at the valuation age, and for disablement pay after the first six months of illness for a period of two years. Then, again, it was stated in the Paper that in many approved societies the allowance for administration was insufficient and that therefore the scheme could not rely on an abatement of contributions from the management fund. On the other hand, it had to be remembered that a large number of societies, especially new societies in connection with the great industrial insurance institutions, found the allowance more than sufficient. In any case, he did not think the contributions to the management fund of the voluntary side should be kept up because the expenses of the State section were excessive. The author also stated that, except in the case of societies with a surplus, the value of the released contribution must be such that the value of the release of contributions did not exceed the total value of the release of liabilities. Unless a society was fully solvent the release of contributions must always be less than the release of liability, the ratio of the two sums released depending on and always being less than the ratio of solvency of the society. Otherwise, the solvency of the society would be prejudicially affected.

It might be interesting to note, in connection with Table E, that the author had used the mortality of the Manchester Unity experience 1893–97 in Area 2. Area 2, he understood, had now practically ceased to apply, the rates in that area now approximating to rates in Area 3 of the Manchester Unity experience. It was stated by the author that when the mortality was heavy and was given effect to in the valuation, the abatement in contributions could be greater than in an exactly similar society valued on a table giving

a light rate of mortality. That was a perfectly general statement, but it surely depended on the age distribution of the excess mortality of the members reducing, and also on the progression of the sickness rates. With regard to the statement in the Paper that a society holding assets to the extent of 85 per-cent of its liabilities can only relieve its members of a percentage of the full amount of abatement, but that as a result of sickness rates, &c., this proportionate abatement may amount to the full 3*d.* per week, the fact really was that the sum of 3*d.* was not a full equivalent in all cases of the liabilities under the Act, and it was for that reason that a society which was not fully solvent might be able to allow that amount to reducing members, especially if part was taken from the management contribution. A little later on the author referred to the reduced provision for aliens and suggested that the full release should be made, less an extra reserve in respect of the reduction in benefits, and that the reduction in contributions should be proportionate to the value of the reduced benefits. Strictly speaking, of course, those reductions should apply throughout to the first three days, waiting periods, chronic sickness, &c. He noticed generally in giving his formulas the author had always assumed the society to be solvent. He agreed with Mr. Maudling that a compulsory scheme for reducing benefits and contributions would be most unpopular. Many people resented being compelled to insure and he was inclined to think that a very much larger number of people would resent being compelled to discontinue an existing insurance.

Turning to Part II, and the proposed Belgian National Insurance Act, he felt that since that Act avowedly had no actuarial basis, it was somewhat difficult to discuss it, but he would have been glad had the author adopted the Manchester Unity 1893-97 experience as a standard of comparison instead of the earlier 1866-70 experience.* He should be also glad if he would give the rationale of his exposed-to-risk formula, and state whether the age he had taken was exact, or the age last birthday or next birthday. Personally, he did not think any importance could be attached to the figures in Table F. The results for the two years 1910 and 1911 were very different, and the method of taking the exposed-to-risk as the mean of the numbers existing at two specific points was likely in any case to give incorrect results where the membership was rapidly increasing, as was evidently the case in the present instance, and that was accentuated when those two points were both taken prior to the year of exposure and no allowance was made for the net movement during that year. That seemed to be a likely explanation of the extraordinary figures for ages 16–20. With reference to the author's comparisons of the Belgian Act with the British Act, if there was no maximum assurance fixed by the Belgian Act, over insurance was not avoided. As he had already said, such insurance might not be an unmixed evil from the national standpoint and the present system did give in effect the liberty in fixing contributions and benefits to which the author

* In the Paper as finally printed, effect has been given to this suggestion.—Eds.

attached great value, except that it provided a minimum which he did not think anyone would contend was adequate to meet the full sickness insurance needs of the workers. With regard to the suggestion that subsidies should be given to those societies charging adequate contributions for the risks undertaken, in his opinion that would inevitably mean adopting a standard table of benefits and contributions, and when that standard table came to be applied to individual societies, for various reasons some societies might be found to be insolvent, and therefore in their case the contribution would be inadequate, whereas if the contribution was in practice adequate he failed to see the need of subsidies.

In conclusion, he might say that the system of dealing with a national problem such as sickness insurance through a large number of financially independent local societies seemed to him necessarily to create difficulties. Men and women in different occupations and localities were experiencing widely different rates of sickness and should be presumably dealt with quite separately, both with regard to initial reserves and transfer values as well as in the valuation—unless of course the Commissioners exercized their option to do without one—whilst the contribution had been determined for the nation as a whole. That appeared to him to be the great weakness which had arisen out of the Act, that contributions so determined had been applied to independent societies having widely different conditions. His personal feeling was therefore very strongly against the differentiation advocated by the author. In his opinion, if public funds were to be utilized to finance a scheme, the simplest, cleanest, and most politic way was for the nation insured to be insured as a nation.

Mr. W. O. NASH called attention to the fact that Section 72 applied to the membership of societies in December 1911, more than two years ago. "Double insurance" must be regarded as a *fait accompli*, and it was hardly possible, after two years, to say that it could no longer be allowed. Besides, he thought that double insurance—the old voluntary benefit and the State benefit taken in conjunction—did not form an evil provided that over-insurance was not involved. A man earning from 20*s*. to 30*s*. a week insured for 5*s*. voluntary benefit and 10*s*. under the State was not over-insured, and there was no reason why he should not take the double benefit. The legislation of the future should endeavour to restrict not double insurance, but over-insurance, and the percentage mentioned by Sir Gerald Ryan, namely, 80 per-cent of the wages, should be a statutory limit for full-pay sickness: there should be some sort of restriction debarring any insured person, whether voluntarily insured or State insured, from drawing, while sick, more than 80 per-cent. of his average earnings. That would be similar to the restrictions now imposed in the case of insurance of infants. In considering the equities of schemes under Section 72, a point should be borne in mind which had not been mentioned in the discussion, namely, that the basis of the National Insurance Act was what had been called a flat rate of contribution: insured persons paid the

same rate independently of their age. It would be seen at once that a large present had been made to the older members of the community. Men of 30, 40, and 50 paid the same for the same benefit, and it was on that ground that the Commissioners had quite rightly refused to accept schemes under Section 72 which would give a varying reduction of contribution according to the attained age of the member. A start having been made with a flat rate, the reduction also should be flat. In spite of all that had been said with reference to Section 55 of the Bill and Section 72 of the Act, he thought the latter was the better. The wording of Section 72 was no doubt absolutely unintelligible, but words that were not there were being read into it, and it was being ably administered, and the department which was administering the Act—which seemed to him to be working well—was given much greater scope under the Section as it was finally passed than would have been given under the Section as originally drafted.

With regard to the second factor mentioned by Sir Gerald Ryan, as affecting the future sickness rate, namely, the alteration in the procedure in regard to medical supervision, he thought that eventually a satisfactory medical service would be arrived at, and a satisfactory supervision of sickness claims. As far as he could see, the tendency would be to have State-paid medical officers to supervise sickness claims and that would be getting back more nearly to the old friendly society system. The third point mentioned by Sir Gerald Ryan was the removal of the old fraternal feeling that debarred a man from making unfair claims on his fellow-members. It might be hoped, however, that the fraternal feeling that existed in friendly societies would become a potent influence in the State generally. Moreover, the members would in the course of a few years, probably after the first valuation, see that their pockets were directly touched by excessive claims, just as they were under the old friendly society system. It was no doubt an unfortunate fact that many approved societies, especially those having a large proportion of female members, would show at the first valuation that their experience had been unfavourable, and that the funds would not provide the full benefits. Other societies, like agricultural societies, he believed, would work well within the provision allowed for sickness, and in those cases there might be an addition to the 10*s.* a week and to the other benefits, while the unfortunate societies would have to reduce benefits. That surely would bring home to the members the desirability of seeing that their friends and neighbours did not make unfair claims on their societies on either the statutory or the voluntary side.

Mr. V. A. BURROWS thought the author of the Paper was under the impression that Section 72 was much more effective as a means of reducing deficiencies than Section 55 of the original Bill, but that was not necessarily true in all cases. For example, in the case of a scheme under Section 55 which reduced the contributions of insured members there was a reduction of assets and liabilities depending upon a degree of solvency based on the ratio of the

accumulated funds to the reserve and not on the ratio of the total assets to the total liabilities. As the former method of arriving at the degree of solvency necessarily gave a lower figure than the latter, it followed that the scheme would be more effective in reducing deficiency. However, on the whole, he thought there could be no doubt that Section 72 was much to be preferred.

With regard to the option of "continuing", he could not see how any Government could possibly have refused to give this option. The Bill offered to the man who had not insured—presumably the less thrifty worker—a very good contract. For fourpence a week a man of, say, 45, previously uninsured, was able to obtain a benefit worth very considerably more. That being so, it would certainly have been most unjust if a man of 45 who had been thrifty enough to join a friendly society had not been offered the same terms without having to give up a reserve accumulated on his account to make up a deficiency existing in his society. The omission, therefore, of the word "continuing", however desirable from the actuarial point of view, would have penalized the thrifty. But whatever view might be taken on this point, the option to continue had been allowed and it was now too late to deplore the fact. At a meeting of a large friendly society a resolution was proposed making it compulsory on all members to reduce their contributions under a scheme, and the feeling of the meeting might be gauged from the fact that when the vote was taken the gentleman who seconded the motion voted against it, leaving the proposer to record the only vote in its favour. Whilst such was the attitude of the friendly society members to the question it was not likely any material alteration of the schemes would be brought about, and a discussion in the Institute proposing that all members should reduce reminded one of the old story of settling who should bell the cat.

With regard to the method advocated by the author for obtaining the degree of solvency, it was stated in the Paper that the assets consisted of "accumulated funds, present value of contributions, value of re-assured benefits, and the amount of arrears, accrued interest, &c.", and the author further said that the solvency was measured by the ratio of assets to liabilities. He could not agree with the author in including the values of re-assurances. It might seem a small point, but he thought it was desirable that Public Valuers should be agreed as to the right method of obtaining the degree of solvency. The author's method led to the absurd conclusion that by merely setting up an internal re-assurance arrangement a district could improve the degree of solvency of its lodges. Thus, taking a hypothetical case of a district consisting of two lodges; suppose that the district funeral fund valuation showed the value of the benefits to be £1,000, and that of the contributions to be £400, then with funds of £200 there would remain a deficiency of £400, which would be divided between the two lodges; suppose, further, the two lodges to be precisely similar, the sickness benefits in each case to be worth £1,000, the death benefits £500, and the contributions to the funeral funds £200, and the proportion of deficiency to be £200;

the contributions on the other side of the account being worth £800, funds £300, and re-assurances £500; then the deficiency would be £300. If the degree of solvency were obtained after eliminating the re-assurance items from both sides of the account, they would get what he regarded as the true ratio of solvency, namely, 80 per-cent. According to the author's method, they would get 84 per-cent. It could, however, be easily shown that 80 per-cent was correct, because if the two lodges were taken together and treated as a district of one lodge, the total liabilities would be £3,000, and the total assets £2,400, thus reproducing the rate of solvency of 80 per cent. The author would find himself in the awkward predicament of confessing that, as a result of merely combining the two lodges, the proportion of assets in both had declined from 16*s.* 10*d.* to 16*s.* per £1 of liabilities. Apparently, it was a case of "divided we stand, united we fall."

The author's statement that the scheme "must not be used as a method of distributing surplus, and as a result after the scheme the surplus measured per £1 of liability must be intact" seemed to be somewhat inconsistent. For by merely retaining the surplus per £1 of liability, part of the original surplus would be actually distributed. Again, the author said, "If the tables in the valuation have been adjusted to meet light or heavy sickness rates experienced by the society, then, provided these adjustments are required for ages below age seventy, a similar adjustment should be used for the scheme." It seemed undesirable to load the scheme values and then make a reserve for what was included in the loading. Surely the excess found in sickness was largely due to the additional accident liability, and it seemed a curious way of valuing to load the sickness values to meet that liability and then to make a separate reserve so as to eliminate it. It would be much simpler to reduce the loading on the scheme values, and thus obviate the necessity for the author's empirical method of dealing with compensated accidents. It was a pity no examples were given in the paper of reserves based upon Area 3 of the Manchester Unity Experience. He was surprised that the author should use Area 2, as it was now regarded as out of date. Certainly the continued use of Area 2 was to be deprecated.

Later in the Paper the author said, "I have reason to believe that the Committees of Management of many approved societies find the sum allowed for administration for State Insurance inadequate. This is, I understand, especially the case with the approved sides of many affiliated orders where branches contain from 100—200 insured members." That seemed to be no argument for not allowing a deduction under the head of management expenses under the scheme. Whether the cost of management entailed by approved societies was or was not justified he could not say, but surely the old societies, now that they were able to debit some of their management expenses to the State side, could manage the considerably reduced benefits of members coming under a scheme at a lower cost, and that in itself justified some deduction under the head of management.

With reference to the author's remarks as to the mortality being heavy and being given effect to in the valuation, no doubt the matter

was quite clear to him, and by giving effect to the heavy mortality he meant using a mortality table appropriate to the experience, but one often came across valuations made by unqualified men who had actually reduced the value of the death benefit liability by a percentage, so as to allow for the light mortality experience. It was desirable that a sentence which might easily be construed into giving Institute sanction to such a proceeding should not be allowed to pass without comment.

With regard to deductions under the scheme, as had been already indicated, they were not necessarily limited to 3*d.* per week. About eighteen months ago he had prepared a scheme under which the total deduction was more than 4*d.*, but in that particular case there were very few members who contributed for the doctor, and he gave nearly the full 4*d.* to the sick and funeral deduction, and the one or two contributing to the doctor were able by means of that extra deduction to obtain more than 4*d.* The assumption throughout the paper that 3*d.* was the deduction seemed to be unnecessary.

The author did not seem prepared to say what he would do in the case of reducing members who also came within the provisions of Section 47, under which employers entered into a bond to pay sickness benefits during a specified period. Such members could be dealt with similarly to ordinary reducing members by merely providing that while receiving wages from their employers in accordance with the bond they should be treated as if they were actually receiving the State benefits.

The omission from the paper of the subject of special deductions applicable to women was rather remarkable, because that was probably the most difficult point arising in connection with schemes. The problem was complicated by the fact that an insured woman could on marriage continue to contribute for a reduced benefit of 5*s.* for 13 weeks and 3*s.* thereafter, in accordance with Table D of Part I of the Fourth Schedule to the Act. If women who came under a scheme were dealt with in the same way as men and without reference to the options on marriage a heavy liability would fall upon the private side of friendly societies. It would seem, therefore, that the best course would be to allow in a scheme for a relief of liability to the extent of the benefits of Table D only and to stipulate further that in the case of a married woman suspended from ordinary benefits the independent funds of the society should only be liable to pay the ordinary benefits in sickness reduced by the benefits of Table D, whether she actually became a voluntary contributor or not. This, of course, was a conservative method of obtaining the deduction, but it would be found that it enabled a lodge in a reasonable state of solvency to allow the full deduction of 3d. After all, it was necessary to be conservative in view of the excessive sickness rates which it was anticipated would be experienced in the case of women.

One point of special interest to actuaries was that in the future the valuation would be a much more serious matter. In the past valuations had been more or less simple, but in the future they would have to take into account the special results of schemes. Some

members would be entitled to truncated benefits, and as far as he could see one would have to go through the same process as was necessary under a scheme; first, the full benefits would need to be valued, and then the amounts by which such benefits were reduced. So that, instead of being a single process, it would involve a double valuation.

Mr. J. BURN wished to associate himself with Sir Gerald Ryan in thanking Mr. Nathan for contributing to the Institute a Paper which must of necessity have involved an immense amount of labour. When he was asked to close the discussion, he felt that the subject dealt with was so important that it would be his duty to clear up any difficulty or doubt which had arisen. As, however, the discussion had already lasted nearly two hours, he thought the most acceptable thing he could do was to be as brief as possible. Two of the points to which he had intended to call attention had been already referred to by the last speaker, and they were points of very great importance. The inclusion of re-assurances in the assets was a matter which the Institute should not allow to pass without comment. As the last speaker had said, it was open to very grave objection. He entirely agreed also with the criticisms of Mr. Burrows on the subject of expenses. It might undoubtedly be the case that small societies had a difficulty in meeting their expenses under the National Insurance Act out of the amount allowed, but that was no reason why they should not make some reduction on their own private side. He had failed to understand exactly what the author's meaning was, and he was also rather concerned with regard to his criticism of Mr. Watson. He thought there must be some little misunderstanding by Mr. Nathan as to what Mr. Watson's meaning was. It was quite clear that if on the private side the society was relieved of a considerable amount of its liability and business there should be room for some amount of reduction in the expenses. If they still found they could not reduce the expenses there was certainly a very great hardship. With Mr. Watson's remarks in his admirable lectures he entirely agreed.

The main point in the Paper to which he wished to call attention was one in regard to which the author appeared to him to be under a misapprehension. Mr. Nathan was apparently under the impression that at the present time every member of a registered friendly society had the right to ask for the option of continuing or varying his benefits. It was said in the paper: "In the sixth Lecture on Friendly Society Finance, Mr. Watson explained fully Sections 72 and 73, as at the date the lecture was delivered. Following on a legal ruling of the Law Officers of the Crown, which has up to the present been uncontested, the section has been modified in practice, and options which Mr. Watson suggests might be given voluntarily if funds were available have now to be incorporated as a compulsory part of the Scheme." He had been unable to find that legal ruling, although he had made very careful enquiries, and it was perfectly clear that the members of friendly societies had not the right to demand the option except in so far as

they were part of the friendly society, and could take part in the passing of any decision at a meeting called for the purpose. Having bound the minority in that way, of course the society itself could grant the option. At first, when Mr. Watson was writing his lectures he was of opinion that the option would not be given and that it would be possible to bind all members in one particular way, but it was afterwards decided that friendly societies—not the members but the societies themselves—could, if they liked, give their members, or some of them, the option of continuing or otherwise. That, however, was quite a different thing from what the author appeared to imply in his Paper.

The author stated : " If the society is worth 25*s*. in the £1 prior to the scheme it must retain the same degree of solvency after the scheme has been put in operation." He did not think that was correct. Anyone who desired a really clear explanation of that difficult subject could not do better than read Mr. Watson's sixth lecture, which was admirable in every way. In that lecture Mr. Watson clearly showed the difficulty of attaining the same degree of solvency, but reference to the Act showed that the provision did not apply to societies which had a surplus. There was some difficulty in construing Section 72, but he thought the meaning must be that if a society had a surplus that surplus must first of all be put on one side and then Section 72 must be applied in the ordinary manner ; any surplus which arose could then be dealt with in the ordinary way, but it was not necessary to apply the standard of solvency to societies which already had a surplus. He believed Mr. Nathan would find, if he attempted to apply it, that he would get very curious results. The intention of clause 72, in his opinion, was excellent. It was certainly a hardship that a member who had been fully insured should be compelled to pay an extra 4*d*. for another insurance which he did not want, and the scheme was devised as a means of getting over the difficulty. Yet it very soon became evident to him that very few societies would accept the scheme. The reason he believed to be as follows. In the societies there were a large number of men over 50 who under the National Insurance Act would receive a very much larger benefit than they paid for ; they were perfectly well aware of this fact, and when asked to reduce their benefit refused to do so. They said in effect : " The Government is making me a present of something like £20 : I am very fond of my society and I should like to help it, but why should I give up £20 ? " They were too sharp ; they saw that they had a good thing and they were not going to give it up. In one case, at a meeting of several hundreds of members, one man got up and said— " I have been asked to reduce my benefits by 10*s*. and to get some reduction of the contribution, perhaps 4*d*. Now I want to put it to you straight : I get £2 a week when I am in employment and I am insured in three societies and get 36*s*. a week from them if I am ill. Each society has come and asked me to reduce my benefit. If I accept all three, what will be my position ? I shall pay 8*d*. less a week and shall be £1 less in benefit from sickness."

Of course, the thing was absolutely ridiculous, and it was clear that the position of a man insured in more than one society presented an insuperable difficulty. This one man's speech alone spoilt that scheme.

Further on in the Paper the author referred to the medical administration under the National Insurance Act and criticized the doctors and the administration of the Approved Societies. In a paper read before the Institute of Actuaries it was necessary to be very careful in such matters. It might be that Mr. Nathan had evidence of what he said, and if he had he was probably aware that there was already sitting a Committee before whom he could place that evidence. On the other hand, if he had no evidence then he should not have made the statement. In fact, in any case it was very doubtful whether a criticism like that was in place at the present moment. It might be right or it might be wrong—it was a matter very much contested at the present time—and he thought it his duty to call attention to the manner in which it had been stated.

In his conclusions the author made various suggestions with regard to Clause 72, and they appeared to be a little remarkable. He said, in the first place, that all schemes should be compulsory. That brought him up against the first difficulty, that the scheme could be made compulsory by the society itself, but there was no option to the particular member. The society undoubtedly could bind all its members. He was quite aware that in Messrs. Comyns Carr and Stewart Garnett's work on "The National Insurance Act" they queried whether, on an attempt being made by a society to compel its members to pay the 4*d.*, when such a scheme was submitted to the Registrar, a person who dissented would not have a right to object to the Registrar confirming the scheme. On the other hand, on reference to the actuarial section of the First Annual Report of the Insurance Commissioners, it would be found that a thousand schemes had been submitted to and approved by the Registrar-General for continuing, and among that large number he did not think there was a single case in which an objection had been raised.

The author also suggested as an improvement to Section 72 that the scheme should be such as to enable alterations to be included other than those in respect of overlappings of the National Insurance benefits and the original benefits. That, he took it, was the only intention of Section 72, that wherever a benefit was provided for it should be possible to cancel it and make re-arrangements such as were suggested in the Act. The author said it should be made to deal with sick pay after 70 and various other things, but he could not agree with him at all. Mr. Nathan said these might or might not be advisable alterations. If they were advisable then the society should deal with them in the ordinary way and the Friendly Societies Act gave a very simple method of altering benefits. Mr. Nathan also suggested that it would be very convenient to be able to cancel a large amount of present sickness benefit and give increased death benefits. There might be some objections to that

in certain quarters, but quite apart from that he really failed to see the use of it. The large majority of the population were already insured to a very considerable amount, and why these people should have their sick benefits cancelled and be compelled to take death benefits in their stead he failed to understand.

With regard to the Belgian Act, the author stated that Belgian friendly societies of all descriptions did not appear to be much given to the pageantry and semi-Masonic ceremonies which were such a picturesque feature of British friendly societies, but which added considerably to the cost of working and distracted members' attention from the efficient management of the insurance features of the society. Possibly, when Mr. Nathan had had more experience of some of the friendly societies he might alter his opinion about that. It was easy to make fun of a man going along with a big sash around him and carrying a banner with a peculiar picture upon it; it might not be picturesque, but its utility was beyond doubt. The spirit of the friendly society movement, the feeling that a man was doing something which was really good and which made him feel important, was a feeling to be encouraged, and it was perhaps because he carried that big banner and had a sash around him that he was content to keep some wretched ledgers the whole year through for nothing.

The CHAIRMAN, in moving a vote of thanks to Mr. Nathan for his extremely exhaustive Paper, and for the very large and useful amount of information he had given with regard to the proposed Belgian National Insurance Act, said he had read the essay with great interest, and had certainly obtained a considerable amount of knowledge that he did not possess before. He had meant to make some remarks on the Paper itself, but the hour was so late that he would simply content himself with moving a very hearty vote of thanks to Mr. Nathan.

The resolution was carried with acclamation.

Mr. NATHAN, in reply, said that as it was extremely late and the criticisms had been very full, he could hardly deal with them in detail. He would, however, touch very briefly on the question Mr. Burn had raised, because apparently it had caused some misapprehension. When he dealt with the question of options he was referring not to the right of a member to continue or to reduce, but to the right of the society, acting under actuarial advice, to withdraw from insured members under the scheme the additional benefits represented by the extra reserves. In Mr. Watson's lectures he submitted that the alternatives suggested were whether insured persons under the scheme should receive sick pay for compensated accidents and during the waiting periods, or be excluded from these benefits as a result of the scheme, whereas these benefits had to be provided for these members as a part of the scheme.

With regard to Mr. Burn's reference to his remarks on medical administration, when he wrote that he had in front of him the cases of two societies, and he was afraid they rather influenced his view of the matter. One was an approved society and the other a friendly society. In one a large number of cases were referred to

independent medical men, with the result that all except six of them promptly declared off without going to a doctor. The other was a friendly society which found its sickness so excessive that it instituted the old system of having a society doctor, for which all the members paid 4*s*. a year, and of referring insured persons to that doctor as a sort of check upon the panel doctor, and the result of such check was immediately efficacious and the sick pay fell considerably. In referring to the uniform abatement of 3*d*. per week, he was simply faced with the fact that some abatement had to be dealt with, and he selected that after allowing the abatement of 1*d*. a week for the doctor. He was naturally aware that the abatement might in some cases be different.

Adverting to the question of Table F, the data furnished to him for that table gave no references to new entrants, withdrawals, or deaths, but simply gave the membership at certain successive years. The Federations required a waiting period of six months and the Belgian Friendly Society required the same, and in addition a member could not claim on the Federation until he had been in the society for a period of three or six months. His efforts were directed to estimating as far as possible from the available data, which were scanty, the number of members who were exposed to risk, and for that purpose he considered the membership as half that aged $(x-1)$ in the previous year and half that aged $(x-2)$ in one year further back. Mr. Bacon had suggested that the excessive sickness in the age groups 16–20 of Table F, was probably due to the method of obtaining the exposed to risk. This was not, he thought, the case; a great deal of the excessive sickness remained even if it were assumed that the increased membership in this age group had been exposed to risk for a full year; whereas new entrants were not generally exposed to risk in the Federation until at least a full year had elapsed since joining. In the Paper he suggested that the excessive sickness was due to the introduction of a large number of miners into the experience. In 1911 35 per-cent of the claimants were miners, whereas only 19·19 per-cent of the whole membership were engaged in that occupation. Mr. Maudling had suggested that the Belgian Insurance Act approximated to the German Act and not to the English Act, but he did not agree with him, because the German Act dealt with the whole population of the country, whereas the Belgian Act proceeded entirely on individual lines, lines even more individual than the Act of this country.

[Mr. Nathan has sent us the following Note with reference to the foregoing Paper and the discussion thereon.—Eds., *J.I.A.*]

As the discussion following my paper extended to a late hour, I was unable to reply to several of the points raised by the speakers. Messrs. Maudling and Burrows both alluded to the method I adopted in making a reserve for the accident risk left to a society as a result of the Scheme. They both suggested that they preferred the method I alluded to in the paper, which involves the valuation of the release

of liability by a Standard Table (such as the Manchester Unity A.H.J.), instead of the loaded table employed in the valuation. This method seems to assume that the whole of the extra cost of sickness is due to accidents under the Workmen's Compensation Acts, and on the other hand, that the Standard Table does not involve any claims due to this cause. This is, I think, not entirely in accordance with facts. Many of the heavy industrial employments require a degree of physical fitness that extends the convalescent period following sickness, while many slight maladies may incapacitate from heavy work which might pass unnoticed in a light employment. Moreover, many heavy employments render a member susceptible to illnesses which are in no way covered by the Workmen's Compensation Acts. On the other hand, most of the employments in the Standard Table involve some risk of accident, and in some of them, notably some of those in group A, the accident risk is substantial. If the Standard Tables are employed the accident risk is not automatically eliminated, and a balance of liability may be left over to the society for which no contribution is payable.

Our information as to the cost of accident claims generally is not sufficiently complete to enable us to make anything but an empirical reserve for the liability.

Several speakers alluded to the fact that I had used throughout a uniform abatement of 3*d*. per week. This was, of course, done for illustrative purposes.

Mr. Bacon enquired why I require the ages of insured persons and not only those reducing their contracts under the National Insurance Act. Some societies paid the doctor's bills on a fixed scale, instead of providing medical attendance at so much per head per annum. In these cases the cost of the Medical Benefit varies with the age, and in finding the reduction of contributions for medical benefit, the age of the insured members is needed.

The reserve for the first three days at 1*s*. 3*d*. refers only to the case of a society paying sick pay at the rate of 10*s*. per week.

With reference to the omission of women from the scheme, prior to the passing of the National Insurance Act women members formed a very small proportion of the total membership of friendly societies, and the numbers coming under the scheme would, I think, be very inconsiderable. As the paper extended to a considerable length, I excluded all reference to women under the scheme.

I am afraid Mr. Burn has misunderstood my reference to options under the scheme. As a result he states that I am under the impression that every member of a registered friendly society had a right to ask for the option of continuing or varying his benefits. I can assure Mr. Burn I am under no such misapprehension. The option I refer to is the option of the society to withdraw from the insured member under the scheme the additional benefits represented by the extra reserves. It may be of interest, however, to point out that most societies who adopt schemes seem to give their members the option of continuing or reducing their benefits. The Section states that the effect of a scheme shall not prejudicially

affect the solvency of a society. As several speakers pointed out, the Section is not clear. I think, however, that in view of the sacrifices a reducing member makes he should be permitted to profit under the scheme by a share of the surplus of the society, so long as the degree of solvency is not affected thereby.

My reference to the medical administration of the Act may not have been sufficiently reserved, but the fact that many approved societies employ independent medical referees apart from the panel doctors is a good indication that this part of the Act is not working in an entirely satisfactory manner.

With reference to Table F, the proportion of members receiving sick pay after three months, instead of after six months, may have something to do with the excessive rates shown among Belgian societies in the age group 16–20. I am of the opinion, however, that the large increase in the proportion of mining employments which would especially affect the young ages accounts in a large measure for the excessive sickness in this age group.

Graduation by Reduction of Mean Square of Error. (II). *By* W. F. Sheppard, Sc.D., LL.M.

(*Continued from p.* 185.)

11. For another example, take the data* for the Government Annuitants (1910) ultimate table for women. If we tabulate q_x or colog p_x, the new method does not give the "best" graduation of the table, for there is certainly (*see* Note A, p. 403) some negative correlation between the errors of adjacent values, and the mean square of error also is different at different ages. The method will however give a graduated table, and we can use it by way of illustration of the mechanical process. We shall therefore apply it to the observed values of $u \equiv \text{colog}\,_{10}p_x$ from $x=45$ to $x=92$, for the purpose of obtaining a graduated table through a portion of this range. As the number exposed to risk did not exceed 5,000 at any age, it will be sufficient to take the given u's to 4 places of decimals; but the deduced v's will be taken to 5 places.

For purposes of illustration, we will assume for the present that for constructing a graduated table of colog p_x we can

* "Mortality of Government Life Annuitants", H. of C. Paper 298 (1912), p. 63.

take $j=3$, *i.e.* we can treat differences of U above the 3rd as negligible.

As the u's are rather irregular, we require a wider formula than in Example 1. For 17 u's, writing

$$[\sigma u_r] \equiv \sigma u_{r+8\frac{1}{2}} - \sigma u_{r-8\frac{1}{2}}, \ \{\sigma^2 u_r\} \equiv \sigma^2 u_{r+8} + \sigma^2 u_{r-8},$$

$$[\sigma^3 u_r] \equiv \sigma^3 u_{r+7\frac{1}{2}} - \sigma^3 u_{r-7\frac{1}{2}},$$

the formula is (*see* Table 1, p. 181)

$$323 v_r = -21[\sigma u_r] + 15\{\sigma^2 u_r\} - 2[\sigma^3 u_r].$$

We write $y \equiv 10^4 u$. The successive sums of y, up to the 7th, are given in Part I of Example 2 (pp. 408-9); and the values of $[\sigma y]$, $\{\sigma^2 y\}$, . . , up to $[\sigma^7 y_r] \equiv \sigma^7 y_{r+5\frac{1}{2}} - \sigma^7 y_{r-5\frac{1}{2}}$, are given in Part II (p. 410). For our present purpose we only require to go up to $[\sigma^3 y]$; the result of applying the above formula is shown in cols. (1) and (2) of Part III (p. 411), and the 1st and 2nd differences of $10^5 v$ are given in cols. (3) and (4).

In order to keep the number of figures as small as possible, some of the 1st, 3rd, and 5th sums in Part I are made negative. These are shown in *italic* type.

12. In the above example, it will be noticed that the 2nd differences in col. (4) are rather irregular; and it has been suggested, by way of criticism of the method, that it is not really as valuable as a method which gives a more regular sequence of 2nd or 3rd differences.

This, of course, is not a mathematical question. As is pointed out at the beginning of § 6 (p. 174), the assignment of a criterion of goodness of graduation is quite arbitrary. We have adopted the principle that each v is to be the best possible, having regard to the number of u's available for determining it. But we might quite well have adopted some other principle; *e.g.*, that the v's should be such that (say) their 2nd differences should have the best possible values. This problem can be solved, for the class of cases we are dealing with (§ 8); but, before attacking it, there are some other points to be considered, which are of interest in themselves and also have an indirect bearing on it.

13. *Graduation of First Difference.*—In cases such as that of Example 1, and still more in cases where the 2nd difference of U is certainly negligible and there is doubt as to the nature of the 1st difference, we may be more interested in

the 1st difference of U than in U itself. If we graduate u, as we have done in § 10, the 1st differences of the v's in the resulting table will be an improvement on the 1st differences of the original u's. But we can get much better results by direct graduation of the 1st difference. The general formula, using the $2n+2$ u's from u_{-n} to u_{n+1}, for $j=1$ or 2, is obtained by putting $t=k=1$ in (37) of "Fitting I", § 10, p. 106, and altering n to $n+1$. Writing

$$M \equiv \tfrac{1}{3}(n+1)(2n+1)(2n+3),$$

the formula becomes

$$M\delta v_{r+\frac{1}{2}} = (2n+1)(\sigma u_{r+n+1\frac{1}{2}} + \sigma u_{r-n-\frac{1}{2}}) - 2(\sigma^2 u_{r+n+1} - \sigma^2 u_{r-n})\ ;$$

and, if R^2 is the ratio of the mean square of $\delta v_{\frac{1}{2}}$ to that of $\delta u_{\frac{1}{2}}$, it is shown in "Fitting II" that

$$R^2 = 1/M.$$

If δv were obtained from the graduated u's, j being taken $=2$, the value of R^2 would be

$$R^2 = \frac{3n^2+13n-1}{\frac{1}{3}(2n-1)(2n+1)^2(2n+3)}\,;$$

and the ratio of the former value of R^2 to the latter obviously decreases as n increases. The values of R^2, of R, and of 1/R, for each of the two methods, for values of $2n+2$ up to 32, are given in Table 3 (p. 405).

Since, as will be seen from the table, a small number of u's will give a good result, it is usually simplest to graduate progressively. The above formula can be written

$$M\delta v_{r+\frac{1}{2}} = (u_{r+1} - u_r) + 3(u_{r+2} - u_{r-1}) + 5(u_{r+3} - u_{r-2}) + \ldots$$
$$+ (2n+1)(u_{r+n+1} - u_{r-n}).$$

Hence, denoting by $[2t+1]$ the sum of the particular δu and the t on each side of it, so that, for $\delta u_{r+\frac{1}{2}}$,

$$[2t+1] = u_{r+t+1} - u_{r-t},$$

the successive approximations to $\delta U_{r+\frac{1}{2}}$ are given by

$$\delta v = [1],$$

$$(3^2+1^2)\delta v = [1] + 3[3],$$

$$(5^2+3^2+1^2)\delta v = [1] + 3[3] + 5[5],$$

&c.

The application of this method to the data of Example 1, for the periods 1884–5 onwards, is shown in Example 3 (p. 412). Cols. (1)–(5) give the values of [1], [3], . . . [9], and cols. (6)–(10) give the deduced values of [1], [1]+3[3], [1]+3[3]+5[5], . . . , which have to be divided respectively by 1, 10, 35, . . . As the object is merely to show the progressive approach to regularity, I have not inserted the resulting values of δv.

It must be remembered that these formulæ for δv only hold if $j=1$ or 2.

14. *Preparation of differences for interpolation.*—When we have obtained a graduated table of v, and wish to extend it by introducing intermediate values, the 1st differences used in the interpolation must be the actual differences of the v's; for otherwise we should create discontinuities. But the 2nd, 3rd, ... differences need not be the actual differences of the 1st, 2nd, ... differences; all that is necessary is that for any particular difference $\delta^f v$ we should use a quantity which differs from $\delta^f u$ by differences of order greater than j. We can therefore (if we think it worth while) make the table look neater, for purposes of interpolation, by making an independent graduation of such of the differences (other than the 1st) as we require.

In Example 2, for instance, we have obtained a graduated table of $v \equiv \text{colog}\, p_x$ on the assumption that $j=3$, x being an exact number of years of age. Suppose that we wish to adapt this table for determining* v for intermediate values of x. Then we must leave the 1st differences alone, but we may graduate the 2nd and 3rd differences independently.

It must, however, be remembered, in this particular case, that the formula used is the same for $j=3$ as for $j=2$, and, further, that our statement "$j=3$" means that the 3rd difference of u must not be multiplied by an appreciably large factor. The interpolation-coefficient of the 3rd difference is always relatively small. Moreover, in using a central-difference formula, such as Everett's, the 3rd difference is not shown directly, but is latent as the difference of the two 2nd differences. We can therefore disregard the 3rd difference, and consider only the graduation of the 2nd

* In actual practice we should use the table for constructing a table of colog l_x by successive additions; and therefore the differences which we should probably wish to regularize would be the 1st differences of v. But this is another problem.

differences. This graduation will, like that of the u's, be effected by making the mean square of error, in each case, as small as possible.

In Example 2 each value of v involves 17 u's, and therefore each $\delta^2 v$ involves 19 u's. But it is not the "best" value obtainable from these 19 u's. Strictly, to see the effect of the suggested graduation, we ought to compare the accuracy of the 2nd differences of the v's with that of the best values of $\delta^2 v$ obtainable from 19 u's. But we have had to set out, in Part II of the Example, the values of $\sigma u_{r+8\frac{1}{2}} - \sigma u_{r-8\frac{1}{2}}$, &c.; and these are the values that appear in the formula for improvement of $\delta^2 u$ by means of 17 u's. We shall therefore consider that $\delta^2 v$ is to be determined from the same u's as those employed for determining v.

15. The formula for improvement of $\delta^2 u$ by using $m \equiv 2n+1$ u's ($j=2$ or 3) may, like the formula for v, be expressed in three forms.

(i) The formula for $\delta^2 v_r$ in terms of differences of u_r is obtained in "Reduction", § 15, pp. 359-362, and is also given in "Fitting I", § 7 (i), p. 102, by putting $k=1$, $t=1$ in (18) of the latter. The values of the coefficients for values of m up to 19 are given in "Reduction", Table I.A (i), pp. 378-380. For $m=7$, for instance, the formula is

$$\delta^2 v_r = \delta^2 u_r + \frac{5}{7}\delta^4 u_r + \frac{5}{42}\delta^6 u_r.$$

(ii) The formula in terms of sums is given in "Fitting I", § 11 (ii), p. 106. Modifying this as explained in § 9 (ii), p. 176, of the present paper it becomes

$$\delta^2 v_r = \mathrm{A}(\sigma u_{r+n+\frac{1}{2}} - \sigma u_{r-n-\frac{1}{2}}) + \mathrm{B}(\sigma^2 u_{r+n} + \sigma^2 u_{r-n}) + \mathrm{C}(\sigma^3 u_{r+n-\frac{1}{2}} - \sigma^3 u_{r-n+\frac{1}{2}}),$$

where A, B, C are given by

$$\mathrm{M}' = \frac{1}{30} n(n+1)(2n-1)(2n+1)(2n+3),$$

$$\mathrm{M}'\mathrm{A} = n(2n-1),\ \mathrm{M}'\mathrm{B} = -3(2n-1),\ \mathrm{M}'\mathrm{C} = 6.$$

The values of A, B, C, for values of m up to 31, are given in cols. (2)-(4), taken with col. (1), of

Table 4 of the present paper (p. 406). The values of M in the table are the same as those of M′ as defined above, after removal of any factor common to M′, M′A, M′B, M′C.

(iii) The formula in terms of u's is

$$\delta^2 v_r = b_0 u_r + b_1 u_{r\pm 1} + \ . \ . \ . \ + b_n u_{r\pm n},$$

where, M′ being as above,

$$b_t = b_0 + \frac{1}{2} t^2 \delta^2 b_0,$$

$$M' b_0 = -n(n+1), \ M' \delta^2 b_0 = 6.$$

The values of b_0 and of $\delta^2 b_0$, for values of m up to 31, are given in cols. (5) and (6), taken with col. (1), of Table 4.

The mean square of error of each $\delta^2 u$, taking that of each u to be a^2, is $6a^2$. If the mean square of error of $\delta^2 v$ is $6R^2 a^2$, then

$$R^2 = 1/M'.$$

The values of R and of 1/R are given in cols. (8) and (9) of Table 4.

16. For improvement of $\delta^2 u$ by using 17 u's, the formula as given by Table 4 is

$$3876\delta^2 v_r = +40[\sigma u_r] - 15\{\sigma^2 u_r\} + 2[\sigma^3 u_r].$$

The results of the calculation, applied to the data of Example 2, are given in cols. (5) and (6) of Part III (page 411). It will be seen that the resulting values of $\delta^2 v$ are positive throughout. The values, as given in col. (6), can then be used, for purposes of interpolation, with those of v and of δv given in cols. (2) and (3); or, if we use Everett's formula, we require only cols. (2) and (6).

It is to be remembered that this more regular set of 2nd differences is to be used as being neater. We have not shown that it will give better results for the interpolated values. The question of relative accuracy of interpolation-formulæ is too large to be considered here.

17. The important point to which attention must be called, as regards the results obtained in §§ 13 and 16, is that *these graduated differences of v cannot be summed for the purpose of obtaining graduated values of v.* The formula which gives

col. (10) of Example 3 may (*see* "Reduction", Table I. B (ii), p. 381, $m=10$) be written

$$165\delta v = 165\delta u + 396\delta^3 u + 297\delta^5 u + 88\delta^7 u + 9\delta^9 u.$$

If, starting with a proper value of v, we were to obtain consecutive values by successive additions of 1st differences calculated in this way, these values would be given by

$$165v = 165u + 396\delta^2 u + 297\delta^4 u + 88\delta^6 u + 9\delta^8 u.$$

But this contains a term in $\delta^2 u$, which is not legitimate on the assumption that $\delta^2 U$ is not negligible. Similarly the value of $\delta^2 v$ in col. (6) of Example 2, Part III, is such as would (*see* "Reduction", Table I. A (i), p. 379, $m=17$) be given by

$$\delta^2 v = \delta^2 u + 5\delta^4 u + \frac{55}{6}\delta^6 u + \ldots + \frac{10}{969}\delta^{16} u;$$

and, if we started properly, and performed successive additions, we should get

$$v = u + 5\delta^2 u + \frac{55}{6}\delta^4 u + \ldots + \frac{10}{969}\delta^{14} u.$$

This contains a term in $\delta^2 u$, which is obviously improper.

18. We can now proceed to consider the problem stated in § 12. It will be sufficient (in the first instance) to consider the case of $j=2$ or 3; the principle is the same for other cases. Also we shall assume that, even if $j=3$, we are not concerned about special accuracy of 3rd differences.

The problem then is as follows, the errors of the u's being supposed to be of the standard type (§ 8). We require a formula for v_r, involving the $2n+1$ u's $u_{r-n}, u_{r-n+1}, \ldots u_{r+n}$, such that each of the 2nd differences of the v's shall have the "best" value consistent with the necessary conditions; each of these differences, of course, involving $2n+3$ u's.

The formula for v_r must (since it is obviously symmetrical) be of the form

$$v_r = u_r + c_4\delta^4 u_r + c_6\delta^6 u_r + \ldots + c_{2n}\delta^{2n} u_r;$$

the co-efficient of $\delta^2 u_r$ being zero, since $j=2$ or 3. The 2nd difference will then be

$$\delta^2 v_r = \delta^2 u_r + c_4\delta^6 u_r + c_6\delta^8 u_r + \ldots + c_{2n}\delta^{2n+2} u_r.$$

We require to determine the c's so that the mean square of error of $\delta^2 v_r$ shall be a minimum. The form of $\delta^2 v_r$ shows that the problem is the same as that of determining the "best" value of $\delta^2 v_r$ from $2n+3$ u's for the case of $j=4$ or 5; its solution can, like those of the problems already considered, be expressed in different forms.

(i) The solution in terms of differences is given in the portions of "Reduction" and "Fitting I" already quoted; and the numerical coefficients for certain values of n are given in "Reduction", Table I.A (i). In using this table it must be remembered that there are two more u's involved in $\delta^2 v$ than in v. If, for instance, we wish to use 7 u's for v, the formula for $\delta^2 v$ is that given under $m=9$ in "Reduction", page 378, namely,

$$\delta^2 v_r = \delta^2 u_r - \frac{7}{22}\delta^6 u_r - \frac{28}{429}\delta^8 u_r,$$

which will give

$$v_r = u_r - \frac{7}{22}\delta^4 u_r - \frac{28}{429}\delta^6 u_r.$$

(ii) The formula in terms of sums is given in "Fitting I", § 11 (iv), p. 107; n being replaced by $n+1$. Modifying this as before, we get

$$\delta^2 v_r = \mathrm{A}[\sigma u_{r\pm(n+1\frac{1}{2})}] + \mathrm{B}\{\sigma^2 u_{r\pm(n+1)}\} + \mathrm{C}[\sigma^3 u_{r\pm(n+\frac{1}{2})}]$$
$$+ \mathrm{D}\{\sigma^4 u_{r\pm n}\} + \mathrm{E}[\sigma^5 u_{r\pm(n-\frac{1}{2})}],$$

where—the notation being modified to prevent mistake—$[\sigma u_{r\pm(n+\frac{1}{2})}]$ means $\sigma u_{r+n+\frac{1}{2}} - \sigma u_{r-n-\frac{1}{2}}$, and so on; the values of A, B, C, . . . being given by

$$\mathrm{M}' = \frac{1}{210}(n+1)(n+2)(2n-1)(2n+1)(2n+3)$$
$$(2n+5)(2n+7),$$

$$\mathrm{M}'\mathrm{A} = -(n-1)(n+1)(2n-1)(2n+1),$$

$$\mathrm{M}'\mathrm{B} = 3(4n-1)(2n-1)(2n+1),$$

$$\mathrm{M}'\mathrm{C} = -6(19n-1)(2n-1),$$

$$\mathrm{M}'\mathrm{D} = 270(2n-1), \qquad \mathrm{M}'\mathrm{E} = -540.$$

From this formula, we get

$$v_r = \text{A}[\sigma^3 u_{r\pm(n+1\frac{1}{2})}] + \text{B}\{\sigma^4 u_{r\pm(n+1)}\} + \text{C}[\sigma^5 u_{r\pm(n+\frac{1}{2})}]$$
$$+ \text{D}\{\sigma^6 n_{r\pm n}\} + \text{E}[\sigma^7 u_{r\pm(n-\frac{1}{2})}].$$

(iii) This last formula involves $2n+1$ u's, but it does not correspond in form with those given in §9 (ii) and Tables 1 and 2. By a simple transformation it becomes

$$v_r = \text{A}'[\sigma u_{r\pm(n+\frac{1}{2})}] + \text{B}'\{\sigma^2 u_{r\pm n}\} + \text{C}'[\sigma^3 u_{r\pm(n-\frac{1}{2})}]$$
$$+ \text{D}'\{\sigma^4 u_{r\pm(n-1)}\} + \text{E}'[\sigma^5 u_{r\pm(n-1\frac{1}{2})}] + \text{F}'\{\sigma^6 u_{r\pm(n-2)}\}$$
$$+ \text{G}'[\sigma^7 u_{r\pm(n-2\frac{1}{2})}],$$

where, M′ being as in (ii) above,

$$\text{M}'\text{A}' = -(n-1)(n+1)(2n-1)(2n+1),$$
$$\text{M}'\text{B}' = (-2n^2+12n-1)(2n-1)(2n+1),$$
$$\text{M}'\text{C}' = -(2n^3-47n^2+100n-1)(2n-1),$$
$$\text{M}'\text{D}' = 3(4n-31)(2n-3)(2n-1),$$
$$\text{M}'\text{E}' = -6(38n^2-201n+181),\ \ \text{M}'\text{F}' = 270(2n-5),$$
$$\text{M}'\text{G}' = -540.$$

(iv) The formula for v in terms of u's may be shown to be

$$v_r = b_0 u_r + b_1 u_{r\pm1} + b_2 u_{r\pm2} + \ \dots\ + b_n u_{r\pm n},$$

where

$$b_t = b_0 + \frac{1}{2}t^2\delta^2 b_0 + \frac{1}{24}t^2(t^2-1)\delta^4 b_0$$
$$+ \frac{1}{720}t^2(t^2-1)(t^2-4)\delta^6 b_0,$$

$$\text{M}'b_0 = \{\tfrac{1}{2}(n+1)(n+2)\}^2\{(n+1)(n+2)-3\},$$
$$\text{M}'\delta^2 b_0 = -\tfrac{1}{2}(n+1)(n+2)\{5(n+1)(n+2)-12\},$$
$$\text{M}'\delta^4 b_0 = 6\{7(n+1)(n+2)-15\},\ \ \text{M}'\delta^6 b_0 = -540.$$

The formula for $\delta^2 v$ is then

$$\delta^2 v_r = f_0 u_r + f_1 u_{r\pm 1} + f_2 u_{r\pm 2} + \ . \ . \ . \ + f_{n+1} u_{r\pm(n+1)},$$

where

$$f_t = \delta^2 b_0 + \frac{1}{2} t^2 \delta^4 b_0 + \frac{1}{24} t^2(t^2-1)\delta^6 b_0.$$

19. I have not thought it necessary to give the numerical coefficients in the above formulæ for all values of n that we might be likely to require. The following are some cases in which the coefficients are comparatively small.

$(n=4)$ $M=429$, $1/M=\cdot[2]233100233$;

$$Mv_0 = 135u_0 + 112u_{\pm 1} + 56u_{\pm 2} - 21u_{\pm 4}.$$

$(n=5)$ $M=2431$, $1/M=\cdot[3]411353353$;

$$Mv_0 = 637u_0 + 560u_{\pm 1} + 360u_{\pm 2} + 120u_{\pm 3} - 55u_{\pm 4} - 88u_{\pm 5}.$$

$(n=8)$ $M=22287$, $1/M=\cdot[4]448692063$;

$MA=-357$, $MB=527$, $MC=-302$, $MD=90$, $ME=-12$;

$MA'=-357$, $MB'=-187$, $MC'=395$, $MD'=13$, $ME'=-134$, $MF'=66$, $MG'=-12$;

$Mb_0=3915$, $M\delta^2 b_0=-438$, $M\delta^4 b_0=82$, $M\delta^6 b_0=-12$.

$(n=10)$ $M=144210$, $1/M=\cdot[5]693433188$;

$MA=-1463$, $MB=1729$, $MC=-798$, $MD=190$, $ME=-20$;

$MA'=-1463$, $MB'=-1197$, $MC'=1197$, $MD'=323$, $ME'=-438$, $MF'=150$, $MG'=-20$;

$Mb_0=20812$, $M\delta^2 b_0=-1584$, $M\delta^4 b_0=202$, $M\delta^6 b_0=-20$.

$(n=14)$ $M=237336$, $1/M=\cdot[5]421343580$;

$MA=-1131$, $MB=957$, $MC=-318$, $MD=54$, $ME=-4$;

$MA'=-1131$, $MB'=-1305$, $MC'=465$, $MD'=375$, $ME'=-214$, $MF'=46$, $MG'=-4$;

$Mb_0=25280$, $M\delta^2 b_0=-1056$, $M\delta^4 b_0=74$, $M\delta^6 b_0=-4$.

20. We can now apply the method to Example 2. Using 17 u's, the formula for v_r is, with the original notation,

$$22287v_r = -357[\sigma u_r] - 187\{\sigma^2 u_r\} + 395[\sigma^3 u_r] + 13\{\sigma^4 u_r\}$$
$$-134[\sigma^5 u_r] + 66\{\sigma^6 u_r\} - 12[\sigma^7 u_r].$$

This formula involves seven successive summations. Instead of it, we might use the formula in terms of the u's, which involves eight pairs of u's besides u_r, and would be more convenient if we did not require the successive sums for other purposes. The coefficients of the u's in this formula are obtained as follows, from the values of Mb_0, $M\delta^2 b_0$, . . . given in § 19.

t	b_t	δ	δ^2	δ^3	δ^4	δ^5	δ^6
		+ 219		− 41		+ 6	
0	+3915		−438		+ 82		−12
		− 219		+ 41		− 6	
1	+3696		−397		+ 76		−12
		− 616		+117		−18	
2	+3080		−280		+ 58		−12
		− 896		+175		−30	
3	+2184		−105		+ 28		−12
		−1001		+203		−42	
4	+1183		+ 98		− 14		−12
		− 903		+189		−54	
5	+ 280		+287		− 68		−12
		− 616		+121		−66	
6	− 336		+408		−134		
		− 208		− 13			
7	− 544		+395				
		+ 187					
8	− 357						

The formula is therefore

$$22287v_r = 3915u_r + 3696u_{r\pm1} + 3080u_{r\pm2} + 2184u_{r\pm3}$$
$$+ 1183u_{r\pm4} + 280u_{r\pm5} - 336u_{r\pm6} - 544u_{r\pm7} - 357u_{r\pm8}.$$

The values of $22287v$ and of v, as obtained by either of the above formulæ, are given in cols. (7) and (8) of Example 2, Part III (p. 411); and the resulting 1st and 2nd differences are given in cols. (9) and (10). It will be seen that, while

the 2nd differences are a good deal more regular than those in col. (4), they are not so regular as those in col. (6).

21. Table 5 (p. 407) gives a comparison of the extent to which the different methods (for $j=2$ or 3) reduce the mean square of error of v or of $\delta^2 v$; R^2 being in all cases the ratio of the new mean square of error to the old. The table is framed on the assumption that v in each case involves $2n+1$ u's, and $\delta^2 v$ involves $2n+3$ u's.

(A) Direct graduation by improvement of u (§§ 9–11):

(i) For v, the value of R is given in col. (2); this has already been given in col. (8) of Table 1. The formula for R^2 is

$$R^2 = \frac{3(3n^2+3n-1)}{(2n-1)(2n+1)(2n+3)}.$$

(ii) For $\delta^2 v$, the formula for R^2 is

$$R^2 = \frac{3(2n^2+2n+11)}{(2n-1)(2n+1)^2(2n+3)}.$$

The values of R and of 1/R are given in cols. (5) and (6).

(B) Direct graduation of $\delta^2 u$ (§ 15). The formula for R^2 is

$$R^2 = \frac{30}{(n+1)(n+2)(2n+1)(2n+3)(2n+5)}.$$

The values of R and of 1/R are given in cols. (7) and (8). They are the same as the values in cols. (8) and (9) of Table 4, $2n+1$ being altered to $2n+3$.

(C) Graduation of u so as to give the best values for $\delta^2 v$ (§18):

(i) For v, the formula for R^2 is

$$R^2 = \frac{70(4N-11)(23N^2+3N+45)}{143N(2n-1)(2n+1)(2n+3)(2n+5)(2n+7)},$$

where N denotes $(n+1)(n+2)$. The value of R is given in col. (3).

(ii) For $\delta^2 v$, the formula for R^2 is

$$R^2 = \frac{210(7N-15)}{N(2n-1)(2n+1)(2n+3)(2n+5)(2n+7)}.$$

The values of R and of 1/R are given in cols. (9) and (10).

22. The above-mentioned table shows the relative accuracy of the values of v_r obtained by the (A) and (C) methods. As a rough test of the relative accuracy of the three methods for purposes of interpolation, let us apply them to the calculation of $v_{\frac{1}{2}}$, taking $2n+1=13$. The interpolation-formula is

$$16v_{\frac{1}{2}}=8(v_0+v_1)-(\delta^2v_0+\delta^2v_1) \quad . \quad . \quad . \quad (\mathrm{I}),$$

which, if δ^2v_0 and δ^2v_1 are the actual 2nd differences of the v's, is equivalent to

$$16v_{\frac{1}{2}}=-v_2+9v_1+9v_0-v_{-1} \quad . \quad . \quad . \quad (\mathrm{II}).$$

(A) The formula for direct graduation of v is

$$143v_0=25u_0+24u_{\pm 1}+21u_{\pm 2}+16u_{\pm 3}+9u_{\pm 4}-11u_{\pm 6}.$$

Applying this to (II) we get

$$2288v_{\frac{1}{2}}=396u_{\frac{1}{2}\pm\frac{1}{2}}+364u_{\frac{1}{2}\pm 1\frac{1}{2}}+300u_{\frac{1}{2}\pm 2\frac{1}{2}}+204u_{\frac{1}{2}\pm 3\frac{1}{2}}$$
$$+76u_{\frac{1}{2}\pm 4\frac{1}{2}}-108u_{\frac{1}{2}\pm 5\frac{1}{2}}-99u_{\frac{1}{2}\pm 6\frac{1}{2}}+11u_{\frac{1}{2}\pm 7\frac{1}{2}},$$

so that, if a^2 is the mean square of error of each u, and α^2 that of the deduced value of $v_{\frac{1}{2}}$,

$$\alpha^2/a^2=\cdot 1712683, \quad \alpha/a=\cdot 414.$$

(B) The second method (§ 14) consists in using (I), u and δ^2u being graduated independently. Each second difference of the graduated v's will involve 15 u's; for δ^2v we may (*see* § 14, last par.) use either 13 or 15 u's, the former giving the simpler result numerically. The expressions for v_0 and v_1 are the same as those used in (A); the expressions for δ^2v_0 and δ^2v_1 are to be obtained from Table 4.

(i) With 13 u's for v and 13 for δ^2v we shall get (coefficients only being given on the right-hand side, to avoid printing of $u_{\frac{1}{2}\pm\frac{1}{2}}$, etc.)

$$16016v_{\frac{1}{2}}=2771+2543+2087+1403+491-649-638,$$

$$\alpha^2/a^2=\cdot 1679342, \quad \alpha/a=\cdot 410.$$

(ii) With 13 u's for v and 15 for δ^2v,

$$1089088v_{\frac{1}{2}}=187791+172427+141699+95607+34151$$
$$-42669-43461-1001,$$

$$\alpha^2/a^2=\cdot 1670881, \quad \alpha/a=\cdot 409.$$

(C) The graduation of u by improvement of $\delta^2 u$ will give

$$2956096 v_{\frac{1}{2}} = 648693 + 533423 + 337995 + 123993 \\ - 37807 - 93357 - 39897 + 5005,$$

$$\alpha^2/a^2 = \cdot 1937905,\ \alpha/a = \cdot 440.$$

It will be seen that the (C) method, although the 2nd differences are decidedly more regular than those used in the (A) method, does not give so good a result as the latter; and, further, that the much greater smoothness of the 2nd differences in the (B) method (as shown by comparison of col. (8) in Table 5 with col. (6)) has so little effect in improving the interpolated values as to make it doubtful whether it is worth the extra labour of the independent graduation of these differences.

23. If we want to calculate v directly from the u's, and the coefficients in the formula are large, we can simplify the formula by replacing the fractional coefficients by decimals. In the formula of §20, for instance, the coefficients of $u_r, u_{r\pm 1}, \ldots$ are approximately

$$+ \cdot 17566 + \cdot 16584 + \cdot 13820 + \cdot 09799 + \cdot 05308 + \cdot 01256 \\ - \cdot 01508 - \cdot 02441 - \cdot 01602.$$

Taking these to 3 decimal places, and making small adjustments so that, when v_r is expressed in terms of u_r and its differences, the coefficients of u_r and of $\delta^2 u_r$ shall be respectively 1 and 0, we get such a formula as

$$v_r = \cdot 174 u_r + \cdot 166 u_{r\pm 1} + \cdot 138 u_{r\pm 2} + \cdot 098 u_{r\pm 3} + \cdot 054 u_{r\pm 4} \\ + \cdot 013 u_{r\pm 5} - \cdot 015 u_{r\pm 6} - \cdot 025 u_{r\pm 7} - \cdot 016 u_{r\pm 8}.$$

The resulting values of R for v_0 and for $\delta^2 v_0$ are respectively $\cdot 389$ and $\cdot 0249$, which are (see Table 5) practically the same as those due to the exact formula.

Note A.

The data for the construction of a mortality-table approximate to one or other of two extreme types.

The case of the first type is that in which the data are the death-history of a definite set of persons, all starting from the same age. Here there is a negative correlation between the errors of the deaths at neighbouring ages. Consider,

for instance, the persons who attain age 60. A certain proportion, on the average, of these persons will die at age 65+. They will die at this age because at age 60 their constitutions will be such that they will live for another 5+ years. But, when the numbers are small, the actual proportion will vary from its average value; partly because the number fated to die after 5+ years will vary, and partly because the actual time of death will be affected by external influences such as a mild or severe winter. A positive or negative deviation at one year of age, whichever of these two causes it may be due to, will imply a negative or positive deviation at another year or other years of age, so that the errors will be negatively correlated.

The case of the other type is that in which we start with all the persons (of the class under consideration) alive on a definite day, and see how many are alive on the same day in the next year. Here the first of the two causes mentioned above will not operate, and the second will tend to produce a positive correlation of errors. But this correlation will extend over such a large range of ages that it will be practically impossible to eliminate it by any ordinary process of graduation. Practically, therefore, there will be no correlation.

Actual cases lie somewhere between these two extremes. The 1910 statistics of Government Annuitants, for instance, covered a period of 29 years; but there were new entrants throughout this period, so that the periods of observation of individuals varied from 29 years to 1 year. There was, therefore, some negative correlation between errors of q_x for successive years. In cases of this kind the correlation increases (numerically) as the number of years, over which the observations extend, increases. I am not aware that an attempt has ever been made to measure the correlation, even for the extreme case.

J.I.A., VOL. XLVIII.—ERRATA.

P. 172, 3rd line of § 4; for b_r read b_2.

P. 175, last line of text; for $\delta^{2k}u_r$ read $\delta^{2n}u_r$.

P. 177, middle of page; the first formula should be

$$143v_7 = -2(\sigma^3 u_{12\frac{1}{2}} - \sigma^3 u_{1\frac{1}{2}}) + 11(\sigma^2 u_{13} + \sigma^2 u_1) - 11(\sigma u_{13\frac{1}{2}} - \sigma u_{\frac{1}{2}}).$$

P. 181, Table I, heading of col. (0); for M read m.

TABLE 3.—*Improvement of* 1*st difference* ($j=1$ *or* 2).

$2n+2$ *u's.*

$$R^2 = \frac{\text{mean square of error of } \delta v}{\text{mean square of error of } \delta u}.$$

$2n+2=$ number of u's in δv	From Graduated Values of u			By Direct Graduation of δu		
	R^2	R	1/R	R^2	R	1/R
(0)	(1)	(2)	(3)	(4)	(5)	(6)
4	1	1·0000	1	1/10	·3162	3
6	37/175	·4598	2	1/35	·1690	6
8	13/147	·2974	3	1/84	·1091	9
10	1/21	·2182	5	1/165	·0778	13
12	139/4719	·1716	6	1/286	·0591	17
14	37/1859	·1411	7	1/455	·0469	21
16	79/5525	·1196	8	1/680	·0383	26
18	59/5491	·1037	10	1/969	·0321	31
20	359/42959	·0914	11	1/1330	·0274	36
22	143/21413	·0817	12	1/1771	·0238	42
24	101/18515	·0739	14	1/2300	·0209	48
26	587/129375	·0674	15	1/2925	·0185	54
28	1/261	·0619	16	1/3654	·0165	60
30	769/234639	·0572	17	1/4495	·0149	67
32	79/27869	·0532	19	1/5456	·0135	74

TABLE 4.—*Improvement of 2nd difference* ($j = 2$ *or* 3).

$2n+1$ u's.

$$\delta^2 v_r = A(\sigma u_{r+n+\frac{1}{2}} - \sigma u_{r-n-\frac{1}{2}}) + B(\sigma^2 u_{r+n} + \sigma^2 u_{r-n}) + C(\sigma^3 u_{r+n-\frac{1}{2}} - \sigma^3 u_{r-n+\frac{1}{2}})$$

$$= b_0 u_r + b_1(u_{r+1} + u_{r-1}) + \ldots + b_n(u_{r+n} + u_{r-n}).$$

$$b_t = b_0 + \frac{1}{2} t^2 \delta^2 b_0.$$

$$R^2 = \frac{\textit{mean square of error of } \delta^2 v}{\textit{mean square of error of } \delta^2 u} = \frac{1}{6}(b^2{}_0 + 2b^2{}_1 + 2b^2{}_2 + \ldots + 2b^2{}_n).$$

$m \equiv 2n+1 =$ number of u's in $\delta^2 v_r$	M = multiplier	$M\delta^2 v_r$ in terms of sums			$M\delta^2 v_r$ in terms of u's		1/M	$R = \sqrt{\lambda_{2,2}/6}$	1/R
		MA	MB	MC	$Mb_0 = M\lambda_{0,2}$	$M\delta^2 b_0 = M\lambda_{2,2}$			
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		+	−	+	−	+			
3	1	1	3	6	2	6	1·	1·	1
5	7	2	3	2	2	2	·142857143	·218	5
7	42	5	5	2	4	2	·[1] 238095238	·0890	11
9	462	28	21	6	20	6	·[2] 216450216	·0465	21
11	429	15	9	2	10	2	·[2] 233100233	·0279	36
13	1001	22	11	2	14	2	·[3] 999000999	·0182	55
15	6188	91	39	6	56	6	·[3] 161603103	·0127	79
17	3876	40	15	2	24	2	·[3] 257997936	·00927	108
19	6783	51	17	2	30	2	·[3] 147427392	·00701	143
21	33649	190	57	6	110	6	·[4] 297185652	·00545	183
23	17710	77	21	2	44	2	·[4] 564652739	·00434	230
25	26910	92	23	2	52	2	·[4] 371609067	·00352	284
27	118755	325	75	6	182	6	·[5] 842069808	·00290	345
29	56637	126	27	2	70	2	·[4] 176563024	·00243	412
31	79112	145	29	2	80	2	·[4] 126403074	·00205	487

TABLE 5.—*Reduction-ratio for v and $\delta^2 v$ by different methods* ($j = 2$ or 3).

$2n+1$ *u's for* v; $2n+3$ *for* $\delta^2 v$.

Method (A) = *Direct graduation by improvement of* u.

Method (B) = *Direct graduation of* $\delta^2 u$.

Method (C) = *Graduation of* u *by improvement of* $\delta^2 u$.

$$R^2 = \frac{\text{mean square of error of new value}}{\text{mean square of error of original value}}.$$

n	R for v			R and 1/R for $\delta^2 v$						
	$2n+1$ = number of u's in v	Method (A)	Method (C)	$2n+3$ = number of u's in $\delta^2 v$	Method (A)		Method (B)		Method (C)	
		R	R		R	1/R	R	1/R	R	1/R
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	3	1·000	1·000	5	1·000	1	·218	5	1·000	1
2	5	·697	·702	7	·363	3	·0890	11	·341	3
3	7	·577	·591	9	·218	5	·0465	21	·171	6
4	9	·505	·524	11	·157	6	·0279	36	·101	10
5	11	·455	·477	13	·123	8	·0182	55	·0652	15
6	13	·418	·441	15	·101	10	·0127	79	·0452	22
7	15	·389	·412	17	·0861	12	·00927	108	·0328	30
8	17	·365	·389	19	·0751	13	·00701	143	·0248	40
9	19	·345	·369	21	·0667	15	·00545	183	·0192	52
10	21	·328	·352	23	·0600	17	·00434	230	·0153	65
11	23	·313	·338	25	·0545	18	·00352	284	·0124	81
12	25	·300	·325	27	·0500	20	·00290	345	·0102	98
13	27	·289	·313	29	·0461	22	·00243	412	·00852	117
14	29	·279	·303	31	·0429	23	·00205	487	·00721	139
15	31	·270	·293	33	·0400	25	·00176	570	·00616	162

Example 2.—*Government Annuitants* (1910) *ultimate table* (*Women*).

$u \equiv colog_{10} p_x . \qquad y \equiv 10^4 u . \qquad j = 3.$

Part I—(*Construction of successive sums*).

x (0)	y (1)	σy (2)	$\sigma^2 y$ (3)	$\sigma^3 y$ (4)	$\sigma^4 y$ (5)	$\sigma^5 y$ (6)	$\sigma^6 y$ (7)	$\sigma^7 y$ (8)
		2 400						
45	32		38 000					
		2 368		*350 000*				
6	65		35 632		2 210 000			
		2 303		*314 368*		*10 000 000*		
7	50		33 329		1 895 632		33 000 000	
		2 253		*281 039*		*8 104 368*		0
8	100		31 076		1 614 593		24 895 632	
		2 153		*249 963*		*6 489 775*		24 895 632
9	41		28 923		1 364 630		18 405 857	
		2 112		*221 040*		*5 125 145*		43 301 489
50	47		26 811		1 143 590		13 280 712	
		2 065		*194 229*		*3 981 555*		56 582 201
1	83		24 746		949 361		9 299 157	
		1 982		*169 483*		*3 032 194*		65 881 358
2	37		22 764		779 878		6 266 963	
		1 945		*146 719*		*2 252 316*		72 148 321
3	49		20 819		633 159		4 014 647	
		1 896		*125 900*		*1 619 157*		76 162 968
4	70		18 923		507 259		2 395 490	
		1 826		*106 977*		*1 111 898*		78 558 458
5	61		17 097		400 282		1 283 592	
		1 765		*89 880*		*711 616*		79 842 050
6	71		15 332		310 402		571 976	
		1 694		*74 548*		*401 214*		80 414 026
7	75		13 638		235 854		170 762	
		1 619		*60 910*		*165 360*		80 584 788
8	66		12 019		174 944		5 402	
		1 553		*48 891*		9 584		80 590 190
9	90		10 466		126 053		14 986	
		1 463		*38 425*		135 637		80 605 176
60	71		9 003		87 628		150 623	
		1 392		*29 422*		223 265		80 755 799
1	107		7 611		58 206		373 888	
		1 285		*21 811*		281 471		81 129 687
2	109		6 326		36 395		655 359	
		1 176		*15 485*		317 866		81 785 046
3	100		5 150		20 910		973 225	
		1 076		*10 335*		338 776		82 758 271
4	141		4 074		10 575		1 312 001	
		935		*6 261*		349 351		84 070 272
5	154		3 139		4 314		1 661 352	
		781		*3 122*		353 665		85 731 624
6	143		2 358		1 192		2 015 017	
		638		*764*		354 857		87 746 641
7	142		1 720		428		2 369 874	
		496		956		355 285		90 116 515
8	151		1 224		1 384		2 725 159	
		345		2 180		356 669		92 841 674
9	171		879		3 564		3 081 828	
		174		3 059		360 233		95 923 502

EXAMPLE 2—*continued.*

PART I—(*Construction of successive sums*)—(*continued*).

y (1)	σy (2)	$\sigma^2 y$ (3)	$\sigma^3 y$ (4)	$\sigma^4 y$ (5)	$\sigma^5 y$ (6)	$\sigma^6 y$ (7)	$\sigma^7 y$ (8)
	496		956		355 285		90 116 515
151		1 224		1 384		2 725 159	
	345		2 180		356 669		92 841 674
171		879		3 564		3 081 828	
	174		3 059		360 233		95 923 502
180		705		6 623		3 442 061	
	6		3 764		366 856		99 365 563
237		711		10 387		3 808 917	
	243		4 475		377 243		103 174 480
229		954		14 862		4 186 160	
	472		5 429		392 105		107 360 640
257		1 426		20 291		4 578 265	
	729		6 855		412 396		111 938 905
293		2 155		27 146		4 990 661	
	1 022		9 010		439 542		116 929 566
315		3 177		36 156		5 430 203	
	1 337		12 187		475 698		122 359 769
355		4 514		48 343		5 905 901	
	1 692		16 701		524 041		128 265 670
383		6 206		65 044		6 429 942	
	2 075		22 907		589 085		134 695 612
415		8 281		87 951		7 019 027	
	2 490		31 188		677 036		141 714 639
467		10 771		119 139		7 696 063	
	2 957		41 959		796 175		149 410 702
499		13 728		161 098		8 492 238	
	3 456		55 687		957 273		157 902 940
541		17 184		216 785		9 449 511	
	3 997		72 871		1 174 058		167 352 451
629		21 181		289 656		10 623 569	
	4 626		94 052		1 463 714		177 976 020
669		25 807		383 708		12 087 283	
	5 295		119 859		1 847 422		190 063 303
805		31 102		503 567		13 934 705	
	6 100		150 961		2 350 989		203 998 008
812		37 202		654 528		16 285 694	
	6 912		188 163		3 005 517		220 283 702
868		44 114		842 691		19 291 211	
	7 780		232 277		3 848 208		239 574 913
1 025		51 894		1 074 968		23 139 419	
	8 805		284 171		4 923 176		262 714 332
999		60 699		1 359 139		28 062 595	
	9 804		344 870		6 282 315		290 776 927
1 297		70 503		1 704 009		34 344 910	
	11 101		415 373		7 986 324		325 121 837
1 219		81 604		2 119 382		42 331 234	
	12 320		496 977		10 105 706		
1 265		93 924		2 616 359			
	13 585		590 901				
1 351		107 509					
	14 939						

NOTE.—Numbers printed in *italic* type are negative.

EXAMPLE 2—(*continued*).

PART II—*Differences and sums of pairs of successive sums.*

x (0)	$[\sigma y]$ (1)	$\{\sigma^2 y\}$ (2)	$[\sigma^3 y]$ (3)	$\{\sigma^4 y\}$ (4)	$[\sigma^5 y]$ (5)	$\{\sigma^6 y\}$ (6)	$[\sigma^7 y]$ (7)
53	1 115	45 611	320 578	2 297 628	10 135 637	33 014 986	80 590 190
4	1 192	41 958	292 557	1 953 838	8 327 633	25 046 255	55 709 544
5	1 227	38 479	265 554	1 650 988	6 771 246	18 779 745	37 454 310
6	1 318	35 150	239 628	1 385 540	5 443 011	13 936 071	24 547 486
7	1 372	32 062	214 779	1 154 165	4 320 331	10 272 382	15 903 688
8	1 474	29 169	191 107	953 675	3 381 545	7 578 964	10 609 950
9	1 569	26 466	168 719	781 070	2 605 981	5 675 999	7 907 304
60	1 637	23 988	147 675	633 587	1 974 014	4 410 507	7 173 166
1	1 771	21 698	128 080	508 643	1 467 183	3 653 466	7 904 591
2	1 902	19 628	110 036	403 846	1 068 285	3 297 135	9 702 489
3	2 069	17 808	93 644	317 025	761 447	3 252 590	12 256 886
4	2 237	16 286	79 023	246 241	532 216	3 447 463	15 333 312
5	2 423	15 064	66 339	189 806	367 659	3 823 903	18 760 387
6	2 641	14 174	55 746	146 344	256 468	4 336 783	22 418 681
7	2 890	13 643	47 435	114 774	189 131	4 952 153	26 230 953
8	3 155	13 517	41 609	94 362	158 071	5 646 020	30 153 859
9	3 467	13 817	38 512	84 738	157 832	6 403 428	34 171 295
70	3 775	14 607	38 392	85 954	185 265	7 217 902	38 289 497
1	4 133	15 921	41 523	98 526	239 734	8 091 294	42 534 046
2	4 532	17 802	48 220	123 453	323 371	9 034 044	46 948 971
3	4 932	20 323	58 809	162 290	441 318	10 065 937	51 598 124
4	5 407	23 539	73 635	217 213	601 988	11 217. 397	56 569 028
5	5 933	27 527	93 096	291 040	817 389	12 531 339	61 979 438
6	6 596	32 326	117 679	387 272	1 103 481	14 065 630	67 986 888
7	7 257	38 081	147 902	510 190	1 480 566	15 896 200	74 801 540
8	7 954	44 819	184 399	664 915	1 973 746	18 120 865	82 702 663
9	8 799	52 605	227 802	857 553	2 613 412	20 863 959	92 059 103
80	9 561	61 653	278 742	1 095 259	3 435 812	24 281 872	103 354 136
1	10 629	71 929	338 015	1 386 285	4 483 634	28 569 622	117 215 144
2	11 591	83 759	406 363	1 740 165	5 806 617	33 968 496	134 448 662
3	12 563	97 101	484 790	2 167 725	7 462 283	40 774 852	156 081 315
4	13 602	112 023	574 200	2 681 403	9 516 621	49 350 261	183 407 198

Example 2—(*continued*).

Part III—*Graduation of* u *and* $\delta^2 u$.

x	Direct improvement of u from 17 u's				Direct improvement of $\delta^2 u$ from 17 u's		Graduation of u (from 17 u's) by improvement of $\delta^2 u$ $(j=5)$ from 19 u's			
	$323.10^4 v$	v	$10^5\delta v$	$10^5\delta^2 v$	$3876.10^4\delta^2 v$	$10^5\delta^2 v$	$22287.10^4 v$	v	$10^5\delta v$	$10^5\delta^2 v$
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
			+		+	+			+	+
53	19 594	·00607	*12*	...	1 591	4	1 301 600	·00584	10	...
4	19 224	·00595	34	+ 46	3 424	9	1 323 699	·00594	29	19
5	20 310	·00629	0	− 34	3 003	8	1 387 548	·00623	36	7
6	20 316	·00629	69	+ 69	4 726	12	1 468 884	·00659	43	7
7	22 560	·00698	56	− 13	3 508	9	1 565 054	·00702	47	4
8	24 367	·00754	70	+ 14	3 639	9	1 669 413	·00749	66	19
9	26 603	·00824	108	+ 38	3 208	8	1 815 472	·00815	80	14
60	30 093	·00932	62	− 46	1 010	3	1 995 685	·00895	86	6
1	32 119	·00994	71	+ 9	1 530	4	2 185 328	·00981	94	8
2	34 406	·01065	61	− 10	1 732	4	2 395 620	·01075	90	*4*
3	36 383	·01126	90	+ 29	2 928	8	2 596 385	·01165	86	*4*
4	39 267	·01216	97	+ 7	3 236	8	2 786 997	·01251	84	*2*
5	42 399	·01313	101	+ 4	3 638	9	2 976 052	·01335	85	1
6	45 657	·01414	106	+ 5	4 522	12	3 165 561	·01420	93	8
7	49 085	·01520	130	+ 24	5 825	15	3 373 024	·01513	110	17
8	53 282	·01650	128	− 2	6 663	17	3 617 745	·01623	133	23
9	57 424	·01778	174	+ 46	8 449	22	3 913 556	·01756	167	34
70	63 046	·01952	183	+ 9	8 679	22	4 285 116	·01923	196	29
1	68 976	·02135	200	+ 17	9 551	25	4 722 211	·02119	229	33
2	75 418	·02335	255	+ 55	10 690	28	5 232 429	·02348	262	33
3	83 655	·02590	267	+ 12	10 053	26	5 815 942	·02610	276	14
4	92 268	·02857	305	+ 38	10 465	27	6 430 976	·02886	289	13
5	102 120	·03162	275	− 30	10 607	27	7 075 802	·03175	298	9
6	111 016	·03437	371	+ 96	14 308	37	7 740 477	·03473	330	32
7	123 014	·03808	417	+ 46	14 869	38	8 476 740	·03803	381	51
8	136 453	·04225	378	− 39	14 673	38	9 323 939	·04184	422	41
9	148 692	·04603	553	+175	18 489	48	10 266 451	·04606	489	67
80	166 530	·05156	407	−146	15 129	39	11 354 181	·05095	513	24
1	179 696	·05563	637	+230	22 255	57	12 498 722	·05608	577	64
2	200 248	·06200	707	+ 70	19 981	52	13 784 724	·06185	657	80
3	223 112	·06907	718	+ 11	15 585	40	15 248 127	·06842	726	69
4	246 303	·07625		...	12 135	31	16 866 660	·07568		...

Note.—Differences in *italic* type, in cols. (3) and (10), are negative.

Example 3.—*Infantile Mortality* (*progressive graduation of 1st difference*).

δu = excess of proportionate number of deaths (*see Example* 1) *over number in preceding year.*

Year	$\delta u \equiv [1]$	[3]	[5]	[7]	[9]	Approximations to $\delta v \times$				
						1	10	35	84	165
(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1885	+1					+1				
6	+2	+1				+2	+5			
7	−2	−2	+2			−2	−8	+2		
8	−2	−1	+8	+10		−2	−5	+35	+105	
9	+3	+8	+7	+6	+5	+3	+27	+62	+104	+149
1890	+7	+11	+6	+2	−2	+7	+40	+70	+84	+66
1	+1	+5	+6	−2	+4	+1	+16	+46	+32	+68
2	−3	−4	−3	+8	0	−3	−15	−30	+26	+26
3	−2	−11	−2	−1	0	−2	−35	−45	−52	−52
4	−6	0	−9	−10	−5	−6	−6	−51	−121	−166
5	+8	−4	−8	−13	−12	+8	−4	−44	−135	−243
6	−6	0	−8	−10	−10	−6	−6	−46	−116	−206
7	−2	−10	−2	−5	−14	−2	−32	−42	−77	−203
8	−2	−4	−7	−6	−13	−2	−14	−49	−91	−208
9	0	+1	−8	−15	−11	0	+3	−37	−142	−241
1900	+3	−4	−7	−13	−18	+3	−9	−46	−137	−299
1	−7	−5	−9	−10	−20	−7	−22	−67	−137	−317
2	−1	−12	−8	−16	−24	−1	−37	−77	−189	−405
3	−4	−4	−19	−22	−18	−4	−16	−111	−265	−427
4	+1	−11	−18	−21	−23	+1	−32	−122	−269	−476
5	−8	−13	−13	−19	−30	−8	−47	−112	−245	−515
6	−6	−10	−14	−22	−27	−6	−36	−106	−260	−503
7	+4	−7	−19	−22	−24	+4	−17	−112	−266	−482
8	−5	−5	−15	−21		−5	−20	−95	−242	
9	−4	−13	−7			−4	−43	−78		
1910	−4	−6				−4	−22			
1	+2					+2				

LEGAL NOTES.

By WILLIAM CHARLES SHARMAN, F.I.A., *Barrister-at-Law.*

THE decision of Mr. Justice Astbury in the case of *Hewson* v. *Shelley*, reported in these Notes, vol. xlvii, page 563, has since been reversed by the Court of Appeal, 83 L.J. Ch. 607.

Administration revoked on subsequent discovery of will. Position of purchaser from administrator.

The case was concerned with the validity of the sale of an estate by an administratrix whose administration was subsequently revoked upon the discovery of a will, and the facts are fully reported in the previous note upon the case.

The Master of the Rolls, in his judgment allowing the appeal, said : "The contention which prevailed in the Court below leads "to startling results. A debtor of the deceased man might pay "the administrator and would get a good discharge. If sued "by the administrator, he would not be allowed to challenge the "title of the administrator. The purchaser could not resist a "decree for specific performance of a contract for sale by "the administrator. In short, it seems to me that the person "for the time being clothed by the Court of Probate with the "character of legal personal representative is the legal personal "representative, and enjoys all the powers of a legal personal "representative unless and until the grant of administration is "revoked or has determined. If this view is not right, no person "could safely deal with, or accept, a title from an administrator, "for it is impossible to prove that there may not be a will. The "decision of Mr. Justice Barnes in the *Goods of Wright* (1893, "P.D. 21) is a direct authority in favour of the view which I "have taken. In that case administration was granted until "the original will or an authentic copy should be brought into "the registry for the very purpose of enabling the sale of lease-"holds to be completed, and limited to that purpose. In my "opinion *Graysbrook* v. *Fox*, 1 Plowden 275 ; *Abram* v. "*Cunningham*, 2 Lev. 182, and *Ellis* v. *Ellis* (1905), 1 Ch. 613 "must be regarded as no longer law. It remains to consider "the effect of the Land Transfer Act, 1897, Section 2. I think "it confers upon the legal personal representative for the time "being the same powers with reference to real estate as he would "have as to personal estate, subject to some exceptions not "material for the present purpose. If, as I hold, the adminis-"tratrix could have sold and conveyed a leasehold, I think she

"could convey, as she did, the freehold property to Sir John "Shelley. I prefer to base my decision on the general ground "above stated, and not upon any special provisions in the "Probate Act, 1857, although I think there is a good deal in that "Act which assists the appellant. There is, however, a separate "point under Section 70 of the Conveyancing Act, 1881. It "seems to me that the order granting administration to the "widow was a judicial act, and even if that grant could be held "void on the ground of want of jurisdiction, the title of the "purchaser would be protected under that section."

Lords Justices Buckley and Phillimore also delivered judgments allowing the appeal.

The recent case of the *Australian Widows' Fund Life Assurance Society, Limited*, v. *The National Mutual Life Association of Australasia, Limited*, 83 L.J.P.C. 289 deals with the liability of an insurance company under a policy of re-insurance.

Liability under Policy of Re-insurance.

The case is of considerable interest, and, in view of the fact that it was held that the policy of re-insurance in question was an independent contract and not a contract of indemnity, shows the importance which attaches to the terms of a policy drawn up for the purpose of re-insuring a risk.

The case was an appeal to the Judicial Committee of the Privy Council from a judgment of the High Court of Australia, affirming a decision of the Supreme Court of Victoria, which had set aside a judgment of Chief Justice Madden, of Victoria. It was held that the appellants were not liable under the policy.

The material facts are as follows: The *National Mutual Life Association of Australasia* having granted to one Patrick Moran a policy of assurance on his life for £5,000 with profits, re-insured his life with the *Australian Widows' Fund Life Assurance Society* for the same amount without profits, the liability of the re-insurers being expressly limited to what was paid, irrespective of bonus additions, under the original policy.

The original policy was dated 2 January 1908. It recited that the assured had lodged with the association a proposal and declaration and had made a personal statement to a Medical Officer of such association, which proposal, declaration, and personal statement formed the basis of this contract. By the operative part of the policy the association contracted to pay

the sum assured or other the moneys payable thereunder within one calendar month after the death of the assured, with a proviso postponing payment until such proof of the identity of claimant, the validity of the claim, and the age of the assured, as the directors should consider necessary had been deposited with the association. The policy contained a clause to the effect that the policy should be avoided and all moneys paid thereunder forfeited to the association in any of the events therein specified, that was to say (*a*) if any premium should be unpaid for 30 days after it became payable, but so that if the policy had a surrender value such surrender value should be applied by the directors in payment of the premium in arrear ; (*b*) if the proposal or any document on the faith of which the policy was granted contained any untrue statement, or if the person making the proposal had, with a view of obtaining the policy, made any false statement or been guilty of any concealment or misrepresentation. The policy also contained a clause reducing the sum assured and the amount payable in respect of profits, if the age of the assured had been understated.

The policy of re-insurance was dated 29 January 1908. It recited that the respondent association, having an interest in the life of the assured, had, by a proposal and declaration dated 2 January, 1908, applied to the appellant society to have such life assured in the appellant society by effecting a policy on such life payable within one month after proof of the death of the assured. It also contained a recital that the statements in the proposal and declaration, together with the statements contained in the personal statements made to two doctors, were the basis of the contract, and were deemed to be part of the contract therewith. It further contained a recital that the appellant society had agreed to accept the proposal of the respondent association. By the operative part of the policy the appellant society contracted that, in the event of the death of the assured while the premiums under the policy were duly paid, the society would pay to the association £5,000 within one calendar month after such evidence as the board of directors of the appellant society might consider necessary to establish the age, identity, and death of the assured had been supplied to the society.

Patrick Moran died, and the respondent association having, notwithstanding the protest of the appellant society, paid to his legal personal representative £5,000, sued the appellant society for that amount under the policy of re-insurance.

The appellant society contended that its liability under the policy of re-insurance, as also the liability of the respondent association under the original policy, was conditional on the truth of certain statements made by Moran when he effected the original policy, and that these statements were false to his knowledge. The respondent association put the falseness of these statements in issue, and further alleged that whether the statements in question were true or false it had acted reasonably and in good faith in admitting and settling the claim on the original policy, and that the appellant society was, under the terms of the policy of re-insurance, bound by such settlement, and could not rely on the untruth of the statements in question.

The action was tried before the Chief Justice of Victoria and a special jury, who found the statements in question to have been false to the knowledge of Moran, but that the respondent association in settling the claim on the original policy acted reasonably and in good faith. On these findings the Chief Justice dismissed the action, holding that on the true construction of the policy of re-insurance the liability of the appellant society was conditional on the truth of the statements which the jury had found to be false, and that the appellant society was not bound by the settlement effected by the respondent association of the claim against it on the original policy. On appeal the Full Court of Victoria by a majority reversed the decision of the Chief Justice, and directed judgment to be entered for the respondent association for the amount claimed. The High Court of Australia by a majority confirmed the decision of the Full Court, and the appellant society was by special leave appealing from the order of the High Court.

The Judicial Committee of the Privy Council allowed the appeal. In delivering their Lordships' judgment, Lord Parker said, "The result of the appeal depended entirely upon the " construction to be placed on the two policies, and in particular " on the policy of re-insurance.

"Apart from any inference to the contrary to be drawn " from the recital that the appellant society had agreed to accept " the proposal of the respondent association, it was not, and " indeed it could not be, disputed that the liability of the " appellant society under the policy of re-insurance was con- " ditional on the truth of the statements made the basis of the " contract. Further, apart from any effect to be attributed " to this recital, the terms of the policy of re-insurance differed

"in almost every particular from the terms of the original "policy. The basic conditions were different, the premiums "were different. The original policy allowed, but the policy "of re-insurance did not allow, a period of grace for the payment "of premiums. The moneys assured differed in amount, and "were payable at different dates. The persons to determine "the sufficiency of the evidence as to the age, identity, and "death of the assured were different. The original policy "contained a number of special provisions which were not "contained in the policy of re-insurance. Everything pointed, "therefore, to the policy of re-insurance being an independent "contract of assurance rather than a contract of indemnity. "Even the provision limiting liability under the policy of "re-insurance to the amount paid under the original policy would "be unnecessary if the contract were one of indemnity only. . . . "Having regard, however, to the admission in the pleadings, "their Lordships would assume that the recital had the effect "of incorporating in the contract the terms and conditions of the "document of 2 January 1908, which contained the following "clause:

"'It is understood that in accepting the risk under this "'re-insurance the *Australian Widows' Fund Life Assurance* "'*Society* (*Limited*) (*i.e.*, the appellant society) does so on the "'same terms and conditions as those on which the *National* "'*Mutual Life Association of Australasia* (*Limited*) (*i.e.*, the "'respondent association) have granted a policy, and by "'whom in the event of claim the settlement will be "'made.'

"Suppose, then, that that clause had actually been repeated "in the policy itself, what would be its effect? It would be "contrary to all sound canons of construction to reject or modify "the expressed terms of the policy in order that it might be made "to conform to the general words of the clause in question. "Such clause would be almost necessarily construed as if it "were prefaced with the words 'except as herein otherwise "'provided.' It would be only less difficult to maintain that "the effect of the clause was to introduce into the policy of "re-insurance provisions relating to (*a*) application of surrender "value towards payment of premiums in arrear, or (*b*) forfeiture "of premiums already paid, if the basic conditions of the "contract were not fulfilled, or (*c*) the allowance of days of "grace. But it was enough to say that the incorporation in the

"policy of the clause could not be allowed to contradict the "express provision of the policy.

"In their Lordships' opinion, having regard to the facts "found by the jury, the appellant society was not, and never "was, liable under the policy of re-insurance."

The Moratorium. Although the declaration of a Moratorium is not an unknown expedient in some foreign countries, yet its adoption in Great Britain is an event which a short time ago would have been considered almost inconceivable.

The object of moratory laws, which are only declared in times of emergency, is to prevent a too rapid conversion of credit into cash, and this is accomplished by legalising a period of grace for the payment of certain debts. It is evident that a moratorium, such as is in operation at the time of writing, is of practical importance to Life Office officials, and may possibly give rise to some interesting legal decisions. It therefore seems advisable that a résumé of its terms should be placed on record in these Notes.

The first moratorium proclamation dated 2 August 1914, was of a partial nature, and applied only to Bills of Exchange, other than cheques, or bills on demand accepted before 4 August 1914. Payment on these on re-acceptance was postponed for one calendar month, provision being made for interest to be charged on the payment thus deferred.

This proclamation was confirmed by the Postponement of Payments Act, 1914, which Act also gave power for the postponement by Proclamation of any other payment in pursuance of any contract.

The Proclamation dated 6 August 1914 was of much wider scope, and applied to all payments which became due and payable before 6 August 1914, or which would become payable before 4 September 1914, in respect of (*a*) Bills of Exchange (being a cheque or bill on demand) drawn before 4 August 1914; (*b*) Negotiable Instruments (not being a Bill of Exchange) dated before that date; (*c*) Any contracts made before that date.

Such payments shall be deemed to be due and payable on a day one calendar month after the date on which the payment originally became due, or on 4 September 1914, whichever is the later date, provision being made for payment of interest if a specific demand for payment is made and payment is refused.

The following classes of payments were exempted from the operation of the Proclamation :

(1) Any payment in respect of wages or salary.

(2) Any payment in respect of a liability which when incurred did not exceed five pounds in amount.

(3) Any payment in respect of rates or taxes.

(4) Any payment in respect of maritime freight.

(5) Any payment in respect of any debt from any person resident outside the British Islands, or from any firm, company or institution whose principal place of business is outside the British Islands, not being a debt incurred in the British Islands by a person, firm, company, or institution having a business establishment or branch business establishment in the British Islands.

(6) Any payment in respect of any dividend or interest payable in respect of any stocks, funds, or securities (other than real or heritable securities) in which trustees are, under Section 1 of the Trustee Act, 1893, or any other Act for the time being in force, authorized to invest.

(7) Any liability of a bank of issue in respect of bank notes issued by that bank.

(8) Any payment to be made by or on behalf of His Majesty or any Government Department, including the payment of old age pensions.

(9) Any payment to be made by any person or society in pursuance of the National Insurance Act, 1911, or any Act amending that Act (whether in the nature of contributions, benefits or otherwise).

(10) Any payment under the Workmen's Compensation Act, 1906, or any Act amending the same.

(11) Any payment in respect of the withdrawal of a deposit by a depositor in a trustee savings bank.

A Proclamation dated 12 August 1914 extended the moratorium to Bills of Exchange, which had not been re-accepted under the Proclamation of 2 August, unless such re-acceptance was expressly refused, and also to payments in respect of any debt from any bank whose principal place of business is in any part of His Majesty's Dominions or any British Protectorate, although the debt was not incurred in the British Islands and the bank had no branch in the British Islands.

By a further Proclamation dated 3 September 1914, the period of deferment for the payment of Bills of Exchange re-accepted under the Proclamation of 2 August 1914, was extended from one calendar month to two calendar months, and the Proclamations of 6 August and 12 August were extended to apply to payments due and payable on or after 4 September 1914 and before 4 October 1914, whether they became so due and payable by virtue of the said Proclamations or otherwise. This is equivalent to extending the period of deferment by a calendar month.

It will be seen that the terms of the Proclamation of 6 August 1914 are very wide, and cover practically all payments, unless specifically excluded, which become due and payable within the specified period under contracts made before 4 August 1914. Life Offices are affected with regard to interest, rent and payments due to them in the United Kingdom in respect of their invested funds. Advantage could be taken of the moratorium in regard to the payment of claims and annuities.

At the time of writing, the question as to whether premiums under life policies are included under the moratorium has not been decided. The payment of a life assurance premium is a condition precedent to the continuance of the contract, but the company has no power to enforce payment of such premium. There are strong grounds for the opinion that, on a strict interpretation of the words of the Proclamation, it only applies to obligatory payments, that is, payments for the recovery of which the law gives the right to sue. If this opinion be confirmed, it would appear that, generally speaking, life assurance premiums are exempt from the operation of the moratorium.

On the other hand, the Postponement of Payments Act uses the very wide words : "postponement of payments in pursuance of any contract," and in view of the fact that the Proclamation was issued on the ground of public policy and in a time of emergency, it might be held to have a more liberal interpretation than that suggested, and to include non-obligatory payments in its scope.

Courts (Emergency Powers) Act.

The Courts (Emergency Powers) Act, 1914, 4 & 5 Geo. 5, Chapter 78, which is one of the measures recently passed to deal with the unusual situation brought about by the present war, is intended to prevent, to some extent, the harsh exercise of legal powers by creditors in certain cases.

The Act contains some remarkable provisions, and one in particular which will materially affect industrial life assurance companies. The important sections are as follows :

1.—(1) From and after the passing of this Act no person shall—

(*a*) Proceed to execution on, or otherwise to the enforcement of, any judgment or order of any Court (whether entered or made before or after the passing of this Act) for the payment or recovery of a sum of money to which this subsection applies, except after such application to such Court and such notice as may be provided for by rules or directions under this Act ; or

(*b*) Levy any distress, take, resume, or enter into possession of any property, exercise any right of re-entry, foreclose, realize any security (except by way of sale by a mortgagee in possession), forfeit any deposit, or enforce the lapse of any policy of insurance to which this subsection applies, for the purpose of enforcing the payment or recovery of any sum of money to which this subsection applies, or, in default of the payment or recovery of any such sum of money, except after such application to such court and such notice as may be provided for by rules or directions under this Act.

This subsection shall not apply to any sum of money (other than rent not being rent at or exceeding fifty pounds per annum) due and payable in pursuance of a contract made after the beginning of the fourth day of August nineteen hundred and fourteen.

This subsection applies to life or endowment policies for an amount not exceeding twenty-five pounds, or payments equivalent thereto, the premiums in respect of which are payable at not longer than monthly intervals, and have been paid for at least the two years preceding the fourth day of August nineteen hundred and fourteen.

(2) If, on such application, the Court to which the application is made is of opinion that time should be given to the person liable to make the payment on the ground that he is unable immediately to make the payment by reason of circumstances attributable, directly or indirectly, to the present war, the Court may, in its absolute discretion, after considering all the circumstances of

the case and the position of all the parties, by order, stay execution or defer the operation of any such remedies as aforesaid, for such time and subject to such conditions as the Court thinks fit.

(6) The powers given under this Act shall be in addition to, and not in derogation of, any other powers of any Court.

2.—(4) His Majesty may, by Order in Council, at any time determine the operation of this Act, or provide that the Act shall have effect subject to such limitations as may be contained in the Order; but, subject to the operation of any such Order in Council, this Act shall have effect during the continuance of the present war, and for a period of six months thereafter.

REVIEW.

The English Convict (a Statistical Study). By Charles Goring, M.D., B.Sc. Lond. Pp. 440.

Published by H.M. Stationery Office, 1913. Price 9s.

The official publication of an attempt to unravel statistics about criminals marks a great step forward; it is an effort to replace the untabulated "practical experience" of the partisan or theorist by a tabulated actual experience uninfluenced by personal bias. Sooner or later Dr. Goring's work will, we hope, be extended, and will be followed by similar official work on other social matters. When we gradually learn to expect such information from official sources we may become ashamed of holding definite opinions without definite evidence, and may find that we do not mind discarding cherished opinions when we see that they do not describe the facts that investigation reveals.

The work before us upsets a few opinions that are commonly held, and in one or two places is perhaps a little severe in its criticisms, but in the process of destruction. and also independently of it, much valuable information is recorded which ought to form the basis of a far more scientific criminology than the guesswork of the past has given us.

The first part of Dr. Goring's work is concerned with a criticism of the doctrine, preached by Lombroso and his followers, that the criminal is different in kind from the non-criminal. The criticism is based on an examination of the statistics obtained from 3,000 English male convicts, and relating to over thirty characters, which include head length, breadth and circumference, occipital projection, chin projection, inclination of nose, &c., and the conclusion reached is that any differences in the measurements of various classes of

criminals, or between the criminal and non-criminal, were so small as to betoken a quantitative, not a qualitative, difference. In reaching this conclusion allowance had to be made for many disturbing influences, such as age, stature, personal equations of different observers, occupation, class, health, &c.; the existence of crude physical differences before these influences were eliminated proved nothing as regards the relative measurements of criminals, but their elimination enables us to say that criminals are not physically differentiated because they are criminals, but because of differences in age, stature, intelligence, &c., and of different social classes from which they are drawn.

In the second part of the work we find information about the physique, fertility, age distribution, vital statistics, and mental differentiation of criminals, and the influence of force of circumstances and heredity on crime. We learn that the criminal has somewhat defective physique and mental capacity, and that alcoholism, venereal disease, epilepsy and insanity appear to be constitutional determinants of crime, but are apparently accidental associations depending on the high degree of relationship between defective intelligence and crime. On the other hand, such conditions as illiteracy, parental neglect, poverty, lack of employment, &c., seem to have no significant relationship with crime.

Imprisonment itself has no apparent effect on physical well-being; it has both a direct and indirect effect on the fertility of the criminal, but apart from this the criminal is not relatively sterile.

These, broadly, are the results of an extremely interesting and careful enquiry, but in reviewing the book for actuaries, we may make rather fuller mention of the chapters dealing with age and vital statistics than would otherwise be necessary. Considering first the remarks about the age distribution of criminals at first offence, according to the kind of crime committed, Dr. Goring gives full information of the age incidence, and deduces the following mean ages at the first conviction:

	Mean Age
General population, males over 15	37·50
Criminals convicted of—	
Damage to property	41·15
Stealing and burglary...	34·36
Sexual offences	34·05
Violation to the person	33·96
Forgery and fraud	41·68
All crimes	36·21

As a result of an examination of the figures from which these means are obtained, Dr. Goring suggests that the tendency to be convicted of serious crime for the first time seems to centre round three age periods, early manhood (20–25), a less pronounced middle age

period (35–45) and a transient period between 55 and 65. An interesting discussion of an unsuccessful attempt to fit a frequency curve to the age-population statistics of the habitual criminals is also given.

Turning now to Dr. Goring's remarks on the vital statistics of criminals, we find a comparison of the sickness experience of all inmates of State prisons from 1900 to 1909 with that of the Manchester Unity (A. W. Watson), which shows that the criminals have about ·08 weeks less sickness per annum per individual than the members of the Manchester Unity, although the latter are to some extent "select," and are probably drawn from a superior class to the criminal. This result affords evidence that the criminals are well looked after—one wonders, however, to what extent malingering affects the comparison: there is little doubt that it is present in both the experiences.

The causes of death of criminals show that whereas only 11 per 1,000 of the deaths in the general population are due to epilepsy and insanity, 26 per 1,000 of the deaths among prisoners are due to these diseases. Alcoholism, combined with a weak nervous system, and syphilis and its sequelæ, also account for a far larger proportion of deaths among criminals than among the general population. The statistics, however, do not support the theory that tuberculosis and crime are related. The prevalence of insanity among prisoners seems to show that crime and insanity are related, but rather more definite evidence is needed than that available.

Dr. Goring reaches a rather surprising result in his investigation into the mortality of the criminals, and gives a table of which the last two columns are shown below:

Mortality in Prisons and in General Population.

(*Males.*)

Ages	Expected Number of Deaths in Prisons per annum	Actual Number of Deaths in Prisons per annum
10–	·1	·1
15–	22·8	21·1
25–	34·1	34·1
35–	38·5	37·4
45–	33·1	30·3
55–	35·0	26·4
65–	22·5	13·9
75–	4·9	3·6
85–	1·1	·4
Totals	192·1	167·3

It is a little hard to follow Dr. Goring's method, and we think it may have slightly overestimated the expected number of deaths, but not sufficiently to be of any importance or to account for the large differences between the expected and actual deaths above age 50. Dr. Goring suggests as an explanation that only the robust are able to continue a criminal career in old age, and the inferior drift into workhouses, hospitals, &c.; in other words, the "prisoner" is to some extent a select life especially in old age. This explanation may also apply to the sickness rates which we have already mentioned.

The discussion of vital statistics closes with an attempt to enumerate the proportion of individuals in one generation who are imprisoned for crime at least once: the total criminal offenders amount to about 7·2 per-cent of the population; this percentage is chiefly made up of those convicted of stealing and burglary (4·2 per-cent), and violence against the person (1·7 per-cent). In addition to the criminal offences, a man may be convicted for drunkenness or for breach of municipal law, &c. (13·7 per-cent), so that the total offenders are about 21 per-cent of the population.

One or two terms, expressions, &c., in the chapter on vital statistics are open to criticism, but Dr. Goring's conclusions appear to be correct, and we will not dwell on these little blemishes lest we give them an exaggerated importance.

It may seem rather strange to review in our *Journal* an official publication about criminals, but the work is largely statistical, is thoroughly scientific, and contains much of interest to actuaries. It gives incidentally an excellent account in clear language of the methods of correlation and contingency, and in solving the intricate problems that arise it provides interesting examples of modern statistical method: in fact, it would be hard to give a student of the subject a better series of examples.

Anyone reading the book will be impressed by the clear and logical way in which the subject is treated; he will be pleased by many illuminating passages, and he will admire the energy and concentration that Dr. Goring has brought to bear with such success on the great task which the Commissioners entrusted to him.

W. P. E.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, 1914.

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Examination for Admission to the Class of Associate (Part I).

First Paper.

1. Solve the equations

(1) $4x^3-32x^2-x+8=0$, having given that the sum of two roots is zero.

(2) $x^4-7x^3+14x^2-7x+1=0$

2. One-ninth of the gross premium income of an office consists of reassurance premiums which must be deducted to arrive at the net premium income, but the proportion of reassurance premiums to gross premiums in the case of new business is $\frac{11}{5}$ times as high as in the case of renewal business. The total expenses are $12\frac{1}{2}$ per-cent of the total net premium income. If all the expenses are divided between new business and renewal business on the assumption that the ratio between the expenses on account of new premiums and the amount of the net new premiums is 5 times as great as the ratio between the expenses on account of renewal premiums and the amount of the net renewal premiums, those ratios are $37\frac{1}{2}$ per-cent and $7\frac{1}{2}$ per-cent respectively. The total gross premiums amount to £540,000.

Divide this amount into new and renewal premiums, and find the amount of the net new and renewal premiums, respectively, after deducting reassurances.

3. Calculate the value of $\log_{10}e$ correct to 5 places of decimals.

4. Give without proof, the expansion of $\log_e(1+x)$ in a series of ascending powers of x, and state for what range of values of x the expansion is permissible.

If $a+b+c=0$, show that

(1) $$a^6+b^6+c^6=\frac{(a^3+b^3+c^3)^2}{3}+\frac{(a^2+b^2+c^2)^3}{4}$$

(2) $$\frac{a^5+b^5+c^5}{5}=\frac{a^3+b^3+c^3}{3}\cdot\frac{a^2+b^2+c^2}{2}$$

5. Find the sum of the series

(1) $$\frac{13}{30}+\frac{13.18}{30.40}+\frac{13.18.23}{30.40.50}+\ldots \text{ to infinity}$$

(2) $4+5+8+16+36+84+\ \ldots$ to n terms.

6. A and B cut a pack of cards, the player who wins the cut six times to be the winner. A, having won four times to B's once, cuts a five. Find the chance that A will be the winner.

7. Two companies, A and B, make simultaneous issues each of 1,000 bonds. Those of company A are redeemable by equal drawings spread over 20 years, and those of company B by equal drawings spread over 40 years. Find, in the case of two definite bonds, one of each issue,

(1) The probability that the bond of company B will be redeemed before the bond of company A.

(2) The probability that the bond of company B will be redeemed before the bond of company A and within fifteen years of issue.

8. If a coin be tossed 1,000 times, how many times would you expect to get a sequence of exactly five heads?

9. If there are six ways of going from the Bank to the Marble Arch (2 by Tube, 3 by Bus, and 1 by Taxi), four ways of going from the Marble Arch to Waterloo (1 by Tube, 2 by Bus, and 1 by Taxi), and nine ways of going from Waterloo to the Bank (3 by Tube, 4 by Bus, and 2 by Taxi), in how many ways can the round journey be made:—

(*a*) using a Tube once, a Bus once, and a Taxi once?

(*b*) using a Bus at least twice?

(*c*) using a Tube at least as often as a Taxi?

Second Paper.

1. If $u_1=1$. $u_3=11$, $u_4=20$ and $u_{10}=4$, complete the series, assuming third differences are constant, and find the value of x which makes u_x a maximum.

2. Given that when $x=0,\ f(x)=0$

$$x=1,\ f(x)=100$$

$$x=2,\ f(x)=2000$$

find x when $f(x)=1900$ by Lagrange's formula of interpolation (applied inversely) and explain why the result does not agree with that found by using the formula $f(x)=(1+\Delta)^x f(0)$ and solving the quadratic.

3. Find formulæ, true to 3rd differences, for the bisection of an interval:

(1) In terms of the two nearest values of the function and of differences of the function;

(2) In terms of values of the functions only.

Apply either formula to find P_{35}, given

$$P_{20}=\cdot01313$$

$$P_{30}=\cdot01727$$

$$P_{40}=\cdot02392$$

$$P_{50}=\cdot03493$$

4. Given

$$f(1)=(x-2)(x-3)$$

$$f(2)=(x-7)(x-5)$$

$$f(3)=(x-10)(x+1)$$

$$f(4)=63$$

obtain a value for x, assuming second differences are constant.

5. Discuss the error in assuming that

$$\int_0^1 f(x)dx=f\left(\frac{1}{2}\right)=\frac{1}{2}\left(f(0)+f(1)\right)$$

if (1) $f(x)=a+bx+cx^2$

or (2) $f(x)=100\times 4^x$

6. Differentiate

(1) $a^x\left(\frac{x}{a}\right)^{ax}\log\frac{x}{a}$, with respect to $\log\frac{x}{a}$

(2) $\log\frac{xe^x}{e^x-1}$, with respect to x

(3) $x^{(\log x)^2}$, with respect to x.

7. Evaluate

(1) $\int x^n e^x dx$

(2) $\int_0 \frac{dx}{1-x^2}$

(3) $\int_0^1 (1-x^2)^n dx$

8. A point is taken at random within the area bounded by the curve $y=x\log x$, the x axis, and the ordinates at the points $x=1$ and $x=4$. Find the probability that the distance of the point from the y axis is less than 2.

Third Paper.

*1. Prove the relations

$$1+i=e^{\delta}$$

$$a_{\overline{n}|}^{(p)}=\frac{i}{j_{(p)}}a_{\overline{n}|}$$

Find the value at an effective rate of interest of 4 per-cent of a continuous annuity of 1 per annum for 25 years.

2. Find the present value of a perpetuity payable yearly, deferred n years, the successive payments of which are

$$1,\ \frac{1}{2},\ \frac{1}{3},\ \frac{1}{4},\ \ldots\ldots\ldots,$$

if the rate of interest diminishes in such a way that if i, i' represent the rates of interest in two successive years

$$\frac{i-i'}{(1+i)(1+i')}\text{ is constant.}$$

*3. The shareholders of a Railway receive guaranteed interest at the rate of 3 per-cent payable on the 30 June and 31 December

in each year, together with a dividend depending on profits payable on 31 December. Find the yield, expressed as a nominal rate of interest convertible half-yearly, to an investor who purchases on 1 January at the price of 95, on the assumption that future dividends will be at the rate of 1 per-cent.

*4. A bond for £100, redeemable on 31 December 1924, bears interest as follows:

> Up to and including 31 December 1919, 4 per-cent per annum payable half-yearly, and for the remainder of the term 5 per-cent per annum payable half-yearly.

Find the approximate yield to an investor who buys on the 28 February 1914 at the price of 109.

*5. An issue of £1,000,000 stock, bearing interest at $4\frac{3}{4}$ per-cent, payable annually, is to be redeemed by purchase in the open market by means of a cumulative sinking fund of £50,000 per annum. If the stock is issued at par, and purchases are made at the end of each year at a price of 95, find approximately the effective rate of interest paid by the borrowers on the whole transaction.

*6. Find, at 4 per-cent interest convertible half-yearly the value of a £100 debenture redeemable in $5\frac{1}{4}$ years at a premium of 4, and bearing interest at 5 per-cent per annum payable half-yearly. Construct a schedule showing the division of the half-yearly receipts between interest and repayment of capital.

*7. What annual premium per-cent should be charged for a capital redemption assurance for a term of 30 years, on the basis of interest at the rate of 4 per-cent per annum for the first 10 years, $3\frac{1}{2}$ per-cent per annum for the next 10 years, and 3 per-cent per annum for the last 10 years, assuming that 25 per-cent of the first premium and $2\frac{1}{2}$ per-cent of the renewal premiums will be absorbed in expenses?

*8. A syndicate wishes to take up an issue of £100,000 debentures, bearing interest at 5 per-cent payable annually and redeemable in 25 years by quinquennial drawings of £20,000. What price should it offer in order to realize 4 per-cent on its initial outlay during the entire 25 years and replace its capital at the end by the accumulations, at three per-cent, of the surplus receipts?

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Examination for Admission to the Class of Associate (Part II).

First Paper.

*1. Given that

$$l_0 = 60{,}000$$
$$l_1 = 50{,}000$$
$$l_2 = 47{,}000$$
$$l_3 = 45{,}500$$

Find as accurately as possible L_1 and hence show the error in p_1 if it is assumed that $p_1 = \dfrac{2 - m_1}{2 + m_1}$ where $m_1 = d_1 \div L_1$

2. If $\mu_x = ab^x + cd^x$ find expressions for l_x and q_x.

*3. Find by the H^M Table, in the case of two lives aged 24 and 22 respectively,

(*a*) the probability that one at least will die before attaining 40;

(*b*) the probability that the last survivor will die aged 40.

4. The probability that exactly one life out of three lives aged 20, 35 and 50, will survive 15 years is ·092; the probability that all will die within 15 years is ·006.

If ${}_{15}p_{20} = \cdot 9$, find ${}_{30}p_{20}$ and ${}_{45}p_{20}$.

5. In a provincial town, unaffected by migration, the number of births had increased by k per-cent per annum during the 20 years ending 31 December 1900. The education committee therefore provided school accommodation sufficient for the next 10 years, at the rate of 9 square feet for each child between the ages of 5 and 14, on the assumption that the number of births would continue to increase at the same rate.

On the 31 December 1910 it was found that the number of births had continuously fallen by k per-cent per annum during each of the preceding 10 years.

Find an expression for the excess accommodation at that date, assuming that the rate of mortality has remained unchanged throughout the whole period.

*6. A society issues each year 1,000 endowment assurances for £100 each on lives aged 40, 250 of the policies effected each year

maturing at age 55, 500 at age 60, and 250 at age 65. When the society reaches a stationary condition, what will be the total number of policies in force, and what sum will be paid each year (*a*) in respect of death claims, and (*b*) in respect of policies maturing? Give numerical answers, assuming that each year's new assurances are equally distributed throughout the year, that there are no withdrawals, and that the mortality is in accordance with the $O^{[NM]}$ Table.

7. Express in commutation symbols:

(1) The office annual premium for an assurance under which, in the event of death within n years, the office premiums paid are to be returned plus £10 in respect of each premium paid, while in the event of survival to the end of the n years the office premiums paid alone are to be returned;

(2) The corresponding net-premium policy-value after m years by the prospective and retrospective methods respectively.

Assume that the office premium is equal to the net premium increased by one-ninth.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Second Paper.

*1. A man aged 45 who wishes to effect a whole-life policy with an office which charges $O^{[NM]}$ 3 per-cent net premiums, is rated up 7 years. He offers to pay the ordinary premium for his true age and to deposit a sum of money with the office, which is to be forfeited by him in the event of his death during the first 25 years after entry, but to be returned to him at the end of the 25 years without interest, if he is then alive. What sum should he so deposit in respect of a policy for £1,000, on the basis of the $O^{[NM]}$ Table with 3 per-cent interest, assuming that the rate of mortality in each year of age may be taken to be the normal rate for a life 7 years older?

2. Three pensions of equal amount are paid out of a certain und. There is a "waiting list", containing the names in order of those who are entitled to subsequently become pensioners. As soon as a pensioner dies, the first man then alive on the "waiting list" will take his place; the second man then on the "waiting list" will become the first man on that list, and, if alive when the next death of a pensioner occurs, will take that pensioner's place, and so on.

How would you find the approximate value of the interest of the fourth man on the "waiting list"?

3. If Makeham's law hold, show that

$$\bar{A}_x = -\log s\,\bar{a}_x + (\mu_x + \log s)\bar{a}'_x$$

where $\bar{a}'_x$ is calculated at a rate j such that

$$j = \frac{1+i}{c} - 1$$

4. Show that

$${}_{n+t}V_{x:\overline{r}|} - {}_{n+t}V_x = [t + (1-t)vp_{x+n}](P_{x\overline{r}|} - P_x)\frac{N_{x-1} - N_{x+n}}{D_{x+n+1}}$$

where n is an integer and t a fraction.

5. Assuming that money is earning no interest, what is the value of an annuity of 1 on a life (x), with the provision that at the end of the year of death the excess (if any) of purchase money over the annuity payments shall be refunded.

6. If $\pi_{t\ x}$ represents the net yearly premium payable for t years to secure an annuity of 1, deferred t years, on a life aged x, with a return of the office premiums paid in the event of death within the t years, show algebraically that the policy-value (calculated on the same basis as the premium) after n years $(n<t)$ is equal to $\frac{\pi_{t|x}}{\pi_{n\ x}} \cdot a_{x+n}$, provided the office premium is obtained by adding a fixed percentage to the net premium, and give an interpretation of this result.

*7. A child's deferred assurance, subject to yearly premiums, is effected at age 5, the sum assured being payable in the event of death after age 21, and the premiums paid accumulated at 3 per-cent compound interest being returnable in the event of death before age 21. A reversionary bonus at the rate of 35*s.* per-cent on the sum assured is to be added at age 21 and a similar bonus yearly thereafter. Find the amount of paid-up policy which may be granted at age 30, just before payment of the premium then due, it being understood that the bonuses previously added will be reduced in the same proportion as the sum assured and that future bonuses will be reckoned on the reduced sum assured. Work on the H^M Table with 3 per-cent interest, and ignore the question of loading.

8. How would you calculate a table of net single premiums for reversionary annuities of the type $\mathring{\hat{a}}^{(4)}_{y|x}$ for all values of x and y, such that the difference between the two ages does not exceed 15 years

and that neither age is less than 20, using one mortality table for y, another for x during y's lifetime, and another for x after y's death?

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Third Paper.

1. Explain fully the methods you would adopt to construct and check tables of

$$A^{1}_{[x][y]} \text{ and } P^{1}_{[x][y]}$$

*2. A B proposes for an endowment assurance of £1,000 with profits for 30 years and is accepted at an increased annual premium. A B objects that the total amount he can pay under the contract is more than the sum assured. It is suggested that he should pay decreasing premiums of equal present value to the increased level premium, the first payment being £60. Assuming that the H^{M} 3 per-cent Table is used at age 35, calculate the amount by which the premium will decrease annually so that the maximum premiums he can pay will be £1,000, and find the increased level premium at which he was originally accepted.

*3. Find the annual premium by H^{M} 3 per-cent for an endowment assurance of £100 effected at age 30 and maturing at 60, with the additional benefit that the policyholder shall in no case receive less than £25 more than all the net premiums paid.

4. Show how to construct a table of annual premiums for whole-life assurances deferred n years, all premiums with compound interest at rate j to be returned in the event of death during the deferred period. Explain how the work may be facilitated by the construction of a special commutation column in the form of R_x.

5. A whole-life assurance is effected on a life (x) for a sum of £a with the proviso that £b will be paid at the end of n years, if the assured be alive at that time, £c at the end of $2n$ years and £d at the end of $3n$ years, the amounts so paid being deducted from the original sum assured.

Find the net annual premium and the reserve at the end of the period of n years immediately after payment of the first instalment of the sum assured.

What limitations must be placed on the values of b, c and d?

6. Show how the value of an endowment assurance at the end of $n+1$ years can be obtained from the value at the end of n years, and hence show how endowment assurances can be valued without grouping, if the mortality is constant at all ages.

7. Compare the policy-values of two tables which are based on rates of mortality such that the l_x column from 20 onwards, by one table, is equal to the D_x column at $2\frac{1}{2}$ per-cent by the other table.

8. How would you classify, for valuation purposes, endowment assurance policies by limited premiums? Draft forms of class-book suitable for the valuation of such policies.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Fourth Paper.

1. A life office collects the greater part of its premiums through branches. Each branch is debited at the beginning of each month with the total renewal premiums due through that branch in the month, and is debited with new premiums as soon as acceptances of new proposals through the branch are issued. Each branch renders an account in respect of each month's premiums six weeks after the end of the month, taking credit for any unpaid premiums, for any premiums paid direct to the head office, and for commission and expenses paid. The cash balance at the branches is remitted to the head office once a week. Give a short account of the books required to be kept by the head office to carry out these arrangements.

2. State what are the necessary entries in respect of the following transactions:

(1) Received a composition, amounting to £20, in respect of A B's agency balance, the whole of which had previously been written off as a bad debt.

(2) Received £100 in respect of a bond drawn at par, standing in the company's books at 105 per-cent

(3) The company cancels a policy on which it has made an advance of £100, which with accumulated interest now amounts to the full surrender-value, namely, £125.

3. Enumerate the assets which usually occur in the balance sheet of a life office other than invested funds and cash, and explain each carefully.

4. On 1 December 1911, a company purchased a rent charge of £100 per annum payable by 20 half-yearly instalments at an inclusive price of £798. 3*s.* 8*d.*, *i.e.*, on a $4\frac{1}{2}$ per-cent interest basis. On 1 February 1913, £750 is tendered and accepted for the redemption of the remaining instalments. What entries will have appeared in the company's books?

5. Explain the following terms:

Coupon.
Talon.
Free market.
Marking a bargain.
Contract note.
Floating charge.

6. Give a brief account of the items comprised in the Weekly Return issued by the Bank of England.

7. Explain the meaning of the following terms as applied to Stock Exchange securities?

(1) "Registered."
(2) "Inscribed."
(3) "Bearer."

In which of the above forms can the following securities be held?

$2\frac{1}{2}$ per-cent Consolidated Stock
India $3\frac{1}{2}$ per-cent ,,

8. Draft a form of register for giving details of all bonds and debentures held by a company.

Examination for Admission to the Class of Fellow (Part III).

First Paper.

1. Give a concise account of one only of the following:

(*a*) A recent investigation into the probabilities of marriage.
(*b*) A recent investigation into the probabilities of issue.
(*c*) The medico-actuarial investigation of the Actuarial Society of America.

2. How is the mean population for the various age-groups calculated in the construction of mortality tables from census returns? What are the limitations of and objections to the method you give? Can you suggest any improvement?

3. A very large office tabulates its policies for valuation according to nearest age at 31 December. A record is kept

according to office year of entry of the number of policies in force on each 31 December in each age group. The claim register gives the calendar year of entry and office valuation group. How would you obtain select rates of mortality from the facts so given?

4. Describe the basis and methods of graduation of the life tables adopted in computing reserve values for the purposes of the National Insurance Act, 1911. How do the values of q_x compare with the rates shown by the Manchester Unity Whole Society Experience, and English Life Table No. 6?

*5. From the following abstracts of a Society's experience for the five years ending 31 December 1913, calculate the expected deaths according to Table VIII of the Short Collection of Actuarial Tables:

Statement showing the number of members on 1 *January* 1909 *and the number of entrants in each of the years under review.*

Calendar year of Birth	Number of Members, 1 Jan. 1909	Number of Entrants					
		1909	1910	1911	1912	1913	Total
1848	224	48	52	52	60	62	274
1849	214	46	50	48	58	54	256
1850	208	40	46	48	52	50	236
1851	206	48	38	40	50	50	226
1852	194	38	40	36	44	48	206
1853	184	38	36	44	40	42	200
...	1,230	258	262	268	304	306	1,398

Statement showing the number of withdrawals and deaths in each of the years under review.

Calendar Year of Birth	Number of Withdrawals						Number of Deaths					
	1909	1910	1911	1912	1913	Total	1909	1910	1911	1912	1913	Total
1848	16	18	14	18	20	86	8	4	6	8	8	34
1849	14	14	16	16	18	78	8	6	4	8	10	36
1850	14	16	18	14	18	80	10	10	8	6	4	38
1851	14	14	16	16	16	76	8	8	10	6	8	40
1852	16	16	14	14	16	76	10	8	8	8	6	40
1853	14	16	18	12	14	74	10	10	10	10	8	48
...	88	94	96	90	102	470	54	46	46	46	44	236

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Second Paper.

1. In graduating mortality rates by summation formulæ what advantages are gained by graduating separately the exposed to risk and the deaths?

Mention the principal objections to graduation by means of summation formulæ.

*2. How would you graduate by Makeham's formula a table represented by the following facts? Find c approximately and mention an alternative method of finding c.

Age Group	Exposed to Risk	Deaths
30–	201	2
40–	603	6
50–	2,018	35
60–	5,070	135
70–	4,127	250
80–	977	150
90–	127	52
Total	13,123	630

3. Discuss concisely the provision to be made in valuations for the future profits and expenses of whole-life policies by single and limited premiums.

4. An office allows policies to be revived within twelve months of the expiration of the days of grace on payment of over-due premiums with a fine, but without medical examination. What reserves should strictly be kept for such policies in a valuation, and what in your view is the most practical way of dealing with these policies in a valuation?

*5. It has been stated that an office valuing at 3 per-cent, when it is making 4 per-cent, roughly provides for maintaining its compound reversionary bonus in future. Mention the limitations involved in such a statement and, after valuing the following endowment assurances which constitute the whole business of a company, say (1) how much surplus is available if the fund is £781,000, and (2) how much bonus you would declare. Use your valuation and the particulars to explain the limitations you have indicated in your answer.

In making your investigation you may assume that (1) the investments of the office realize £4. 0*s.* 10*d.* per-cent net, (2) the expenses are 15 per-cent of the premium income, (3) new business

is slowly increasing, (4) the office is proprietary and the shareholders take $12\frac{1}{2}$ per-cent of the divisible surplus, (5) bonuses are declared annually, (6) the policies are grouped according to calendar year of exit and the maturity ages are found by adding the number of years of the endowment term to the age next birthday at entry. State the formulæ that should be used for valuing sums assured and premiums.

NOTE.—As temporary life annuities are not published in the tables, premiums may be valued by $a_{\overline{n}|}$ at one per-cent higher rate than that required, and sums assured by v^n at one-half per-cent lower rate than that required.

Table of Endowment Assurances.

No. of Years of term unexpired	Sums Assured	Reversionary Bonuses	Office Premiums	Net Premiums 3 per-cent
5	430,000	100,000	26,000	20,000
15	880,000	110,000	43,000	35,000
25	550,000	41,000	19,500	16,000
35	150,000	2,000	3,500	3,000
Total	2,010,000	253,000	92,000	74,000

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Third Paper.

1. An office grants with-profit policies to officers in the army at the following annual extra premiums payable until retirement :

Officers in the Home Service ... ... (*a*) per-cent.
,, engaged on Foreign Service ... (*a*) + (*b*) per-cent.

At the option of the assured (*a*) may be foregone in consideration of the policy not participating in profits until retirement.

How would you treat these policies at a valuation?

2. An office gives a compound reversionary bonus to whole life policies and a simple reversionary bonus to endowment assurances. Describe a practical method of allocating the divisible surplus between the two sections.

*3. An office which values at H^M 3 per-cent allots its surplus upon a contribution method. It is able at each of a succession of

quinquennial valuations to allot its participating policies (1) excess interest at the rate of £1. 2*s.* 6*d.* per-cent per annum on the values of the policies and bonuses at the beginning of the quinquennium, and (2) 15 per-cent of the office premiums paid during the quinquennium. The cash apportionment is then turned into reversion at the valuation rate. Upon this basis, what would be the amount payable at maturity in respect of a 15-year endowment assurance, effected in the first year of a quinquennium, at age 35. Assume for the purposes of the question that the office premiums are the H^M 3 per-cent net premiums loaded 20 per-cent.

4. It is the practice of the office referred to in question (3) to allot interim bonus at the cash rate apportioned to the policy at the previous distribution. A proposer, aged 35, is considering taking out, in the 3rd year of a quinquennium, a 15-year endowment assurance, but raises the objection that he will lose in interim bonus as compared with a similar policy effected in the first year of a quinquennium. What are the relative advantages to the proposer of the two policies?

5. An office, giving a simple reversionary bonus, finds that a large percentage of its with-profit endowment assurances consists of reassurances from other offices carrying the bonus of the principal office. What points have to be taken into consideration in determining how to deal with these policies as regards reserve and allocation of surplus? What course would you adopt?

*6. From the following information of a mutual life office, analyze approximately the profit of the quinquennium:

Consolidated Revenue Account *for the five years from 1 January* 1909, *to* 31 *December* 1913.

Life Assurance Fund, 1 January 1909		...£8,000,000	Claims paid and outstanding ...	...£3,000,000
Premiums—			Surrenders	300,000
Annual	£4,150,000		Expenses of management & Commission	600,000
Single	50,000		Written off Investments	250,000
		4,200,000	Life Assurance Fund, 31 December 1913..	9,550,000
Interest, Dividends and Rents (less Income Tax)		1,500,000		
		£13,700,000		£13,700,000

The valuation at 31 December 1908 was made on the O^M 3 per-cent basis, and after providing for the bonuses then declared, there remained an undivided surplus of £50,000 which was carried forward unappropriated. The 1913 valuation, on the same basis, disclosed a surplus of £600,000, and, in addition, £30,00 was paid as interim bonuses on claims. The office rates are practically

O^M 3 per-cent net rates with the following loadings: annual premiums, 25 per-cent; single, 5 per-cent. Surrender-values are allowed after payment of one premium and are practically two-thirds of the O^M 3 per-cent values. The growth of the office has been steady.

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Fourth Paper.

*1. An office has on its books a large number of special endowment assurances with and without profits, maturing at age 60, under which the payment of the sum assured (and bonuses) is postponed till the death of the life assured, and in the meantime 5 per-cent is paid each year on the total amount so postponed. The premiums were fixed in 1890 and have not altered since.

The following particulars show the position of the office:

Date	1890	1913
Valuation basis	H^M and $H^{M(5)}$ 3%	O^M 3 %
Compound reversionary bonus...	25/-	30/-
Net rate of interest earned ..	£3. 15/- %	£4. 1*s*. 6*d*. %
Expense ratio	16·5	14·5

New business from 1890 to 1913 stationary.

Discuss with the help of a numerical example:

(1) Whether the premiums require alteration;

(2) What reserves should be made.

*2. An insurance office decides to grant endowment assurances for twenty-year terms at ages 20 to 40 with guaranteed simple reversionary bonuses of £2 per-cent per annum. What basis for the calculation of premiums would you recommend if (1) the office gives a 30/- per-cent compound reversionary bonus to its with-profit policies, (2) the with-profit premiums are roughly $O^{[NM]}$ 3 per-cent plus $17\frac{1}{2}$ per-cent, (3) the non-profit premiums are $O^{[NM]}$ 3 per-cent plus 5 per-cent, and (4) the office earns $4\frac{1}{4}$ per-cent gross interest on its funds?

3. State the bases on which you would calculate office annual premiums for the following benefits:

1. Reversionary annuities.
2. Whole life assurances with profits by limited payments.
3. Joint life assurances with and without profits.

Mention any special points that you would bear in mind in fixing the bases.

4. How would you calculate a scale of decreasing debts in lieu of extra? Mention the assumptions your method makes as regards extra mortality.

For what classes of risks, if any, do you consider (1) a decreasing debt, (2) a level debt, a suitable method of extra rating?

*5. How would you calculate premiums for the following?

(1) Pure endowments at 21 with return of premiums in the event of the child's death, the premiums to cease on death of the parent. Work out a rate by taking the parent as 35 and the child as 5.

(2) Deferred annuities to female lives with return of all the premiums paid in the event of prior death.

6. What data would you employ if you were called upon to calculate scales of premiums for assurances on lives of Europeans resident in India?

A policy is effected on a life resident in India, the rate of premium and reserves being calculated by means of tables based on Indian mortality. After n years the life comes to England and the premium is reduced to the ordinary English rate for the original age at entry. On the assumption that the life assured will thereafter be subject to the ordinary English mortality, what must be the relation of the annuities-due under the Indian and English mortality tables to make this correct?

7. What extra premiums would you quote for the following, and what information would you require before accepting the cases?

(1) A civil servant in the Sudan who may work anywhere in the Sudan, but is unlikely to be stationed in any one place for more than a year.

(2) A person who states that he is resident in Nairobi.

(3) A resident in Rhodesia.

(4) A "publican."

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

Examination for Admission to the Class of Fellow

(Part IV.)

First Paper.

1. What do you understand by the term "contracts *uberrimæ fidei*?" Give some examples and mention the chief characteristics of such contracts.

2. Is an insurance company responsible and, if so, to what extent, for the due payment of estate duty in respect of the amount payable under a policy which has become a claim by death?

3. What is meant by constructive notice, and how may priority as between a legal and equitable mortgagee be affected by the doctrine of constructive notice to the lender or his solicitor?

4. In what forms as regards grantee and persons entitled to exercise options are children's deferred assurances granted? What are the objections to each form? Can anyone borrow on the policy or surrender it before or after the deferred age, and, if so, who? Set out your answer concisely.

5. Give a short account of the law relating to insurable interest. What modifications of the law were made by the Friendly Societies Acts, and the Assurance Companies Act, 1909?

6. State in what ways a mortgage of freehold or leasehold property may be transferred. Is it necessary for the mortgagor to join in the transfer? Give reasons.

7. Discuss briefly the respective rights and duties of executors and administrators. What differences exist between the offices of executor and administrator? Under what circumstances is interest payable on a legacy?

8. What constitutes a contract?

"A" makes a proposal for assurance and is accepted. Whilst still in good health he posts a remittance of cash to the office, and immediately afterwards (before his letter is delivered) is killed in a street accident. Is the office liable?

Would your opinion be altered if the remittance for the first premium had been by cheque?

Second Paper.

1. State concisely the distinctive features of collecting friendly societies.

2. A friendly society, in addition to sickness benefits, insures to each member a fixed sum on the death of his first wife, and a sum of varying amount on the death of each child. It also grants to widows of members (by first marriage) the right to continue the insurance of the death benefit on payment of a nominal subscription. State what formula you would use in calculating the premiums to be charged for

these benefits (in addition to the contribution for the sickness benefits and expenses of management) and how the liability should be provided for in a valuation.

3. A friendly society has a state section for both men and women insured under the National Insurance Acts, and a voluntary section for men only, admission to the society being limited to those employed in a certain occupation. The committee wish to add a voluntary section for the women clerks, giving sickness and old age benefits, and you are asked to advise them generally on this subject. To what points would you specially call their attention, and what general recommendations would you make?

4. State the relative merits of

(*a*) the reserve system,

(*b*) assessment system,

as applied to any scheme of State insurance.

Discuss the methods adopted under the National Health Insurance and Old Age Pension Schemes in this country.

5. A chartered professional institute with 600 members and an accumulated fund of £10,000 proposes that a portion of this fund be utilized to start a widows' fund, the widow of each member receiving an annuity of £50, and the annual subscriptions being increased by £5. 5*s*. for bachelor members and £10. 10*s*. for married members. The promoters desire you to report whether the scheme is practical, and what additional subscriptions should be imposed on new entrants either in the form of an entrance fee or marriage tax.

State what particulars you would require to enable you to report, and the lines on which you would proceed with the investigation.

6. Set out concisely (*a*) what classes of assurance can be accepted without medical examination; (*b*) the reasons leading to your conclusion in each case; (*c*) what enquiries should be made and questions asked; and (*d*) if a general rule is made not to ask for examinations for any class of assurance, in what exceptional circumstances (if any) you would ask for such an examination.

7. An office on valuing its securities finds a serious depreciation in the values. Two methods of dealing with the matter are suggested: (1) To set up an "Investment Reserve Fund"; (2) To write down the book values. If either method necessitates a reduction in the rate of bonus, would you, in order to maintain previous bonus rates, recommend an increase in the valuation rate of interest? State which method you prefer and why? Explain clearly the book entries necessary to give effect to your recommendations.

8. A life office is considering the question of transacting sinking fund business. Write a report to the directors dealing with the desirability or otherwise of commencing the business, giving bases of rates and suggesting provisions as to surrender values and policy conditions.

9. Draft a form of proposal for an assurance against the risk of issue to A (who is now married) by his present or any future wife. What additional enquiries would you make?

Third Paper.

1. State briefly the causes which, in your opinion, have led to the depreciation in the market values of securities during the year 1913.

2. What information would you require before lending, and on what terms would you lend at the present time, on:

(*a*) Landed property in good agricultural districts?

(*b*) Leasehold shop and office property in large provincial cities?

(*c*) Small suburban residential property?

3. What are the objections to making loans by way of (*a*) sub-mortgages, (*b*) contributory mortgages? How can the objections be overcome by insurance offices?

4. Give your opinion on the following as investments for a Life Fund, and state what approximate rate of interest you would expect to receive under each class:

(*a*) Trust Company Debentures.

(*b*) American Municipal Bonds.

(*c*) Mortgage of High-class Office Property in a large Canadian City.

(*d*) Short term notes such as

(1) Kansas City Terminal Railway Company 3 years 5 per-cent secured sterling notes.

(2) Canadian Northern Railway Company 5 years 5 per-cent secured notes.

5. Give a short description of the various classes of securities included under the term Public Utility Bonds. What points in each class would you investigate when considering their suitability for investments of a life office.

6. Discuss the salient points of difference between the methods adopted for financing English and American railroads. An English railway company proposes to issue terminable Debenture Stock. Give your opinion on such a course.

7. Discuss the monetary situation in Mexico during the recent crisis, and its effect upon English trading companies operating there.

8. How are British insurance companies assessed for income tax (*a*) imposed by the British Government, (*b*) by the United States Government, in respect of income arising from investments made in the United States?

Fourth Paper.

1. How would you deal, as regards commutation of premium, paid-up policy and surrender-value, with policies that have been taken out at an increased premium equivalent to an addition to the age at entry?

*2. The undermentioned fund is left in trust to provide annuities of £200 and £300 to ladies aged 53 and 58 respectively, any balance of income being distributed among the reversioners. A B is entitled to one moiety of the fund subject to these annuities. Value his interest:

£4,000 Great Central $3\frac{1}{2}$ per-cent Second Debenture Stock (quoted at 82-84).

£200 per annum East Indian Railway "C" Annuity.

£3,000 Western Australia 4 per-cent Inscribed Stock.

£4,000 on mortgage of house property at $4\frac{1}{2}$ per-cent.

*3. The reversion to the following property, expectant on the death of a man aged 60, is offered for sale. At what price would you value it?

Well secured freehold ground rents amounting to £262. 10*s.* per annum on 15 houses in a good residential district near London, with reversion to rack rentals in 1969.

An annuity bond in a good office for £30 a year, payable by half yearly instalments until 1969 inclusive.

4. It is wished to raise the sum of £20,000 to be repaid by way of a reversionary charge secured on the interest of the borrower, aged 25, as tenant in tail in remainder of estates of ample value, subject to the successive life interests of a lady aged 73, and of the borrower's father, aged 50.

Indicate the methods you would employ to calculate the amount of the necessary reversionary charge,

(1) If the father will assist the borrower in every way and charge his life interest,

(2) If the father will not help the borrower in any way.

*5. Advances have been made to A B, who was born in January 1859, on security of the reversionary interest to which he is entitled contingently on himself or any of his children surviving a lady born in December 1833, and of fully paid policies for £7,200, payable in the event of the borrower and all his issue predeceasing the tenant for life. Capitalization of interest was provided for until April 1914, and the accumulated advances amount to £5,100. It is asked that a further £500 be now advanced, and capitalization of interest on the existing and new advances be allowed for a further five years. Calculate the amount of the further policy that should be set up, assuming that it may be effected at a single premium of £2. 2*s*. per-cent, the single premium being advanced by the lender.

*6. A B wishes to sell his one-half share in a trust fund invested in

£15,000 Consols,

£10,000 Midland Railway $2\frac{1}{2}$ per-cent Debenture Stock,

expectant on the death of a married woman aged 60 whose husband is aged 70, and subject to deduction of £5,000 if the tenant for life leaves a husband surviving her.

What would you advise should be given for the reversion?

*7. A B, aged 38 next birthday, is entitled to the income from the following fund:

Well-secured leasehold ground rents with 30 years to run, producing £500 per annum.

30 recently built freehold suburban houses in a newly developed district, let at £50 a year each, on 3-years agreements.

£10,000 Consols.

£5,000 Great Central 5 per-cent Preference Stock, 1894.

£5,000 Midland Railway $2\frac{1}{2}$ per-cent Preference Stock.

How much would you lend and on what conditions? What enquiries would you make before lending?

* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

PROCEEDINGS OF THE INSTITUTE.—Session 1913-1914.

First Ordinary Meeting, 24 *November* 1913.

The President (Mr. Frederick Schooling) in the Chair.

A paper entitled "Approximate Valuation of Endowment Assurances," was read in abstract by the Author, Mr. W. Palin Elderton.

The following gentlemen took part in the discussion:—Messrs. W. Penman, D. C. Fraser, R. E. Underwood, J. Bacon, A. Levine, R. Todhunter, H. E. Raynes, N. Blanchard, H. J. Rietschel, and H. J. Baker.

Second Ordinary Meeting, 22 *December* 1913.

The President (Mr. Frederick Schooling) in the Chair.

Messrs. William Lindsay Crawford, F.F.A., Alexander Mackintosh Hogg, F.F.A., and George Watson Paterson, F.F.A., were duly elected Associates of the Institute.

A paper entitled "Whole Life Non-Profit Assurances" was read in abstract by the Author, Mr. Finlay J. Cameron.

The following gentlemen took part in the discussion:—Messrs. H. J. Reitschel, C. R. V. Coutts, E. C. Thomas, C. C. Monkhouse, V. Marr, J. Mayhew Allen, N. J. Carter, N. Blanchard, C. H. Ashley, E. J. Hancock, and S. G. Warner.

Third Ordinary Meeting, 26 *January* 1914.

The President (Mr. Frederick Schooling) in the Chair.

A paper entitled "On the extension of existing Valuation Methods of grouping Policies by the employment of a system of weights," was read in abstract by the Author, Mr. A. E. King.

The following gentlemen took part in the discussion:—Messrs. H. E. Melville, S. E. Macnaghten, A. Henry, R. E. Underwood, W. Palin Elderton, G. J. Lidstone, and F. T. Mason Byers.

Fourth Ordinary Meeting, 23 *February* 1914.

The President (Mr. Frederick Schooling) in the Chair.

Messrs. David Malcolm Carment, B.A., F.F.A., James Hogg, F.F.A., and Colin Strathern Penn, F.F.A., were duly elected Associates of the Institute.

A paper entitled "On the Valuation of Benefits dependent upon promotion to a higher status," was read in abstract by the Author, Mr. E. C. Thomas.

The following gentlemen took part in the discussion:—Messrs. L. E. Clinton, Thomas G. Ackland, J. Bacon, T. Tinner, G. M. Reeve, and V. Marr.

Fifth Ordinary Meeting, 30 *March* 1914.

Mr. Geoffrey Marks (Vice-President) in the Chair.

Messrs. James Wilson More, F.F.A., and David Goldie Young, F.F.A., were duly elected Associates of the Institute.

A paper entitled "On the Treatment of the Depreciation in Assets due to an enhanced rate of interest," was read in abstract by the Author, Mr. R. R. Tilt.

The following gentlemen took part in the discussion:—Messrs. L. A. Mouat Jones, E. W. Phillips, H. J. Rietschel, E. B. Nathan, S. G. Warner, H. J. P. Oakley, and the Chairman.

Sixth Ordinary Meeting, 27 *April* 1914.

The President (Mr. Frederick Schooling) in the Chair.

A paper entitled "Section 72 of the National Insurance Act. Some other features of Friendly Societies and National Insurance, including a note on the proposed Belgian National Insurance Act," was read in abstract by the Author, Mr. E. B. Nathan.

The following gentlemen took part in the discussion:—Sir Gerald H. Ryan, Messrs. R. G. Maudling, J. Bacon, W. O. Nash, V. A. Burrows, and J. Burn.

The Sixty-Seventh Annual General Meeting, 8 *June* 1914.

The President (Mr. Frederick Schooling) in the Chair.

The proceedings at the Annual General Meeting will be found on page 457.

REPORT, 1913–1914.

The Council have the pleasure to report to the Members upon the progress of the Institute during the Session of 1913–1914, the sixty-sixth year of its existence.

There has been an *increase* of 2 in the total number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by Royal Charter the number of Members was 434; twenty years later, at 31 March 1905, it was 881. Since that time the numbers have been as follows:

On 31 March	Fellows	Associates	Students	Honorary and Corresponding Members	Total
1906	232	301	367	22	922
1907	248	303	383	22	956
1908	253	313	421	22	1,009
1909	254	325	400	19	998
1910	259	335	348	21	963
1911	267	339	308	20	934
1912	278	354	268	20	920
1913	282	355	252	19	908
1914	295	358	238	19	910

The following schedule shows the additions to, and the changes and losses in the membership which have occurred during the year ending 31 March last:

Schedule of Membership, 31 *March* 1914.

	Fellows	Associates	Students	Corresponding Members	Total
i. Number of Members in each class on 31 March 1913	282	355	252	19	908
ii. Withdrawals by					
(1) Death	2	3	3	...	47
(2) Resignation or otherwise	1	9	29	...	
	279	343	220	19	861
iii. Additions to Membership					
(1) By Election	...	8	...	...	49
(2) By Order of Council	...	...	30	...	
(3) By Re-instatement	...	3	8	...	
	279	354	258	19	910
iv. Transfers					
(1) By Examination:					
from Associates	...	12	...	...	...
to Fellows	12	...	...	...	...
	291	342	258	19	910
(2) By Examination:					
from Students	...	...	4	...	...
to Fellows	4	...	...	...	...
	295	342	254	19	910
(3) By Examination:					
from Students	...	...	16	...	...
to Associates	...	16	...	...	...
v. Number of Members in each class on 31 March 1914	295	358	238	19	910

There are also 200 candidates admitted as Probationers, and 67 as Students conditionally on their passing Part I of the Examination. These are not included in the above Schedule of Membership. The numbers in these two classes since 31 March 1908 have been as follows:

On 31 March	Probationers	Conditional Students
1909	128	39
1910	141	42
1911	160	58
1912	181	59
1913	197	55
1914	200	67

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of one Fellow, Mr. C. J. Harvey; three

Associates, Messrs. D. Burke, J. A. Humphreys, and C. Stevenson; and three Students, Messrs. G. C. Reynell, A. J. Welman, and R. H. Younger.

The Annual Subscriptions, together with admission and other fees, amounted to £2,527. 17*s.* 6*d.*, as compared with £2,491. 2*s.* 6*d.*, received in the previous year. The net Income and Expenditure for the year were £2,551. 9*s.* 7*d.*, and £2,131. 7*s.* 1*d.* respectively.

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
23,712	Parts of *Journal*.
748	Index to Vols. 1 to 40.
64	*Text-Book*, Part I (New Edition).
280	,, Part II (Second Edition).
641	Government Joint-Life Annuity Tables.
740	Select Life Tables.
484	A Short Collection of Actuarial Tables (New Edition).
1,204	Frequency-Curves and Correlation (W. P. Elderton).
10	Messenger Prize Essay (Friendly Societies).
40 *in cloth* 2,343 *in paper*	Lectures on Finance and Law (Clare and Wood Hill).
1,535	Lectures on the Companies Acts (A. C. Clauson).
1,229	Lectures on the Law of Mortgage (W. G. Hayter).
723	Lectures on the Measurement of Groups and Series (A. L. Bowley).
1,482	Lectures on the Construction of Tables of Mortality, &c. (Sir G. F. Hardy, K.C.B.).
993	Lectures on Stock Exchange Investments (J. Burn).
1,550	Lectures on Friendly Society Finance (A. W. Watson).
330	South African War Mortality (F. Schooling and E. A. Rusher).
282	Life Assurance Law (A. R. Barrand).
689	British Offices' Valuation Tables.
648	Transactions of the Second International Congress of Actuaries.
850	Index to Transactions of Seven International Actuarial Congresses.
1,500	Examination Questions, 1910–13.

The following papers were submitted at the sessional meetings of the Institute, namely:

24 *November* 1913.—"Approximate Valuation of Endowment Assurances."—Mr. W. Palin Elderton.

[*Continued on page* 454.]

Dr. *Revenue Account for th*

1913. £ s. d.		£ s. d.	1914. £ s. d
	Amount of Funds at the beginning of the year—		
10,025 0 4	General Fund (including Stock of Publications, other than *Journal*)	9,523 7 9	
374 5 11	Messenger Legacy Fund	385 10 6	
321 11 9	Brown Prize Fund	331 4 8	
10,720 18 0			10,240 2 1
	Subscriptions—		
838 19 0	Fellows	873 12 0	
714 0 0	Associates	720 6 0	
276 3 0	Students	258 6 0	
107 12 6	Probationers	109 4 0	
1,936 14 6		1961 8 0	
31 10 0	*One Annual Subscription compounded*	...	
8 8 0	Fines on Reinstatement	6 6 0	
1,976 12 6			1,967 14
	Application Fees—		
2 2 0	Associates	12 12 0	
16 5 6	Students	42 0 0	
41 9 6	Probationers	42 0 0	
59 17 0			96 12
147 18 11	Balance of Publications Account	....	127 1 1
	Dividends and Interest—		
324 10 7	General Fund	338 11 8	
11 4 7	Messenger Legacy Fund	11 11 3	
9 12 11	Brown Prize Fund	9 18 9	
345 8 1			360 1
£13,250 14 6			£12,791 12

Publications Account for th

£ s. d.		£ s.
371 19 8	Stock (excluding *Journal*) at the beginning of the year	343 8
140 11 8	*Expenditure on Account of A. W. Watson's Lectures on Friendly Societies*	...
...	Cost of Index to Transactions of Seven International Actuarial Congresses	31 18
43 8 9	Cost of Reprint of Text-Book, Part I	23 2
6 14 0	Binding and Advertising	20 1
147 18 11	Balance	127 1 1
£710 13 0		£545 13

Balance Shee

£ s. d.	LIABILITIES.	£ s. d.	£ s. d.	£ s.
9,523 7 9	General Fund		9,922 0 3	
233 9 2	Messenger Legacy Fund	233 9 2		
152 1 4	Accumulated Dividends	163 12 7		
385 10 6			397 1 9	
200 0 0	Brown Prize Fund	200 0 0		
131 4 8	Accumulated Dividends	141 3 5		
331 4 8			341 3 5	
				10,660 5
80 17 0	Examination Fees for year 1914			122 17
50 10 4	Sundry unpaid Accounts			47 6
£10,371 10 3				£10,830 8

year ending 31 *March* 1914. **Cr.**

1913. £	s.	d.		1914. £	s.	d.	£	s.	d.
			Journal—						
523	*11*	*10*	Printing of Nos. 253-4, 255, 256, 257	707	12	4			
86	*5*	*0*	Editorial Expenses	86	5	0			
609	*16*	*10*		793	17	4			
260	*12*	*8*	*Less* Sales during the year	323	14	0			
349	*4*	*2*					470	3	4
62	*14*	*8*	Library—Binding, Purchases, &c.				67	15	3
66	*11*	*9*	Meetings				76	10	8
357	*10*	*1*	Examination charges	334	1	7			
208	*19*	*0*	*Less* Fees received from Candidates (1913)	214	4	0			
148	*11*	*1*					119	17	7
329	*14*	*0*	Tutors for classes in Parts I and II	333	7	6			
245	*14*	*0*	*Less* Fees received from Students	249	7	6			
84	*0*	*0*					84	0	0
600	*0*	*0*	Office Expenditure—Rent	600	0	0			
404	*2*	*1*	Salaries	393	14	0			
72	*10*	*2*	House expenses	78	8	3			
16	*0*	*6*	Corporation Duty	17	1	2			
30	*8*	*2*	Fire and other Insurance	30	9	11			
133	*4*	*5*	Stationery and Printing	134	18	2			
45	*12*	*3*	Postage and Telegrams	44	3	5			
3	*8*	*1*	Sundries	4	5	4			
1,305	*5*	*8*					1,303	0	3
...			Donation to Napier Tercentenary Celebration				10	0	0
994	*4*	*3*	*Amount written off in respect of decrease in value of Stock Exchange Securities*				...		
10,240	*2*	*11*	Amount of Funds at the end of the year as per Balance Sheet				10,660	5	5
£13,250	*14*	*6*					£12,791	12	6

Examined and found correct, 30 *April,* 1914.
H. LUCEY,
W. G. TITMUSS, } *Auditors.*
ARTHUR TAYLOR,

year ending 31 *March* 1914.

£	s.	d.		£	s.	d.
367	*4*	*4*	Sales (excluding *Journal*)	292	16	7
343	*8*	*8*	Stock (excluding *Journal*) at the end of the year	252	16	6
£710	*13*	*0*		£545	13	1

Examined and found correct, 30 *April,* 1914.
H. LUCEY,
W. G. TITMUSS, } *Auditors.*
ARTHUR TAYLOR,

31 *March* 1914.

£	s.	d.	ASSETS.	£	s.	d.
2,370	*0*	*0*	£3,000 Natal 3 per-cent Inscribed Stock	2,370	0	0
1,008	*0*	*0*	£1,200 Metropolitan Railway 3½ per-cent Debenture Stock	1,008	0	0
1,980	*0*	*0*	£2,000 Great Eastern Railway 4 per-cent Debenture Stock	1,980	0	0
835	*0*	*0*	£1,000 Great Northern Railway Preferred Ordinary Stock	835	0	0
1,431	*0*	*0*	£1,350 Great Western Railway 4¼ per-cent Debenture Stock	1,431	0	0
465	*0*	*0*	£1,000 Dominion of Canada 3½ per-cent Registered 1930-50 Stock	937	8	0
890	*0*	*0*	£1,000 New South Wales 3½ per-cent Inscribed 1930-50 Stock	890	0	0
...			£600 Belgian Government 3 per-cent Sterling Loan of 1914	480	16	0
343	*8*	*8*	Stock of Publications (excluding *Journal*) in hand	252	16	6
350	*0*	*0*	*Cash on Deposit Account*	...		
699	*1*	*7*	Cash on Current Account and in hand	645	8	0
£10,371	*10*	*3*		£10,830	8	6

Examined and found correct, 30 *April,* 1914.
H. LUCEY,
W. G. TITMUSS, } *Auditors.*
ARTHUR TAYLOR,

22 *December* 1913.—"Whole Life Non-Profit Assurances."—Mr. Finlay J. Cameron.

26 *January* 1914.—"On the extension of existing Valuation Methods of grouping Policies by the employment of a system of weights."—Mr. Albert E. King.

23 *February* 1914.—"On the Valuation of Benefits dependent upon promotion to a higher status."—Mr. Ernest C. Thomas.

30 *March* 1914.—"On the Treatment of the Depreciation in Assets due to an enhanced rate of interest."—Mr. R. R. Tilt.

27 *April* 1914.—"Section 72 of the National Insurance Act. Some other features of Friendly Societies and National Insurance, including a note on the proposed Belgian National Insurance Act."—Mr. Eric B. Nathan.

For the Examinations held in the United Kingdom and the Colonies on 20, 21, 22, 23, 24 and 25 April 1914, 277 entries were received, namely:

131	for	Part	I.
8	,,	,,	I, (§) 3.
55	,,	,,	II.
53	,,	,,	III.
30	,,	,,	IV.

The results will be duly announced. The Council warmly acknowledge the valuable services of the Board of Examiners, and also those of the Honorary Supervisors at centres other than London.

The Council have to record their thanks to Mr. R. Todhunter and Mr. J. Spencer for their services as Joint Editors of the Journal during the past year. They regret that Mr. Barrand has found it necessary, owing to increasing demands on his time, to discontinue the contribution of the Legal Notes to the Journal. In contributing these Notes Mr. Barrand has most usefully supplemented his valuable papers on Legal Aspects of Life Assurance Practice, and the Council desire to record their appreciation of his services to the Institute in this connection. The Notes are being continued by Mr. W. C. Sharman.

There has been published during the year a Combined Index to the Transactions of the Seven Actuarial Congresses held in the period 1895 to 1912. The Council believe that this Index, by facilitating reference to the many valuable papers included in the Transactions, will be of considerable service to the profession. The compilation of the Index was suggested, and has been effected by Mr. Jarvis, the Assistant Secretary of the Institute.

The Council have, by resolution, conveyed to Sir George Francis Hardy, K.C.B., their congratulations on the signal honour conferred upon him by His Majesty. The terms of the Resolution appeared in the April number of the *Journal*.

THE INSTITUTE OF ACTUARIES.

EXAMINATIONS, 1914.

Examinations were held on the 20th, 21st, 22nd, 23rd, 24th, and 25th of April 1914, in the United Kingdom, the Colonies, and India, at London, Liverpool, Edinburgh, Dublin, Melbourne, Sydney, Perth, Wellington, Dunedin, Montreal, Toronto, Ottawa, Winnipeg, Singapore, Bombay, Calcutta, with the following results.

The successful candidates are placed in two classes only, the names being printed in alphabetical order in each class.

Part I.

One hundred and thirty-one candidates sent in their names, of whom one hundred and twenty-one presented themselves (ninety-nine in the United Kingdom, and twenty-two in the Colonies), and thirty-eight passed, namely:—

Class I:

Brown, S. P.
Mukerji, N.

Class II:

Beardsworth, R. E. L.
Bolton, H. A.
Britton, C. E.
Bryson, S. F. A.
Cann, F. E.
Carter, G. L. L.
Childe, E. R.
Christopher, R. C.
Dicken, H.
Fielden, G. S.
Garland, W. E.
Grantham, G. H.
Hammond, H. J.
Hocking, W. S.
Houseman, D.
Klagge, O. C. J.
Little, R.
Lobb, M. E.
Lochhead, R. K.
McConney, E. M.
Maddex, G. H.
Majumder, N. K.
Middleton, T.
Miles, G.
Mills, H. J.
Mytton, S. H.
Rowland, F. S.
Rutherford, C. D.
Scholfield, C. N.
Segal, M.
Shaw, E. H.
Sheehan, P. F.
Thompson, F. A.
Tope, W. S.
Watson, H.
Woodhead, F. T. C.

Part II.

Fifty-five candidates sent in their names, of whom fifty presented themselves (thirty-nine in the United Kingdom, and eleven in the Colonies), and twenty-one passed, namely:—

Class I:

Haalmeyer, B. P.
Holmes, H.
Moore, W. R.
Woffindin, R. H.

Class II:

Andras, J. B.
Atkins, F. C.
Barrett, C. C. C.
Blake, W. T. C.
Chase, P. C.
Chatham, E. F.
Corble, E.
Gostelow, C.
Horner, B. S.
Johnson, A. N.
McConaghy, C. A.
Menzler, F. A. A.
Morton, F. W.
Mukerji, N.
Segal, M.
Tutill, H. L.
Weyer, D.

Part III.

Fifty-three candidates sent in their names, of whom forty-nine presented themselves (forty-five in the United Kingdom, and four in the Colonies), and twenty passed, namely :—

Class I:

Evans, A. W.

Class II:

Brenton, W. P.
Brown, B. G. H.
Drake, C. C. H.
Edwards, A. J. C.
Fielder, T. L.
Gopp, J. I.
Hogg, J.
†Hudson, A. J.
Hurley, J. C.
Hustwitt, W. E.
Jackson, H. M.
Keable, H. B.
More, J. W.
Paton, A. G.
Penn, C. S.
Pollard, E. C.
Preston, J. E.
Williamson, W. W.
Young, D. G.

Part IV.

Thirty candidates sent in their names, of whom twenty-eight presented themselves (twenty-seven in the United Kingdom, and one in the Colonies), and thirteen passed, namely :—

Class I:

None.

Class II:

†Alison, S. H.
†Anderson, R. D.
†Hawes, E. E.
†Jones, E. S.
†Linton, M. A.
†Monkhouse, C. C.
†Perry, S. J.
†Pickup, J. R.
†Robertson, B.
†Rowland, S. J.
†Sharp, H. G.
Spiegel, E. W. R.
†Thomson, F. R. T.

Those marked (†) have now completed the Examination for the Class of Fellow.

PART I, § 3.

(COMPOUND INTEREST AND ANNUITIES.)

Eight candidates, who had already passed, or been exempted from, Part I of a Syllabus prior to 1908, entered for this section alone and presented themselves (seven in the United Kingdom, and one in the Colonies), of whom six passed, namely :—

PART I.—THIRD PAPER ONLY.

Ayscough, I.	Hill, F. W.
Crang, J. S.	Houston, C. C.
Curtis, A. T. G.	Thwaites, F. G.

By Order of the Council,

W. PALIN ELDERTON,
Chairman of Board of Examiners.

R. TODHUNTER,
A. D. BESANT,
Joint Honorary Secretaries.

10th June 1914.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Sixty-seventh Annual General Meeting of the Institute was held at Staple Inn Hall on Monday, 8 June 1914, the President (Mr. Frederick Schooling) being in the Chair.

The Report of the Council having been taken as read,

The PRESIDENT, in moving the adoption of the Report and Accounts, said it would be seen from the Report that the Institute had been going along the even tenour of its useful way. The number of members, which had been falling for some years owing to the introduction of the class of Probationers in the year 1908, was now rising again, the increase in the number of Fellows and Associates during the past year having more than compensated for the decrease in the number of Students. During the session papers had been read extending over a wide range of subjects, and he thought it might be said that the standard of interest and excellence had been well maintained. With regard to the Examinations, as far as the candidates in the United Kingdom were concerned—the results for the Colonies not having yet been published—the percentage of passes was not perhaps so high as could be wished. That fact had been remarked upon by previous occupants of the Chair. It was hoped that as time went on some means would be found of increasing the proportion.

It might be asked by some members why there was no mention of the Research Bureau in the Report. He could only say that steady progress had been made with the project, and he hoped that shortly it would be launched upon its career of usefulness. The point as to whether the investigation should be based on policies or lives had been the cause of considerable delay and discussion. The Institute, adopting the recommendation of the Joint Committee which had been appointed to deal with the whole subject, had agreed to a policy basis, and he believed their Scottish

friends were now prepared to concur. On July 24th and the following days the Napier Tercentenary Celebration would be held in Edinburgh. The Institute had given a donation of £10 towards the expenses, and it was hoped that many members would attend the celebration of the inventor of logarithms, and thus show their appreciation of the memory of one who had done so much to save the labours of the actuary. With reference to the new edition of the first part of the Text-Book, he hoped that during the early part of the coming session it would be possible for members to obtain it.

Perhaps the most important event in the life assurance world during the past year—and anything affecting the life assurance world must naturally affect and interest the Institute—was the introduction of the income tax proposals contained in the Finance Bill now before Parliament, under which it was proposed to tax the interest income of life assurance companies at 1*s*. 4*d*. in the £, whether such income was re-invested abroad or not. Such an impost was undoubtly a tax on most thrifty people, many of them being exempt from paying income tax. When it was remembered that the valuation rate of interest must be earned on the investments of a life assurance company free from any deduction if solvency is to be maintained, it would be realised that the 1*s*. 4*d*. in the £ had to come out of the surplus income earned over and above the valuation rate, or in other words, out of policyholders' bonuses. In the event of a return to the condition of affairs that existed in 1897, when the interest yield on investments was very low, the impost would be even harder to bear. Life assurance companies should be taxed on profits and not on interest income, because the interest income of life assurance companies was really part of their capital.

Mr. Geoffrey Marks seconded the motion, which was put to the meeting and carried unanimously.

ELECTION OF OFFICERS.

Messrs. E. A. Rusher and C. P. Dawson were appointed Scrutineers of the ballot for the election of officers; and the President subsequently announced that the following gentlemen had been duly elected:

President.

Ernest Woods.

Vice-Presidents.

George James Lidstone.
William Peyton Phelps, M.A.
Lewis Frederick Hovil.
Robert Ruthven Tilt.

Council.

Thomas Gans Ackland.
Henry James Baker.
Arthur Digby Besant, B.A.
Joseph Burn.
Frederick Timothy Mason Byers.
*Charles Ronald Vawdrey Coutts.
Robert Cross.
*William Palin Elderton.
Sir George Francis Hardy, K.C.B.
James Robert Hart.
Lewis Frederick Hovil.
George King.
Abraham Levine, M.A.
George James Lidstone.
Geoffrey Marks.
Vyvyan Marr.
Alfred Moorhouse.
*Harry Ethelston Nightingale.
William Peyton Phelps, M.A.
Sir Gerald Hemmington Ryan.
*Richard George Salmon.
Frederick Schooling.
*John Spencer.
William Richard Strong.
Robert Ruthven Tilt.
George Todd, M.A.
Ralph Todhunter, M.A.
Harold Moltke Trouncer, M.A.
Samuel George Warner.
Ernest Woods.

Treasurer.
GEOFFREY MARKS.

Honorary Secretaries.
RALPH TODHUNTER, M.A. | ARTHUR DIGBY BESANT, B.A.

* New Members of the Council.

MR. ERNEST WOODS said that after the announcement which had just been made it was his duty to return thanks in the names of the Vice-Presidents, the Council, the Treasurer, the Honorary Secretaries and himself. He thanked the members for the distinction they had conferred upon him by giving him, for the time being, the highest honour in the profession. It was one of the privileges of that position that the President could always count on the loyalty and kindly help and friendly counsel of his colleagues sitting on the Council. In the year to come he was sure that the Council and every officer would do their best to forward the interests of the profession, of the Institute, and of the individual members thereof.

Mr. W. M. HAYCROFT proposed the election of Mr. W. G. Titmuss, Mr. Arthur Taylor, and Mr. R. S. B. Savery as Auditors for the ensuing year.

Mr. W. MOUAT JONES seconded the motion, which was carried unanimously.

Mr. COCKBURN, in proposing "That the thanks of the Institute be given to the President, Vice-Presidents, Council, Officers, Examiners, and Honorary Supervisors at centres other than London, for their excellent services during the past year," said the report showed that many subjects had engaged the attention of all those gentlemen. Owing to the wide sphere of usefulness of the actuary, especially in connection with new legislation, the President and Council had many matters to watch and consider, and the members might feel sure that during the past year anything and everything likely to affect the interests of actuaries had had their best and closest attention. At the Annual Meeting last year they had the pleasure of hearing Mr. Teece, from Australia. He was sure that if the members in the Colonies were present that evening that would heartily join in carrying the motion, as they formed a very important section of the Institute.

Mr. ANDRAS had great pleasure in seconding the motion. He congratulated the Examiners on now receiving something which might be almost designated as fees, when in his own time the Examiners had to be content with nothing at all.

The motion was carried unanimously.

THE PRESIDENT replied on behalf of the Council, the various officers and himself. He said the members had been, and he hoped always would be, a happy family, and during his term of presidency, which had now come to a close, he had received the most loyal assistance from the Honorary Secretaries. He wished also to express his indebtedness to the Assistant Secretary (Mr. Jarvis) for the able way in which he had always performed his duties. In case there should be any misapprehension as to the terms of the vote of thanks, which said—"And Honorary Supervisors at centres other than London," he might mention that there were no Honorary Supervisors in London, so they had not been omitted.

Mr. W. PENMAN moved—"That the thanks of the Institute be given to Messrs. H. Lucey, W. G. Titmuss, and Arthur Taylor for their services as Auditors during the past year."

Mr. H. J. P. OAKLEY seconded the motion which was carried unanimously.

The PRESIDENT then adjourned the meeting to Monday, the 30th November, at 5 o'clock.

Additions to the Library.

The following works have been added to the Library since the publication of the *Journal* for October 1913:

By whom presented (when not purchased).

Accountants and Auditors, Society of Incorporated.
List of Members, &c., 1913–14. — *The Society.*

Accountants, Institute of Chartered, in England and Wales.
List of Members, 1914. — *The Institute.*

Actuarial Society of America.
Transactions, 1913–14. — *The Society.*
Containing *inter alia*—
- "The Income Tax Law of the United States as it affects Life Assurance Companies", by E. E. Rhodes.
- "A Theory of Sub-standard Lives", by A. W. Whitney.
- "Graduation by the Summation Method. Some Elementary Notes", by J. B. Maclean.
- "Life Term and Endowment Mortality Experience of the Provident Life and Trust Company of Philadelphia; with special reference to the relative vitality according to Age at Entry", by M. A. Linton.
- "The influence of medical re-examamination on insured lives. Health conservation", by C. W. Jackson.
- "Annuities with Participation", by D. P. Fackler.
- "An interpretation of some of the results of the Medico-Actuarial Investigation", by Dr. O. H. Rogers and A. Hunter.
- "On the principles governing the valuation of disability benefits by a regular Life Insurance Company", by G. Bohlman.
- "Criteria for testing the adequacy of rates for Workmen's Compensation Insurance", by A. H. Mowbray.
- "Premiums and Reserves upon Life Insurance Contracts that provide for maturity at total and permanent disability as at death", by E. B. Morris.

Actuarial Society of Sweden.
Transactions, 1914. — *The Society.*

Actuaries, Faculty of.
Transactions, 1913-14. — *The Faculty.*
Containing *inter alia*—
- "Inaugural Address", by W. Hutton.
- "Female Mortality", by A. J. C. Fyfe.
- "Valuation with allowance for Initial expense", by A. T. Maclean.

American Mathematical Society.
Transactions, 1913–14. — *The Society.*

American Statistical Association.
Transactions, 1913-14. — *The Association.*

By whom presented (when not purchased).

Assecuranz-Almanach.

Handbuch für Versicherungs-Recht und Technik. Herausgegeben von Dr. A. F. Elsner's Erben. 8vo. Berlin. 1914. — *The Publishers.*

Assecuranz Jahrbuch.

Herausgegeben von A. Ehrensweig. Vol. 35. 8vo. Wien. 1914. — *Purchased.*

Association des Actuaires Belges.

Bulletin, 1913–14. — *The Association.*

Associazione Italiana Attuari.

Bolletino, 1913–14. — *The Association.*

Australian Mutual Provident Society.

Sixty-fifth annual report, 1914. — *The Society.*

Austria-Hungary.

Bericht der Arbeiter-Unfall-Versicherungs-Anstalt für das Königreich Böhmen, 1912.
Die Privaten Versicherungsunternehmungen in den im Reichsrathe vertretenen Königreichen und Ländern, 1911. — *The Austrian Government.*

Versicherungswissenschaftliche Mitteilungen der Mathematisch-Statistische Vereinigung des Österreichisch-ungarischen Verbandes der Privat-Versicherungs-Anstalten, 1913–14. — *The Society.*

Zeitschrift für öffentliche und private Versicherung, 1913-14. — *The Editor.*

Barriol (A.).

Note sur le calcul du taux effectif d'intérêt de l'emprunt $3\frac{1}{2}$ per-cent. Paris. 1914.
Théorie et Pratique des opérations financieres. 2nd edit. Sm. 8vo. Paris. 1914. — *The Author.*

Belgium.

Bulletin du Comité central du travail industriel, 1913–14. — *Le Comité.*

Bulletin du Syndicat des Compagnies d'Assurances-vie populaires opérent en Belgique, 1913–14. — *Le Syndicat.*

Compte Rendu des Opérations et de la Situation de la Caisse Générale d'Epargne et de Retraite. 1913. — *The Belgian Government.*

"Biometrika."

Volume IX, Parts III and IV, and Volume X, Part I. — *Purchased.*

Containing *inter alia*—

"On the Expectation of Life in Ancient Rome, and in the provinces of Hispania, and Lusitania and Africa", by W. R. Macdonnell.

"On the calculation of Intra-class and Inter-class Coefficients of correlation from class moments when the number of possible combinations is large", by J. A. Harris.

"Studies in the meanings and relationships of Birth and Death Rates. Notice of a Paper by John Brownlee", by W. Palin Elderton.

"Note on Infantile Mortality and Employment of Women", by Ethel M. Elderton.

"The elimination of spurious correlation due to position in time or space", by 'Student.'

By whom presented (when not purchased).

"Biometrika"—(*continued*).

Vol. IX—(*continued*).

"Note on the essential conditions that a population breeding at random should be in a stable State", by Prof. Karl Pearson.

"On the probability that two independent distributions of Frequency are really samples of the same population, with special reference to recent work on the identity of Trypanosome Strains", by Prof. Karl Pearson.

Blaschke (Dr. E.).

Die Todesursachen bei österreichischen Versicherten nach fünf jährigen Geschäftsperioden im Zeitraume. 1876–1900. Wien. 1914. — *The Author.*

Bowley (A. L.).

A general course of pure Mathematics from Indices to Solid Analytical Geometry. 8vo. Oxford. 1913. — *Purchased.*

Braun (H.).

Die Gesetze der menschlichen Lebensdauer.

Eine graphische Darstellung der bei Sterblichkeitsmessung an Versicherten vorkommenden Gesamtheiten.

Übersicht über die Entwicklung der Lebensversicherungstechnik.

Die Sterblichkeits-und Invaliditätsverhaltnisse des Lokomotivpersonals deutscher Eisenbahnen. Berlin. 1911.

Die Abhängigkeit einiger bei der Pensionsversicherung vorkommender Wahrscheinlichkeitswerte von der Versicherungsdauer.

Die Behandlung aussereuropäischer, insbesondere von Tropen-Risiken in der deutschen Lebensversicherung. Berlin. 1912.

Die Tropensterblichkeit und ihre ursachen, mit besonderer berücksichtigung der verhältnisse in den deutschen Kolonien. 1912.

Ein Beitrag zur technik der Lebensrückversicherung. Berlin. 1914. — *The Author.*

Bunyon (C. J.).

The Law of Life Assurance. 5th edit. By J. V. Vesey FitzGerald, Arthur R. Barrand, and Cecil A. Hunt. 8vo. 1914. — *Purchased.*

Canadian Annual Financial Review.

Compiled by W. R. Houston. With Appendix. Ob. 8vo. Toronto. 1913–1914. — *Purchased.*

Chartered Insurance Institute, Journal of the

Vol. 16. 8vo. 1913. — *The Institute.*

Containing *inter alia*—

"Bonds of Caution to Executors Dative and other Bonds of the Scottish Courts", by R. Mackay.

"Insurance: the Offices, the State, and the Community", by G. W. Richmond.

"Rejected Lives", by Dr. I. Beattie.

"The applicability of Life Assurance to modern needs", by S. Jackson.

"The disability features in Life Policies", by S. H. Pipe.

By whom presented (when not purchased).

Chartered Insurance Institute, Journal of the—(*continued*).

Vol. 16—(*continued*).

"The Life Assurance Contract", by E. W. Snyder.

"The necessity of accumulated Funds in Life Assurance", by W. A. Robertson.

"The underwriting of Employers' Liability risks", by E. E. B. Eldridge.

"Workmen's Compensation from a medical point of view", by Dr. J. Young.

Convention of State Insurance Commissioners, 1871.

Report of Committee on winding-up of Insolvent Companies. — *D. P. Fackler.*

Denmark.

Beretning fra Forsikringsraadet for aaret, 1912. — *The Danish Government.*

Deutscher Verein für Versicherungs-Wissenschaft. — *The Society.*

Schriften der Zentralstelle für die gemeinsamen deutschen Sterblichkeits-Untersuchungen. Heft 1. Wirkungen der Auslese in der Versichertensterblichkeit der deutschen Lebensversicherung. Von Dr. A. Abel. 8vo. Berlin. 1914.

Veroffentlichungen, 1913–14.

Zeitschrift für die gesamte Versicherungs-Wissenschaft, 1913–14.

Dobrée (A.).

Chinese characters. Their structure and method of indexing them. 1909. — *W. Schooling.*

Drapers' Company Research Memoirs. — *Purchased.*

Studies in National Deterioration. 4to.

IX. A statistical study of oral temperatures in school children, with special reference to parental environmental and class differences. By M. H. Williams, Julia Bell and Prof. Karl Pearson.

Questions of the Day and of the Fray. 8vo.

VII. Mendelism and the problem of mental defect. 1. A criticism of recent American work. By D. Heron. 1913.

VIII. Mendelism and the problem of mental defect. 2. On the continuity of mental defect. By Prof. Karl Pearson and G. A. Jaederholm. 1914.

IX. Mendelism and the problem of mental defect. 3. On the graduated character of mental defect and on the need for standardising judgments as to the grade of social inefficiency which shall involve segregation. By Prof. Karl Pearson. 1914.

Dublin (Dr. L. I.).

Possibilities of reducing mortality at the higher age groups. New York. 1913. — *The Author.*

Dublin (Dr. L. I.) and Koff (E. W.).

An experiment in the compilation of mortality statistics. New York. 1913. — *The Authors.*

	By whom presented (when not purchased).
Economic Society (Royal).	
Journal of the, 1913–14.	*Purchased.*
Elderton (W. Palin) and Fippard (R. C.).	
The construction of Mortality and Sickness Tables. A Primer. Sm. 8vo. 1914.	*Purchased.*
Eugenics Laboratory.	*Purchased.*
Lecture Series. 8vo.	
X. On the handicapping of the First-born. By Prof. Karl Pearson. 1914.	
France.	
Cinquième Rapport sur le fonctionnement du contrôle des sociétés d'Assurances sur la vie et des Sociétés de capitalisation, 1911. 8vo. Paris. 1913.	*The French Government.*
Germany.	
Die Entwickelung des Privaten Versicherungswesens unter Reichsaufsicht in dem Jahrfünft, 1907–11. 4to. Berlin. 1913. Veroffentlichungen des Kaiserlichen Aufsichtsamts für Privatversicherung. 1913–14. Versicherungs-Statistik über die Reichsaufsicht stehenden Unternehmungen. 1912.	*K. A. für Privatversicherung.*
Giffen (Sir Robert).	
Statistics. With an introduction by H. Higgs and G. Udny Yule. 8vo. 1913.	*Purchased.*
Goring (Dr. C.).	
The English Convict. A Statistical Study. Fol. 1913.	*Purchased.*
Gotha.	
Rechenschaftsbericht der Gothaer Lebensversicherungsbank auf Gegenseitigkeit für das Jahr 1913.	*The Gotha Life Office.*
Hamer (Dr. W. H.).	
The London Life Table, based on the Mortality during the decennium, 1901–1910. Fol. 1913.	*Purchased.*
Hoag (G. G.).	
A Theory of Interest. New York. 8vo. 1914.	*Purchased.*
Hobson (Dr. E. W.).	
"Squaring the Circle." A history of the problem. 8vo. Cambridge. 1913. John Napier and the invention of Logarithms, 1614. 8vo. Cambridge. 1914.	*Purchased.*
Hoffman (Dr. F. L.).	
The menace of Cancer. New York. 1913.	*The Author.*
Holland.	
Archief voor de Verzekerings-Wetenschap. 1913–14.	*The Society.*
Mededeelingen der Vereeniging voor Levensverzekering.	*The Society.*

	By whom presented (when not purchased).
Holland—(*continued*).	
Nationale Levensverzekering-Bank. Verslag over het jaar 1913.	*The "Nationale" Company.*
Dordrecht Levensverzekering-Maatschappij. Verslag over het jaar 1913.	*The Company.*
Horsburgh (E. M.).	
Handbook of the Exhibition of Napier Relics and of Books, Instruments and Devices for facilitating Calculation. 8vo. Edinburgh. 1914.	*General Secretary, Napier Tercentenary Celebration.*
Institut des Actuaires Français.	
Bulletin, 1913-14.	*The Institute.*
Institute of Actuaries Students' Society.	
Journal of the. Vol. I. No. 3. 8vo. 1913.	*The Society.*
Other copies.	*Purchased.*
Institute of Bankers.	
Journal of the, 1913–14. List of Members, 1914.	*The Institute.*
Insurance and Actuarial Society of Glasgow. Transactions 1913.	*The Society.*
Containing *inter alia*— "Insurance: The Offices, the State, and the Community", by G. W. Richmond.	
Insurance Institute of America. Proceedings, 1913.	*The Insurance Institute.*
Insurance Institute of Birmingham. Transactions, 1912-13.	*The Insurance Institute.*
Containing *inter alia*— "A few notes on the National Insurance Act, 1911", by J. Murray Laing.	
Insurance Institute of London. Journal of the. Vol. VI. 1912-13.	*The Institute.*
Containing *inter alia*— "Some notes on the settlement of claims under personal accident policies", by J. Swanston. "Some recent legal problems affecting insurance practice", by H. M. Cohen. "Some recent developments of Industrial Assurance Companies", by W. C. Sharman.	
Insurance Institute of Manchester. Report, 1912-13.	*The Institute.*
Containing *inter alia*— "Accident claims and their settlement", by H. A. Strachan. "The why and the wherefore of Life Assurance Practice", by W. Penman.	

By whom presented (when not purchased).

Insurance Institute of Toronto.

Proceedings, 1912–13. — *The Insurance Institute.*
Containing *inter alia*—
"Workmen's Compensation", by N. W. Rowell.
"The disability features in Life Policies", by S. H. Pipe.
"The Relationship of Disease and Accident", by Dr. J. Ferguson.
"Profits from the standpoint of the Life Insurance Company and the public", by P. C. H. Papps.
"The Assignment of Life Insurance Policies", by D. Matheson.

Insurance Institute of Yorkshire.

Proceedings, 1912–13. — *The Insurance Institute.*

Insurance Register, 1914. — *C. & E. Layton.*

International Congresses of Actuaries.

Index to Transactions, 1895–1912. 8vo. 1913. — *Official.*

Bulletin of the Permanent Committee, 1913. — *The Permanent Committee.*

International Congress of Mathematicians.

The Fifth, Cambridge, 1912.

Proceedings. 2 vols. 8vo. Cambridge. 1913. — *Purchased.*

Inwood (W.).

Tables for the purchasing of Estates, Renewing of Leases, &c. 30th edit. By W. Schooling. 8vo. 1913. — *W. Schooling.*

Italy.

Bolletino di Notizie sul Credito e sulla Previdenza. 4to. Rome. 1913–14. — *The Italian Government.*

Kahlo (E.).

Vergleichende Zusammenstellung der Versicherungsbedingungen und Prämiensätze der im Deutschen Reiche arbeitenden Lebensversicherungs-Gesellschaften. Berlin. 1914. — *The Author.*

Life Offices' Association.

Memoranda as to the Meetings of the Standing Committee, 1913–14. — *The Association.*

London Mathematical Society.

Proceedings, 1913–14. — *The Society.*

Maudling (R. G.) and Watson (S.).

Report on the Ninth Valuation of the I.O.O.F. Manchester Unity Friendly Society, and upon the preparation of Schemes under the National Insurance Act, 1911, Section 72. 4to. 1913. — *The Authors.*

Maxwell (Rt. Hon. Sir H.).

Annals of the Scottish Widows' Fund Life Assurance Society during one hundred years, 1815-1914. 8vo. Edinburgh. 1914. — *The Scottish Widows' Fund.*

	By whom presented (when not purchased).
Medico-Actuarial Mortality Investigation.	
Effect of Occupation on Mortality. Vol. III. 4to. New York. 1913. Mortality among insured lives, showing defect in physical condition. Personal history of Family History. Vol. IV. Part 1. 4to. New York. 1914.	*Purchased.*
Milne (W. P.).	
Higher Algebra. 8vo. 1913.	*Purchased.*
Mitchell (W. C.).	
Business Cycles. Large 4to. Berkeley, Cal. 1913.	*The University of California.*
Moir (H.).	
Disability Benefits—Insuring Insurance. New York. 1913.	*The Author.*
Moody (John).	
How to analyse Railroad Reports. 8vo. New York. 1912.	*Purchased.*
Moody's Manual of Railroad and Corporation Securities. 1914.	*Purchased.*
Moxon (T. B.).	
English Practical Banking. 17th edit. 8vo. 1913.	*Purchased.*
Mundy (F. W.).	
The earning power of Railroads. 8vo. New York. 1913.	*Purchased.*
National Health Insurance Commission.	
Statutory Rules and Orders, Regulations, Forms, Circulars, and other papers issued by the Commissioners relative to the National Insurance Acts, 1911, 1913.	*The Insurance Commissioners.*
Nippon Life Assurance Company.	
Report, 1912–1913.	*The Company.*
Parliamentary Papers.	
Acts of Parliament. Bankruptcy and Deeds of Arrangement, 1913. Companies, 1913. Consolidated Fund, 1913. Finance, 1913. Industrial and Provident Societies (Amendment), 1913. National Insurance, 1913. Public Works Loan, 1913. Scottish Assurance Companies (Superannuation Fund) Order Confirmation, 1914.	*Purchased.*
Assurance Companies. Returns to the Board of Trade. 1913.	*Purchased.*
Censuses. 1911. England and Wales. Vols. IX, X, XI, XII. Fol. 1913-14.	*The Registrar-General.*

	By whom presented (when not purchased).
Parliamentary Papers—(*continued*).	
Censuses—(*continued*).	
Other copies.	*Purchased.*
1911. Scotland. Vol. II and III. Fol. 1913-14.	*Purchased.*
1911. India. All India. Report and Tables, including report by Mr. Thomas G. Acland, F.I.A., F.F.A., on the estimated age distribution and rates of mortality, with Life Tables for all India, and for Bengal Presidency, Bombay Presidency, Burma, Madras Presidency, and United Provinces, with discrimination of sex, and for male lives only in the Punjab. 2 vols. Fol. Calcutta. 1913.	*T. G. Ackland.*
The same.	*The Government of India.*
1911. India. Provinces. Reports and Tables. 39 vols. Fol. Calcutta. 1913.	*T. G. Ackland.*
Colonies.	
Australia, Commonwealth of.	
Official Statistics, 1913–14. Official Year Book, 1914. Report of the Royal Commission on Insurance. Part I. Life Assurance. Fol. Melbourne. 1909.	*The Commonwealth Govt.*
Canada.	
Report of the Superintendent of Insurance for the year 1912.	*The Superintendent.*
New South Wales.	
Friendly Societies. Report of the Registrar for 1912. Official Year Book, 1912–13. Statistical Register for 1912 and previous years. Vital Statistics. Report for 1912 and previous years.	*The Government of N.S.W.*
New Zealand.	
Friendly Societies. Thirty-sixth Annual Report of the Registrar, 1912. Government Railways. Superannuation Fund. Report by the actuary, Morris Fox, as at 31 March, 1912. Fol. 1914. Official Year Book, 1913. Statistics of the Dominion for the year 1912.	*The Government of N.Z.*
Victoria.	
Final Report of the Inspectors appointed to investigate the affairs of the Colonial Mutual Life Assurance Society, Ltd., together with Appendices. Fol. Melbourne. 1913.	*A. M. Laughton.*
Friendly Societies. Thirty-fifth Annual Report of the Government Statist, 1912. Twenty-fourth Annual Report of the Registrar, 1913. Statistical Register for 1912.	*The Government of Victoria.*

By whom presented (when not purchased).

Parliamentary Papers—(*continued*).

Colonies—(*continued*).

Western Australia.

Annual Report of the Deputy Government Statistician on the Population and Vital Statistics, 1913. — *The Government of W. Australia.*

Friendly Societies. Report of the Proceedings by the Registrar, 1913. — *The Government of W. Australia.*

Statistical Register for 1912 and previous years. — *The Government of W. Australia.*

Companies.

Twenty-second Annual Report by the Board of Trade. Fol. 1913. — *Purchased.*

East India (Railways).

Administration Report on the Railways of India for 1912. — *Purchased.*

Friendly Societies.

Building Societies.

Eighteenth Annual Report of the Chief Registrar for 1912. — *Purchased.*

Chief Registrar's Report for 1912. Part A. — *Purchased.*

Government Insurance and Annuities.

Account to 31 December 1913. — *Purchased.*

India.

Life Assurance Companies' Act, 1912. — *Purchased.*

Life Assurance Companies. Statements of Accounts and Abstracts of Actuarial Reports. Simla. Fol. 1914. — *G. King.*

Indian Finance and Currency.

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INDEX TO VOL. XLVIII.

END OF VOL. XLVIII.

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